

## ALMOST GP-SPACES

MOHAMMAD REZA AHMADI ZAND

ABSTRACT. A  $T_1$  topological space  $X$  is called an almost GP-space if every dense  $G_\delta$ -set of  $X$  has nonempty interior. The behaviour of almost GP-spaces under taking subspaces and superspaces, images and preimages and products is studied. If each dense  $G_\delta$ -set of an almost GP-space  $X$  has dense interior in  $X$ , then  $X$  is called a GID-space. In this paper, some interesting properties of GID-spaces are investigated. We will generalize some theorems that hold in almost P-spaces.

### 1. Introduction

Let  $X$  be a topological space and  $A \subset X$ ,  $\text{int}_X A$  denotes the interior of  $A$  in  $X$ , and the closure of  $A$  in  $X$  is denoted by  $\text{cl}_X A$ . Where no ambiguity can arise, the interior of  $A$  in  $X$  is denoted by  $A^\circ$ , and the closure of  $A$  in  $X$  is denoted by  $\bar{A}$ . Let  $I(X)$  be the set of all isolated points of  $X$ . If  $I(X) = \emptyset$ , then  $X$  is said to be crowded. Let  $X$  be crowded, if  $\bar{D} = \overline{(X \setminus D)} = X$  for some subset  $D$  of  $X$ , then  $X$  is called resolvable, otherwise  $X$  is called irresolvable.

Let  $X$  and  $Y$  be topological spaces, the set of all functions  $f$  from  $X$  to  $Y$  is denoted by  $F(X, Y)$ , and  $F(X, \mathbb{R})$  is denoted by  $F(X)$ . It is clear that  $F(X)$  with addition and multiplication defined pointwise, is a commutative ring. The set of continuous members of  $F(X)$  is denoted by  $C(X)$ . The set of points at which  $f \in F(X, Y)$  is continuous is denoted by  $C(f)$ . We denote the families of dense, dense open or dense  $G_\delta$  subspaces of a space  $X$  respectively by  $\mathcal{D}(X)$ ,  $\mathcal{DO}(X)$  or  $\mathcal{DG}(X)$ .

Recall that a topological space  $X$  is called weakly Volterra (respectively Volterra) if for each  $f, g \in F(X)$  such that  $C(f), C(g) \in \mathcal{DG}(X)$ , we have  $C(f) \cap C(g) \neq \emptyset$  (respectively  $C(f) \cap C(g)$  is dense in  $X$ ). The class of Volterra spaces was introduced in [9]. See [3], [4], [7], [8], [11] and for more information about Volterra spaces.

A completely regular space  $X$  in which every nonempty  $G_\delta$ -set has nonempty interior is called an almost P-space. Almost P-spaces was first introduced by

---

Received May 9, 2008; Revised August 14, 2008.

2000 *Mathematics Subject Classification*. Primary 54C40, 54F65; Secondary 54G99, 54C30.

*Key words and phrases*. weakly Volterra space,  $G_\delta$ -filter, almost GP-space, GID-space, almost P-space.

A. I. Veksler in [15] and it was also studied further by R. Levy in [13], also see [2], [5] and [12]. A  $T_1$  topological space  $X$  is called an almost GP-space (respectively a GID-space) if every dense  $G_\delta$ -set of  $X$  has nonempty interior (respectively dense interior). In this paper the class of almost GP-spaces and GID-spaces are studied, and basic relationships with the class of weakly Volterra spaces and Volterra spaces are investigated.

In Section 2, new algebraic and topological characterizations of Volterra spaces are given.

In the final section of our paper, we will introduce almost GP-spaces and GID-spaces, and we will give some algebraic characterizations and topological characterizations of GID-spaces.

In this paper, topological spaces are assumed to be  $T_1$ , unless otherwise noted and the reader is referred to [10], [6] and [17] for terms and notations not described here.

## 2. Volterra spaces

**Definition 2.1.** Let  $X$  be a topological space. We define  $V(X)$ ,  $T'(X)$ , and  $TG(X)$  as follows:

$$V(X) = \{f \in F(X) \mid C(f) \text{ is dense in } X\},$$

$$T'(X) = \{f \in F(X) \mid \text{there exists a dense open subset } D \text{ in } X \text{ such that } f|_D \text{ is continuous}\},$$

$$\text{and } TG(X) = \{f \in F(X) \mid \text{there exists a dense } G_\delta\text{-set } D \text{ of } X \text{ such that } f|_D \text{ is continuous}\}.$$

Some properties of  $T'(X)$  is given in [1]. In this section we show that a topological space  $X$  is Volterra if and only if  $V(X)$  is a subring of  $F(X)$ . Let  $X$  be a topological space, since for any  $f \in F(X)$ ,  $C(f)$  is a  $G_\delta$  in  $X$  we have:

$$C(X) \subset T'(X) \subset V(X) \subset TG(X) \subset F(X).$$

Let  $X$  be a nonvoid topological space. Recall that  $\mathcal{DG}(X)$  is called a  $G_\delta$ -filter if  $D, D_1 \in \mathcal{DG}(X)$ , then  $D \cap D_1 \in \mathcal{DG}(X)$ . So  $\mathcal{DG}(X)$  is a  $G_\delta$ -filter if and only if the intersection of two dense  $G_\delta$ -sets of  $X$  is dense in  $X$ .

**Theorem 2.2.** For a topological space  $X$  the following conditions are equivalent:

- 1)  $X$  is Volterra.
- 2)  $\mathcal{DG}(X)$  is a  $G_\delta$ -filter.
- 3)  $V(X)$  is a subring of  $F(X)$ .
- 4) every nonempty proper open subspace of  $X$  is Volterra.
- 5) for every nonempty open subset  $U$  of  $X$ ,  $\mathcal{DG}(U)$  is a  $G_\delta$ -filter.
- 6) for every nonempty open subset  $U$  of  $X$ ,  $V(U)$  is a subring of  $F(U)$ .

*Proof.* 1)  $\Leftrightarrow$  2). It is proved in [7] that a space  $X$  is Volterra if and only if the intersection of any two dense  $G_\delta$ -sets in  $X$  is dense, and this is equivalent with 2).

2)  $\Rightarrow$  3). Let  $f$  and  $g$  be in  $V(X)$ . Then  $C(f)$  and  $C(g)$  are dense  $G_\delta$ -sets of  $X$ . So by 2),  $D = C(f) \cap C(g)$  is a dense  $G_\delta$ -set of  $X$ . But  $D \subset C(f - g)$  and  $D \subset C(fg)$ , so  $f - g, fg \in V(X)$  and thus  $V(X)$  is a subring of  $F(X)$ .

3)  $\Rightarrow$  2). Let  $D$  and  $D'$  be in  $\mathcal{DG}(X)$ . As  $D$  and  $D'$  are dense  $G_\delta$ -sets in  $X$  we may write  $D = \bigcap_{i=1}^{\infty} A_i$ ,  $D' = \bigcap_{i=1}^{\infty} B_i$ , where each  $A_i$  and  $B_i$  are open and dense in  $X$ ,  $A_{n+1} \subset A_n, B_{n+1} \subset B_n$  for each  $n$  and  $A_1 = B_1 = X$ . Define  $f : X \rightarrow \mathbb{R}$  and  $g : X \rightarrow \mathbb{R}$  by

$$f(x) = \begin{cases} 0, & \text{if } x \in D, \\ 1/(2n), & \text{if } x \in A_n \setminus A_{n+1}, \end{cases}$$

and

$$g(x) = \begin{cases} 0, & \text{if } x \in D', \\ 1/(2n + 1), & \text{if } x \in B_n \setminus B_{n+1}. \end{cases}$$

Similar to the proof of Lemma 1 in [7]  $C(f) = D, C(g) = D'$ . By 3)  $f + g \in V(X)$  and so  $C(f + g)$  is dense in  $X$ . Clearly  $C(f + g) \supset D \cap D'$ . If  $t \in D \setminus D'$  or  $t \in D' \setminus D$ , then  $f + g$  is not continuous at  $t$ . Now let  $t \in X \setminus (D \cup D')$ , then there exist  $m, n \in \mathbb{N}$  such that  $f(t) = 1/(2n), g(t) = 1/(2m + 1)$ . Since  $f(X) \cap g(X) = \{0\}$ , without loss of generality we can assume that  $y_0 = (f + g)(t) \notin g(X)$ . Let  $(a, b)$  contains  $y_0$  such that  $(a, b) \cap g(X) = \emptyset$ . Let  $U$  be any open neighborhood of  $t$ . Since  $D$  is dense in  $X$ , there exists an  $x_0 \in D \cap U$ , so  $f(x_0) + g(x_0) = g(x_0) \notin (a, b)$ , i.e.,  $f + g$  is not continuous at  $t$ . Therefore  $D \cap D' = C(f + g)$  is dense. So we are through.

5)  $\Leftrightarrow$  6). By above 2) and 3) are equivalent, so 5) and 6) are equivalent.

1)  $\Leftrightarrow$  4). It is straightforward, see [8, Corollary 4.2, p. 175].  $\square$

### 3. Almost GP-spaces

Recall that a completely regular space  $X$  in which every nonempty  $G_\delta$ -set of  $X$  has nonempty interior is called an almost P-space. In the following definition we give a generalization of almost P-spaces as follows:

**Definition 3.1.** A topological space  $X$  is called an almost GP-space if every dense  $G_\delta$ -set of  $X$  has nonempty interior in  $X$ .

**Example 3.2.** One point compactification  $\alpha\omega$  of  $\omega$  with the discrete topology is an almost GP-space, but it is not an almost P-space.

*Remark 3.3.* If a space  $X$  has an isolated point, then  $X$  is an almost GP-space. Since there exists a crowded P-space [10] and every P-space is an almost GP-space, there are crowded almost GP-spaces. We note that every irresolvable space is an almost GP-space, and by definition, every almost GP-space is weakly Volterra.

In the following proposition we will show that a crowded separable Baire space  $X$  is not an almost GP-space. So we have examples of Baire, Volterra and weakly Volterra spaces which they are not almost GP-spaces.

**Proposition 3.4.** *If  $X$  is a  $T_1$ -crowded separable Baire space, then  $X$  is not an almost GP-space, in particular  $X$  is not an almost P-space.*

*Proof.* Let  $D$  be a countable dense subset of  $X$ . Since  $X$  is Baire,  $X \setminus D = \bigcap_{d \in D} (X \setminus \{d\})$  is a dense  $G_\delta$ -set of  $X$ , so  $\text{int}_X(X \setminus D) = \emptyset$ . Thus  $X$  is not an almost GP-space.  $\square$

The proof of the next result is similar to that of Proposition 2.1 in [13].

**Theorem 3.5.** *If  $X$  is an almost GP-space and  $D \in \mathcal{D}(X)$ , then  $D$  is an almost GP-space.*

*Proof.* Let  $W$  be a dense  $G_\delta$ -set of  $D$ . Then  $W = V \cap D$ , where  $V$  is a dense  $G_\delta$ -set of  $X$ . Since  $X$  is an almost GP-space,  $\text{int}_X V \neq \emptyset$ . So  $\emptyset \neq D \cap \text{int}_X V \subset \text{int}_D W$ . Thus  $D$  is an almost GP-space.  $\square$

By Proposition 2.1 in [13], if  $X$  is a Baire almost P-space and  $D \in \mathcal{D}(X)$ , then  $D$  is Baire. But we do not know that this is true for the class of almost GP-spaces. In the following definition we define a subclass of almost GP-spaces, for which the answer of this question is positive.

**Definition 3.6.** A topological space  $X$  is called a GID-space if every dense  $G_\delta$ -set of  $X$  has dense interior in  $X$ .

Now we will generalize Proposition 2.1 in [13] to a GID-space:

**Theorem 3.7.** *If  $X$  is a (Baire) GID-space and  $D$  is dense in  $X$ , then  $D$  is a (Baire) GID-space.*

*Proof.* Let  $W$  be a dense  $G_\delta$ -set of  $D$ . Then  $W = V \cap D$ , where  $V$  is a dense  $G_\delta$ -set of  $X$ . Since  $X$  is a GID-space,  $\text{int}_X V \in \mathcal{DG}(X)$ , and  $D \cap \text{int}_X V \subset \text{int}_D W$ . So  $\text{int}_D W \in \mathcal{DG}(D)$  and thus  $D$  is a GID-space. Now suppose that  $X$  is a Baire space. If  $U$  is a nonempty open set in  $D$  and  $W = \bigcap V_n$ ,  $V_n \in \mathcal{DG}(D)$  for every  $n \in \mathbb{N}$ , then there exists an open set  $U_1$  in  $X$  such that  $D \cap U_1 = U$ , and there exist  $W_n \in \mathcal{DG}(X)$  such that  $V_n = D \cap W_n$  for every  $n \in \mathbb{N}$ . Since  $X$  is a GID-space,  $W_1 = U_1 \cap \text{int}_X(\bigcap_{n=1}^\infty W_n)$  is nonempty. So  $\emptyset \neq D \cap W_1 \subset U \cap W$ . Therefore  $D$  is a Baire space.  $\square$

Let  $X$  be a topological space. Recall that a subspace  $A$  of  $X$  is semi-open if  $A \subset \text{cl}(\text{int}(A))$ . Let  $SO(X)$  denote the collection of semi-open subspaces of  $X$ . Clearly for any topological space  $X$ ,  $T'(X) \subset TG(X)$ . In the following theorem we show that when the converse is true and we obtain some algebraic and topological characterizations of GID-spaces.

**Theorem 3.8.** *For a topological space  $X$  the following conditions are equivalent.*

- 1)  $X$  is a GID-space.
- 2)  $TG(X) = T'(X)$ .
- 3)  $V(X) = T'(X)$ .
- 4)  $\mathcal{DG}(X) \subset SO(X)$ , i.e., every dense  $G_\delta$ -set of  $X$  is semi-open.
- 5) If  $f \in F(X)$  and the set of points at which  $f$  is continuous is dense in  $X$ , then there exists a dense open set  $D$  of  $X$  such that  $f|D \in C(D)$ .

*Proof.* 1)  $\Rightarrow$  2). Clearly  $T'(X) \subset TG(X)$ . If  $f \in TG(X)$ , then there exists a  $D \in \mathcal{DG}(X)$  such that  $f|D \in C(D)$ . So by 1)  $D^\circ$  is dense in  $X$  and  $f|D^\circ \in C(D^\circ)$ , i.e.,  $f \in T'(X)$ . Thus  $TG(X) \subset T'(X)$ . So  $TG(X) = T'(X)$ .

2)  $\Rightarrow$  3). Since for any topological space  $X$  we have  $T'(X) \subset V(X) \subset TG(X)$ , 2)  $\Rightarrow$  3) holds.

3)  $\Rightarrow$  4). Let  $D \in \mathcal{DG}(X)$ . Then there exists an  $f \in F(X)$  such that  $D = C(f)$  [7], and so  $f \in V(X)$ . But by 3)  $V(X) = T'(X)$ , so  $f \in T'(X)$  and so there exists a dense open subset  $D_1$  of  $X$  such that  $f|D_1 \in C(D_1)$ . So  $D_1 \subset D = C(f)$  and thus  $\text{int}_X D$  is dense in  $X$ , i.e.,  $D$  is semi-open in  $X$ .

4)  $\Rightarrow$  1). It is straight forward.

1)  $\Leftrightarrow$  5). Since for every  $f \in F(X)$ ,  $C(f)$  is a  $G_\delta$  in  $X$  and for every  $D \in \mathcal{DG}(X)$  there exists an  $f \in F(X)$  such that  $C(f) = D$  [7], we are done.  $\square$

If  $X$  satisfies the equivalent conditions of the above theorem, then  $\mathcal{DG}(X)$  is a  $G_\delta$ -filter in  $X$ . So we have the following proposition:

**Proposition 3.9.** *If  $X$  is a GID-space, then  $X$  is a Volterra space.*

*Proof.* It follows from Theorem 3.8 and Theorem 2.2.  $\square$

**Proposition 3.10.** (a) *If  $X$  is an almost  $P$ -space, then  $X$  is a GID-space.*

(b) *If  $X$  is an almost  $P$ -space, then  $X$  is a Volterra space.*

*Proof.* (a) If  $D$  is in  $\mathcal{DG}(X)$ , then by [13]  $D_1^\circ$  is dense in  $D_1$ . Thus  $D_1^\circ$  is dense in  $X$ . Thus  $X$  is a GID-space. Now (b) follows from Proposition 3.9.  $\square$

*Remark 3.11.* By Corollary 2.2  $X$  is Volterra if and only if  $\mathcal{DG}(X)$  is a  $G_\delta$ -filter. So by Proposition 3.9 if  $X$  is a GID-space, then  $\mathcal{DG}(X)$  is a  $G_\delta$ -filter. The converse is not true, for a simple example  $\mathcal{DG}(\mathbb{R})$  is a  $G_\delta$ -filter, but  $\mathbb{R}$  is not a GID-space.

**Proposition 3.12.** *Let  $X$  be an open dense subset of a topological space  $Y$ . Then  $X$  is an almost GP-space if and only if  $Y$  is an almost GP-space.*

*Proof.* Let  $Y$  be an almost GP-space. Then by Theorem 3.5,  $X$  is an almost GP-space. Conversely if  $X$  is an almost GP-space and if  $V$  is a dense  $G_\delta$ -set of  $Y$ , then  $V \cap X$  is a dense  $G_\delta$ -set of  $X$ , so by hypothesis,  $\emptyset \neq \text{int}_X(V \cap X) = \text{int}_Y(V \cap X) \subset \text{int}_Y V$ . Therefore  $Y$  is an almost GP-space.  $\square$

**Proposition 3.13.** *Let  $X$  be an open dense subset of  $Y$ . Then  $X$  is a GID-space if and only if  $Y$  is a GID-space.*

*Proof.* Similar to the proof of Proposition 3.12.  $\square$

Recall that if  $X$  is locally compact, then  $X$  is open in  $\beta X$ , the Stone-Čech compactification of  $X$ .

**Corollary 3.14.** *If  $X$  is locally compact, then  $X$  is an almost GP-space (respectively a GID-space) if and only if  $\beta X$  is an almost GP-space (respectively a GID-space).*

**Example 3.15.** Let  $X$  be the set of reals equipped with the co-finite topology. Then it is well-known that  $X$  is a Baire space, and so it is of the second category and Volterra. Since  $G = \mathbb{X} \setminus \mathbb{N}$  is a dense  $G_\delta$ -set with an empty interior in  $X$ , where  $\mathbb{N}$  is the set of positive integers,  $X$  is not an almost GP-space.

Let  $X$  and  $Y$  be topological spaces, recall that a function  $f : X \rightarrow Y$  is feebly open if for each  $A \subset X$  having nonempty interior we have  $\text{int}_Y f(A) \neq \emptyset$ .

The proof of the next result is modelled of the proof of Proposition 5.1 in [8] and Proposition 2.3 in [2]. Also our next result generalizes Proposition 2.3 in [2].

**Theorem 3.16.** (I) *Let  $X$  be an almost GP-space. If  $f : X \rightarrow Y$  is continuous, onto and feebly open, then  $Y$  is an almost GP-space.*

(II) *Let  $X$  be a GID-space. If  $f : X \rightarrow Y$  is continuous, onto and open, then  $Y$  is a GID-space.*

(III) *If  $\prod_{i \in I} X_i$  is an almost GP-space, then so is each  $X_i$ .*

(IV) *A finite product of almost GP-spaces is an almost GP-space. But no infinite product spaces with more than one point is an almost GP-space.*

*Proof.* (I). If  $A$  is a dense  $G_\delta$ -set of  $Y$ , then as proof of Proposition 5.1 in [8]  $f^{-1}(A)$  is a dense  $G_\delta$ -set of  $X$ . Since  $X$  is an almost GP-space,  $\text{int}_X f^{-1}(A) \neq \emptyset$ . So by hypothesis  $\emptyset \neq \text{int}_Y f(f^{-1}(A)) \subset \text{int}_Y A$ . Thus  $Y$  is an almost GP-space.

(II). Let  $A$  be a dense  $G_\delta$ -set of  $Y$  and let  $X$  be a GID-space, so  $\text{int}_X f^{-1}(A)$  is dense in  $X$ . So by hypothesis,  $f(\text{int}_X f^{-1}(A))$  is open dense in  $Y$ . Since  $f(\text{int}_X f^{-1}(A)) \subseteq \text{int}_Y A$ , we have  $\text{int}_Y A \in \mathcal{DG}(Y)$  and thus we are done.

(III) and (IV). The proofs of (III) and the first part of (IV) are clear. For the second part of (IV), let  $X = \prod_{i \in I} X_i$ , where  $I$  is infinite. Put  $H_{i_n} = \prod_{i_n \neq j \in I} X_j \times G_{i_n}$ , where  $G_{i_n} \neq X_{i_n}$  is dense open of  $X_{i_n}$ , clearly  $\bigcap_{n=1}^{\infty} H_n \in \mathcal{DG}(X)$  has empty interior.  $\square$

*Remark 3.17.* Recall that a space is Baire if and only if each non-empty open subspace is of second category. Gauld, Greenwood, and Piotrowski ([8]) have shown that a space  $X$  is Volterra if and only if each non-empty open subspace is weakly Volterra.

The following theorem emphasizes how the relationship between a GID-space and an almost GP-space parallels that between Volterra and weakly Volterra stated in the above Remark.

**Theorem 3.18.** *For a topological space  $X$  the following conditions are equivalent:*

- (1)  $X$  is a GID-space.
- (2) Every nonempty open subspace of  $X$  is a GID-space.
- (3) Every nonempty open subspace of  $X$  is an almost GP-space.

*Proof.* (1)  $\Rightarrow$  (3). Let  $X$  be a GID-space. If  $U$  is a nonempty open set of  $X$  and if  $V$  is a dense  $G_\delta$ -set of  $U$ , then  $D = V \cup (X \setminus \bar{U})$  is a dense  $G_\delta$ -set of  $X$  and by hypothesis  $\text{int}_X D$  is dense in  $X$ . So  $\emptyset \neq \text{int}_X D \cap U = \text{int}_U(D \cap U) \subset V$ , thus  $\text{int}_U V \neq \emptyset$ . So  $U$  is an almost GP-space.

(3)  $\Rightarrow$  (1). Let each nonempty open subspace of  $X$  be an almost GP-space. If  $A$  is a dense  $G_\delta$ -set of  $X$  and  $U$  is a nonempty open set of  $X$ , then  $U \cap A$  is a  $G_\delta$ -set of  $U$  which is dense in  $U$ . Since (3) holds,  $\emptyset \neq \text{int}_U(U \cap A) = \text{int}_X(U \cap A) = U \cap \text{int}_X A$ . Thus  $\text{int}_X A$  is dense. So  $X$  is a GID-space.

(2)  $\Rightarrow$  (3). It is obvious.

(3)  $\Rightarrow$  (2). Let  $U$  be a nonempty open set of  $X$ . Since every nonempty open set  $W$  of  $U$  is open in  $X$ , by hypothesis  $W$  is an almost GP-space. Thus similar to the implication (3)  $\Rightarrow$  (1)  $U$  is a GID-space.  $\square$

Our next result is similar to Proposition 4.3 in [8].

**Proposition 3.19.** *Let  $\mathcal{U}$  be a collection of open subsets of the space  $X$  whose union is dense in  $X$ . Then:*

- (1) *if there is some nonempty  $U \in \mathcal{U}$  such that  $U$  is an almost GP-space, then  $X$  is an almost GP-space.*
- (2) *if every member of  $\mathcal{U}$  is a GID-space, then  $X$  is a GID-space.*

*Proof.* (1) Let  $D \in \mathcal{DG}(X)$ . Then  $D \cap U$  is a dense  $G_\delta$ -set of  $U$ . Since  $U$  is open and  $U$  is an almost GP-space,  $\emptyset \neq \text{int}_U(D \cap U) = \text{int}_X(D) \cap U \subset \text{int}_X D$ . So  $X$  is an almost GP-space.

(2) Let  $V$  be a nonempty open set of  $X$  and  $D \in \mathcal{DG}(X)$ . So by hypothesis there exists a  $U \in \mathcal{U}$  such that  $U \cap V \neq \emptyset$ . Since  $D \cap U \in \mathcal{DG}(U)$  by (2)  $\text{int}_U(D \cap U) = U \cap \text{int}_X D$  is dense in  $U$ . So  $\emptyset \neq U \cap V \cap \text{int}_X D \subset V \cap \text{int}_X D$  and thus  $\text{int}_X D$  is dense in  $X$ . So  $X$  is a GID-space.  $\square$

**Corollary 3.20.** *The topological sum of a family of GIP-spaces (almost GP-spaces) is a GIP-space (an almost GP-space).*

**Acknowledgement.** The author thanks the referee for his careful reading and valuable comments.

## References

- [1] M. R. Ahmadi Zand, *Algebraic characterization of Blumberg spaces*, in progress.
- [2] F. Azarpanah, *On almost  $P$ -spaces* Far East J. Math. Sci. (FJMS) 2000, Special Volume, Part I, 121–132.
- [3] J. Cao and D. Gauld, *Volterra spaces revisited*, J. Aust. Math. Soc. **79** (2005), no. 1, 61–76.

- [4] J. Cao and H. J. K. Junnila, *When is a Volterra space Baire?*, Topology Appl. **154** (2007), no. 2, 527–532.
- [5] F. Dashiell, A. Hager, and M. Henriksen, *Order-Cauchy completions of rings and vector lattices of continuous functions*, Canad. J. Math. **32** (1980), no. 3, 657–685.
- [6] R. Engelking, *General Topology*, Heldermann Verlag, Berlin, 1989.
- [7] D. Gauld, S. Greenwood, and Z. Piotrowski, *On Volterra spaces. II*, Papers on general topology and applications (Gorham, ME, 1995), 169–173, Ann. New York Acad. Sci., 806, New York Acad. Sci., New York, 1996.
- [8] ———, *On Volterra spaces. III*, Topological operations. Proceedings of the 1998 Topology and Dynamics Conference (Fairfax, VA). Topology Proc. **23** (1998), Spring, 167–182.
- [9] D. Gauld and Z. Piotrowski, *On Volterra spaces*, Far East J. Math. Sci. **1** (1993), no. 2, 209–214.
- [10] L. Gillman and M. Jerison, *Rings of Continuous Functions*, Springer-Verlag, 1976.
- [11] G. Gruenhagen and D. Lutzer, *Baire and Volterra spaces*, Proc. Amer. Math. Soc. **128** (2000), no. 10, 3115–3124.
- [12] M. Henriksen, *Spaces with a pretty base*, J. Pure Appl. Algebra **70** (1991), no. 1-2, 81–87.
- [13] R. Levy, *Almost-P-spaces*, Canad. J. Math. **29** (1977), no. 2, 284–288.
- [14] J. Martinez and W. Wm. McGovern, *When the maximum ring of quotients of  $C(X)$  is uniformly complete*, Topology Appl. **116** (2001), no. 2, 185–198.
- [15] A. I. Veksler,  *$P'$ -points,  $P'$ -sets,  $P'$ -spaces. A new class of order-continuous measures and functionals*, Dokl. Akad. Nauk SSSR **212** (1973), 789–792.
- [16] R. C. Walker, *The Ston-Čech Compactification*, Springer Verlag, Mass, 1970.
- [17] S. Willard, *General Topology*, Addison Wesley, Reading, Mass., 1970.

DEPARTMENT OF MATHEMATICS  
YAZD UNIVERSITY  
YAZD, IRAN  
E-mail address: mahmadi@yazduni.ac.ir