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EXISTENCE THEOREMS FOR FIXED FUZZY POINTS WITH CLOSED α-CUT SETS IN COMPLETE METRIC SPACES

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Abstract. In this paper, some fuzzy fixed point theorems for fuzzy mappings are established by considering the nonempty closed α-cut sets. Some importance observations are also discussed. Our results clearly extend, generalize and improve the corresponding results in the literatures, which have given most of their attention to the class of fuzzy sets with nonempty compact or closed and bounded α-cut sets.

1. Introduction

By using a natural generalization of the concept of set, as fuzzy sets, which was introduced initially by Zadeh [13], considering mathematical programming problems which are expressed as optimizing some goal function given certain constraints can be relaxed by means of a subjective gradation. In 1981, Heilpern [5] used the concept of fuzzy set to introduced a class of fuzzy mappings, which is a generalization of the set-valued mapping, and proved a fixed point theorem for fuzzy contraction mappings in metric linear space. It is worth noting that the result announced by Heilpern [5] is a fuzzy extension of the Banach contraction principle. Subsequently, several other authors have studied existence of fixed points of fuzzy mappings, for examples, Estruch and Vidal [4] proved a fixed point theorem for fuzzy contraction mappings over a complete metric spaces which is a generalization of the given Heilpern fixed point theorem and Sedghi et. al. [10] gave an extended version of Estruch and Vidal [4] theorem (for more examples, see [1, 2, 3, 7, 8, 9, 11]).

Although many kinds of fixed point theorems for fuzzy contraction mappings in complete metric spaces have been studied extensively in recent year, we have to point out that they have given most of their attention to the class of fuzzy sets with nonempty compact α-cut sets in the metric space E, but a few of their attention to the class of fuzzy sets with nonempty bounded or closed, or even bounded closed, α-cut sets. However, it is known that all compact

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sets are bounded closed sets in a general metric space and the converse is not always true. Motivated by the above observations, to create more potential applications, in this paper we have established some fuzzy fixed point theorems for some classes of fuzzy mappings by attention to nonempty closed α-cut sets. Evidently, our results extend, generalize and improve the corresponding results in the literatures.

2. Preliminaries

We now recall some well-known concepts and results.

Let (E,d) be a metric space. For any $x \in E$ and $A \subset X, d(x,A) = \inf\{d(x,y) : y \in A\}$. We denote by $N(E)$ the class of all nonempty subsets of E, by $CL(E)$ the class of all nonempty closed subsets of E and by $K(E)$ the class of all nonempty compact subsets of E.

For all $A,B \in N(E)$, let

$$H(A,B) = \begin{cases} \max\{\rho(A,B), \rho(B,A)\}, & \text{if the maximum exists;} \\ \infty, & \text{otherwise,} \end{cases}$$

where $\rho(A,B) = \sup\limits_{x\in A} \inf\limits_{y\in B} d(x,y) = \sup\limits_{x\in A} d(x,B)$. Such a mapping H is called the generalized Hausdorff metric induced by d.

Remark 2.1. Let $A,B \in CL(E)$. Then it is easy to see that

(1) $\rho(A,B) = 0$ if and only if $A \subset B$;

(2) $H(A,B) = 0$ if and only if $A = B$.

Let $I = [0,1]$. A fuzzy set A of a metric space E is defined by its membership function $A(x)$ which is a mapping from E into I. For any $\alpha \in (0,1]$, the fuzzy point x_α of E is the fuzzy set of E given by $x_\alpha(y) = \alpha$ if $y = x$ and $x_\alpha(y) = 0$ else [12]. The α-cut of A is defined by

$$[A]_\alpha = \{x \in E : A(x) \geq \alpha\},$$

where $\alpha \in (0,1]$, and we separately specify the support $[A]_0$ of A to be the closure of the union of $[A]_\alpha$ for $0 < \alpha \leq 1$.

We denote by $\mathcal{N}(E)$, the totally of fuzzy sets A which satisfy that, for any $\alpha \in I$, the α-cut of A is a nonempty subset in E. Furthermore, for any $A,B \in \mathcal{N}(E)$, we say that A is included in B (denoted by $A \subset B$) if and only if $A(x) \leq B(x)$ for all $x \in E$. Thus we have $A \subset B$ if and only if $[A]_\alpha \subset [B]_\alpha$ for all $\alpha \in I$.

Let $\alpha \in [0,1]$. We define the following denotations:

$$\mathcal{N}_\alpha(E) = \{A : E \to I : [A]_\alpha \text{ is a nonempty subset of } E\},$$

$$\mathcal{CL}_\alpha(E) = \{A \in \mathcal{N}_\alpha(E) : [A]_\alpha \text{ is a closed subset of } E\},$$

$$\rho_\alpha(A,B) = \rho\left([A]_\alpha, [B]_\alpha\right),$$

$$D_\alpha(A,B) = H([A]_\alpha, [B]_\alpha),$$

where $[A]_\alpha, [B]_\alpha \in \mathcal{N}_\alpha(E)$.

For all $x \in E$, we will write $\rho_\alpha(x, B)$ instead of $\rho_\alpha(x_\alpha, B)$.

For any $\alpha \in (0,1]$ and $x_1 \in E$, a sequence $O_\alpha(F, x_1) := \{x_1, x_2, x_3, \ldots\} \subset E$ such that $x_n \in [F(x_{n-1})]_\alpha$ is called an orbit of $F : E \to \mathcal{N}_\alpha(E)$ with respect to x_1.

Definition 2.2. Let $x_1, \omega \in E$. A mapping $g : E \to \mathbb{R}$ is said to be F_α-orbitally lower semi-continuous at ω with respect to x_1 if $x_n \in O_\alpha(F, x_1)$ for all $n \geq 1$ and $x_n \to \omega$ imply $g(\omega) \leq \liminf_{n\to\infty} g(x_n)$.

In a more general sense than the one given by Heilpern [5], we have the following definition:

Definition 2.3 ([11])**.** Let x_α be a fuzzy point of E. We say that x_α is a fuzzy fixed point of the fuzzy mapping F over E if $x_\alpha \subset F(x)$. In particular, according to [5], if $\{x\} \subset F(x)$, we say that x is a fixed point of F.

Remark 2.4. From Definition 2.3, we see that x_α is a fuzzy fixed point of the fuzzy mapping F over E if and only if its the corresponding element $x \in E$ is belong to $[F(x)]_\alpha$.

3. Main results

Now, we are in position to prove the main results.

Theorem 3.1. *Let $\alpha \in (0,1]$. Let (E, d) be a complete metric space and $F : E \to \mathcal{CL}_\alpha(E)$ be a fuzzy mapping. Assume that the following conditions hold*:

(i) *there exist $\kappa \in (0,1)$ and $\varphi : [0,\infty) \to [0,\kappa)$ such that $\limsup_{r\to t^+} \varphi(r) < \kappa$ for any $t \in [0,\infty)$;*

(ii) *for any $x \in E$, there exists $y \in [F(x)]_\alpha$ such that*

$$\kappa d(x, y) \leq \rho_\alpha(x, F(x)) \tag{3.1}$$

and

$$\rho_\alpha(y, F(y)) \leq \varphi(d(x, y))d(x, y). \tag{3.2}$$

Then we have the following:

(1) *For each $x_1 \in E$, there exists an orbit $O_\alpha(F, x_1) := \{x_1, x_2, x_3, \ldots\}$ of F and $\omega \in E$ such that $\lim_{n\to\infty} x_n = \omega$.*

(2) *ω_α is a fuzzy fixed point of F if and only if the function $g_\alpha(x) := \rho_\alpha(x, F(x))$ is F_α-orbitally lower semi-continuous at ω with respect to x_1.*

Proof. Firstly, since, for any $x \in E$, there exists $y \in [F(x)]_\alpha$ satisfies the condition (ii), we observe that, if $y = x$, then we have $x \in [F(x)]_\alpha$. This implies, by Remark 2.4, that x_α is a fuzzy fixed point of F.

Now, let $x_1 \in E$ be arbitrary and fixed. By the conditions (i) and (ii), in view of above observation, without loss of generality, we can find $x_2 \in [F(x_1)]_\alpha,\ x_2 \neq x_1$, satisfying

$$\kappa d(x_1, x_2) \leq \rho_\alpha(x_1, F(x_1)) \tag{3.3}$$

and

$$\rho_\alpha(x_2, F(x_2)) \leq \varphi(d(x_1, x_2))d(x_1, x_2), \quad \varphi(d(x_1, x_2)) < \kappa. \tag{3.4}$$

From (3.3) and (3.4), we obtain

$$\begin{aligned} \rho_\alpha(x_1, F(x_1)) - \rho_\alpha(x_2, F(x_2)) &\geq \kappa d(x_1, x_2) - \varphi(d(x_1, x_2))d(x_1, x_2) \\ &= [\kappa - \varphi(d(x_1, x_2))]d(x_1, x_2) \\ &> 0. \end{aligned}$$

Similarly, for $x_2 \in E$, we can find $x_3 \in [F(x_2)]_\alpha,\ x_3 \neq x_2$, satisfying

$$\begin{cases} \kappa d(x_2, x_3) \leq \rho_\alpha(x_2, F(x_2)), \\ \rho_\alpha(x_3, F(x_3)) \leq \varphi(d(x_2, x_3))d(x_2, x_3), \quad \varphi(d(x_2, x_3)) < \kappa. \end{cases} \tag{3.5}$$

This implies

$$\begin{aligned} \rho_\alpha(x_2, F(x_2)) - \rho_\alpha(x_3, F(x_3)) &\geq \kappa d(x_2, x_3) - \varphi(d(x_2, x_3))d(x_2, x_3) \\ &= [\kappa - \varphi(d(x_2, x_3))]d(x_2, x_3) \\ &> 0. \end{aligned}$$

Moreover, in view of (3.4) and (3.5), we have

$$d(x_2, x_3) \leq \frac{1}{\kappa}\rho_\alpha(x_2, F(x_2)) \leq \frac{1}{\kappa}\varphi(d(x_1, x_2))d(x_1, x_2) < d(x_1, x_2).$$

By induction, we construct a sequence $\{x_n\} \subset E$ such that $x_{n+1} \in [F(x_n)]_\alpha$, $x_{n+1} \neq x_n$, satisfying

$$\begin{cases} \kappa d(x_n, x_{n+1}) \leq \rho_\alpha(x_n, F(x_n)), \\ \rho_\alpha(x_{n+1}, F(x_{n+1})) \leq \varphi(d(x_n, x_{n+1}))d(x_n, x_{n+1}), \quad \varphi(d(x_n, x_{n+1})) < \kappa \end{cases} \tag{3.6}$$

and

$$\begin{aligned} &\rho_\alpha(x_n, F(x_n)) - \rho_\alpha(x_{n+1}, F(x_{n+1})) \\ &\geq \kappa(d(x_n, x_{n+1})) - \varphi(d(x_n, x_{n+1}))d(x_n, x_{n+1}) \\ &= [\kappa - \varphi(d(x_n, x_{n+1})]d(x_n, x_{n+1}) \\ &> 0. \end{aligned} \tag{3.7}$$

Consequently, from (3.6) and (3,7), we have

$$d(x_n, x_{n+1}) \leq \frac{1}{\kappa}\rho_\alpha(x_n, F(x_n)) \leq \frac{1}{\kappa}\varphi(d(x_{n-1}, x_n))d(x_{n-1}, x_n) < d(x_{n-1}, x_n). \tag{3.8}$$

This means that $\{d(x_n, x_{n+1})\}$ is a decreasing sequence and hence it is convergent. From this together with the condition (i), we can find a real number $q \in [0, \kappa)$ such that

$$\limsup_{n\to\infty} \varphi\left(d(x_n, x_{n+1})\right) = q.$$

Therefore, for any $\delta_0 \in (q, \kappa)$, there exists a positive integer n_0 such that

$$\varphi\left(d(x_n, x_{n+1})\right) < \delta_0, \quad \forall n \geq n_0. \tag{3.9}$$

Thus, applying (3.7), we have

$$\rho_\alpha(x_n, F(x_n)) - \rho_\alpha(x_{n+1}, F(x_{n+1})) \geq \xi d(x_n, x_{n+1}), \quad \forall n \geq n_0, \tag{3.10}$$

where $\xi = \kappa - \delta_0$.

Next, using (3.6) and (3.9), we have

$$\begin{aligned}
&\rho_\alpha(x_{n+1}, F(x_{n+1}))\\
&\leq \varphi(d(x_n, x_{n+1}))d(x_n, x_{n+1})\\
&\leq \frac{\varphi(d(x_n, x_{n+1}))}{\kappa}\rho_\alpha(x_n, F(x_n))\\
&\quad \cdots\\
&\leq \frac{\varphi(d(x_n, x_{n+1}))\cdots\varphi(d(x_1, x_2))}{\kappa^n}\rho_\alpha(x_1, F(x_1))\\
&= \frac{\varphi(d(x_n, x_{n+1}))\cdots\varphi(d(x_{n_0+1}, x_{n_0+2}))}{\kappa^{n-n_0}}\\
&\quad \times \frac{\varphi(d(x_{n_0}, x_{n_0+1}))\cdots\varphi(d(x_1, x_2))}{\kappa^{n_0}}\rho_\alpha(x_1, F(x_1))\\
&< \left(\frac{\delta_0}{\kappa}\right)^{n-n_0} \frac{\varphi(d(x_{n_0}, x_{n_0+1}))\cdots\varphi(d(x_1, x_2))}{\kappa^{n_0}}\rho_\alpha(x_1, F(x_1)), \quad \forall n \geq n_0.
\end{aligned} \tag{3.11}$$

Obviously, since $\delta_0 < \kappa$, we have $\lim_{n\to\infty}(\delta_0/\kappa)^n = 0$ and so, by (3.11), we get

$$\lim_{n\to\infty} \rho_\alpha(x_n, F(x_n)) = 0. \tag{3.12}$$

Now, by (3.10), for any $m > n \geq n_0$, it follows that

$$\begin{aligned}
d(x_n, x_m) &\leq \sum_{j=n}^{m-1} d(x_j, x_{j+1})\\
&\leq \frac{1}{\xi}\sum_{j=n}^{m-1}\left(\rho_\alpha(x_j, F(x_j)) - \rho_\alpha(x_{j+1}, F(x_{j+1}))\right)\\
&= \frac{1}{\xi}\left[\rho_\alpha(x_n, F(x_n)) - \rho_\alpha(x_m, F(x_m))\right]\\
&\leq \frac{1}{\xi}\rho_\alpha(x_n, F(x_n)).
\end{aligned} \tag{3.13}$$

Using (3.12) and (3.13), we know that $\{x_n\}$ is a Cauchy sequence in E. Since (E,d) is complete, let $\omega \in E$ be such that $\lim_{n\to\infty} x_n = \omega$. Suppose that $g_\alpha(x) := \rho_\alpha(x, F(x))$ is an F_α-orbitally lower semi-continuous at ω with respect to x_1. Then we have

$$\rho_\alpha(\omega, F(\omega)) = g_\alpha(\omega) \leq \liminf_{n\to\infty} g_\alpha(x_n) = \liminf_{n\to\infty} \rho_\alpha(x_n, F(x_n)) = 0. \tag{3.14}$$

Hence, in view of Remark 2.1, we have $\{\omega\} = [\omega_\alpha]_\alpha \subset [F(\omega)]_\alpha$. Therefore, by Remark 2.4, we conclude that ω_α is a fuzzy fixed point of F.

Conversely, assume that ω_α is a fuzzy fixed point of F and $\{x_n\} =: O_\alpha(F, x_1)$ is an orbit of F such that $\lim_{n\to\infty} x_n = \omega$. Then we have

$$g_\alpha(\omega) = \rho_\alpha(\omega, F(\omega)) = 0 \leq \liminf_{n\to\infty} g_\alpha(x_n).$$

This completes the proof. □

Remark 3.2. The validity of (3.1) in the condition (ii) of Theorem 3.1 is guaranteed by the assumption that the positive number κ, which appeared in the condition (i), must be strictly less than 1. We would like to notice that the assumption on the positive number κ is essential, that is, assuming $\kappa = 1$, there may be no element $y \in [F(x)]_\alpha$ such that $\kappa d(x,y) \leq \rho_\alpha(x, F(x))$. However, if we further assume that $[F(x)]_\alpha \in K(E)$, we see that there always exists an element $y \in [F(x)]_\alpha$ such that $d(x,y) = \rho_\alpha(x, F(x))$.

In view of Remark 3.2, we give another existence theorem for the fuzzy fixed point. To do this, we need the following an important lemma, which is inspired by an idea presented in [6]:

Lemma 3.3. *Let $\alpha \in (0,1]$ and (E,d) be a metric space and $B \in \mathcal{CL}_\alpha(E)$. Then, for any $x \in E$ and $q > 1$, there exists $b \in B$ such that*

$$d(x,b) \leq q\rho_\alpha(x,B). \tag{3.15}$$

Proof. Let $\rho_\alpha(x,B) = 0$. Since $B \in \mathcal{CL}_\alpha(E)$, we have $x \in [B]_\alpha$. Taking $b = x$, we see that (3.15) holds.

Suppose that $\rho_\alpha(x,B) > 0$. Choose $\zeta = (q-1)\rho_\alpha(x,B)$. Then, by the definition of $\rho_\alpha(x,B)$, we can find $b \in [B]_\alpha$ such that

$$d(x,b) \leq \rho_\alpha(x,B) + \zeta \leq q\rho_\alpha(x,B).$$

This completes the proof. □

Using Lemma 3.3, we have the following results:

Theorem 3.4. *Let $\alpha \in (0,1]$ and (E,d) be a complete metric space and $F : E \to \mathcal{CL}_\alpha(E)$ be a fuzzy mapping satisfying*

$$\rho_\alpha(y, F(y)) \leq \varphi(d(x,y))d(x,y), \quad \forall x \in E,\ y \in [F(x)]_\alpha, \tag{3.16}$$

where $\varphi : (0,\infty) \to [0,1)$ such that

$$\limsup_{r\to t^+} \varphi(r) < 1, \quad \forall t \in [0,\infty). \tag{3.17}$$

Then we have the following:

(1) *For each* $x_1 \in E$, *there exists an orbit* $O_\alpha(F, x_1) := \{x_1, x_2, x_3, \ldots\}$ *of* F *and* $\omega \in E$ *such that* $\lim_{n\to\infty} x_n = \omega$.

(2) ω_α *is a fuzzy fixed point of* F *if and only if the function* $g_\alpha(x) := \rho_\alpha(x, F(x))$ *is* F_α*-orbitally lower semi-continuous at* ω *with respect to* x_1.

Proof. Firstly, since for any $x \in E$, we have $[F(x)]_\alpha \neq \emptyset$, there exists $y \in E$ such that $y \in [F(x)]_\alpha$. If $x = y$, then x_α is a fuzzy fixed point of F and our theorem is done. Therefore, without loss of generality, we may assume that such an element y is different from x.

Now, let $x_1 \in E$ and $x_2 \in [F(x_1)]_\alpha$ be such that $x_2 \neq x_1$. Write $d_1 = d(x_1, x_2)$. Choosing $t_1 \in \left(0, \frac{1-\sqrt{\varphi(d_1)}}{d_1}\right)$ and putting $q_1 = \frac{1}{t_1 d_1 + \sqrt{\varphi(d_1)}}$. Since $q_1 > 1$, it follows from Lemma 3.3 that there exists $x_3 \in [F(x_2)]_\alpha$ such that

$$d(x_2, x_3) \leq q_1 \rho_\alpha(x_2, F(x_2)). \tag{3.18}$$

Repeating the above argument, we obtain a sequence $\{x_n\} \subset E$, $x_n \neq x_{n+1}$, such that

$$d_n \leq q_{n-1} \rho_\alpha(x_n, F(x_n)), \tag{3.19}$$

where $x_n \in [F(x_{n-1})]_\alpha$, $d_n := d(x_n, x_{n+1})$ and $q_n = \frac{1}{t_n d_n + \sqrt{\varphi(d_n)}}$ such that

$$t_n \in \left(0, \frac{1-\sqrt{\varphi(d_n)}}{d_n}\right), \quad \forall n \geq 1.$$

Next, using (3.16) and (3.19), we have

$$\begin{aligned} d_n &\leq \frac{\varphi(d_{n-1})}{t_{n-1} d_{n-1} + \sqrt{\varphi(d_{n-1})}} d_{n-1} \\ &\leq \sqrt{\varphi(d_{n-1})} d_{n-1}. \end{aligned} \tag{3.20}$$

If there exists $n > 1$ such that $\varphi(d_{n-1}) = 0$, then it follows from (3.20) that $d_n := d(x_n, x_{n+1}) = 0$. This implies that $x_n = x_{n+1}$, which contradicts a choice of the sequence $\{x_n\}$. Hence $\varphi(d_n) \in (0, 1)$ for all $n \geq 1$. Consequently, $\{d_n\}$ is a decreasing sequence of positive real numbers and so it converges to a nonnegative real number b, say. Moreover, by taking the limit in (3.20), we have

$$b \leq b \sqrt{\limsup_{n\to\infty} \varphi(d_{n-1})}.$$

This implies, in light of (3.17), that $b = 0$. Furthermore, from (3.17), we can choose an $\varepsilon > 0$ and $k \in (0, 1)$ such that

$$\varphi(t) < k^2, \quad \forall t \in (0, \varepsilon). \tag{3.21}$$

Let N be a natural number such that $d_n < \varepsilon$ for all $n \geq N$. Then it follows from (3.20) that

$$\begin{aligned} d_n &\leq \left[\sqrt{\varphi(d_{n-1})}\cdots\sqrt{\varphi(d_0)}\right] d_0 \\ &\leq k^{n-N}\left[\sqrt{\varphi(d_{N-1})}\cdots\sqrt{\varphi(d_0)}\right] d_0 < k^{n-N} d_0, \quad \forall n \geq N. \end{aligned} \tag{3.22}$$

Therefore, for any $m \geq 1$, we have

$$\begin{aligned} d(x_n, x_{n+m}) &\leq d_n + d_{n+1} + \cdots + d_{n+m-1} \\ &< k^{n-N}\left[1 + k + k^2 + \cdots + k^{m-1}\right] d_0 \\ &< \frac{k^{n-N}}{1-k} d_0. \end{aligned} \tag{3.23}$$

Since we have $\lim_{n\to\infty} k^n = 0$ for any $k \in (0,1)$, it follows that $\{x_n\}$ is a Cauchy sequence in E. Since (E,d) is a complete metric space, there exists $\omega \in E$ such that $\lim_{n\to\infty} x_n = \omega$. Furthermore, $x_n \in [F(x_{n-1})]_\alpha$ and it follows from (3.16) that

$$\rho_\alpha(x_n, F(x_n)) \leq \varphi(d_{n-1})d_{n-1} < d_{n-1}. \tag{3.24}$$

Using (3.23) and (3.24), we obtain

$$\lim_{n\to\infty} \rho_\alpha(x_n, F(x_n)) = 0. \tag{3.25}$$

Finally, proceeding as in the proof of Theorem 3.1, one can obtain the required results. This completes the proof. □

As a consequence of Theorem 3.4, we can obtain the following result:

Corollary 3.5. *Let $\alpha \in (0,1]$, (E,d) be a complete metric space and $F : E \to \mathcal{CL}_\alpha(E)$ be a fuzzy mapping satisfying*

$$D_\alpha(F(x), F(y)) \leq \varphi(d(x,y))d(x,y), \quad \forall x \in E,\ y \in [F(x)]_\alpha,$$

where $\varphi : (0,\infty) \to [0,1)$ such that

$$\limsup_{r\to t^+} \varphi(r) < 1, \quad \forall t \in [0,\infty).$$

Then we have the following:

(1) *For each $x_1 \in E$, there exists an orbit $O_\alpha(F, x_1) := \{x_1, x_2, x_3, \ldots\}$ of F and $\omega \in E$ such that $\lim_{n\to\infty} x_n = \omega$.*

(2) *ω_α is a fuzzy fixed point of F if and only if the function $g_\alpha(x) := \rho_\alpha(x, F(x))$ is F_α-orbitally lower semi-continuous at ω with respect to x_1.*

Proof. Since $\rho_\alpha(y, F(y)) \leq D_\alpha(F(x), F(y))$ for any $y \in [F(x)]_\alpha$, by using Theorem 3.4, the required results are followed immediately. □

Remark 3.6. If the function $\varphi : (0,\infty) \to [0,1)$, appeared in the Corollary 3.5, is defined by $\varphi(t) = \kappa$ for some fixed $\kappa \in (0,1)$, then we obtain a generalization of the results in [5] and many others.

4. Conclusion

In this paper we have shown some fuzzy fixed point theorems for fuzzy mappings by considering the nonempty closed α-cut sets. Our results are clearly improve the corresponding results, which have given most of their attention to the class of fuzzy sets with nonempty compact α-cut sets in a complete metric space. Hence, obviously, these results are significant and create a more potentially fruitful area of applications.

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