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# ON FIXED POINT THEOREMS IN INTUITIONISTIC FUZZY METRIC SPACES

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Abstract. In this paper, we give some new fixed point theorems for contractive type mappings in intuitionistic fuzzy metric spaces. We improve and generalize the well-known fixed point theorems of Banach [4] and Edelstein [8] in intuitionistic fuzzy metric spaces. Our main results are intuitionistic fuzzy version of Fang's results [10]. Further, we obtain some applications to validate our main results to product spaces.

## 1. Introduction

In 1965, the concept of fuzzy sets was introduced initially by Zadeh [28]. Since then, many authors have expansively developed the theory of fuzzy sets and applications. Especially, Deng [7], Erceg [9], Kaleva and Seikkala [14], Kramosil and Michalek [15] have introduced the concepts of fuzzy metric spaces in different ways. Mishra et al. [18] and Singh and Tomar [25] obtained some fixed point theorems and these fixed point theorems applied to product spaces.

Alaca et al. [2] using the idea of intuitionistic fuzzy sets [3, 6], they defined the notion of intuitionistic fuzzy metric space (shortly I-FM space) as Park [20] with the help of continuous t-norms and continuous t-conorms as a generalization of fuzzy metric space due to Kramosil and Michalek [15]. Further, they introduced the notion of Cauchy sequences in an I-FM spaces and proved the well-known fixed point theorems of Banach [4] and Edelstein [8] extended to I-FM spaces with the help of Grabiec [11]. Turkoglu et al. [27] introduced the concept of compatible maps and compatible maps of types $(\alpha)$ and $(\beta)$ in I-FM spaces and gave some relations between the concepts of compatible maps and compatible maps of types $(\alpha)$ and $(\beta)$. Turkoglu et al. [26] gave generazation of Jungck's common fixed point theorem [13] to I-FM spaces. They first formulate the definition of weakly commuting and R-weakly commuting mappings in I-FM spaces and proved the intuitionistic fuzzy version of Pant's theorem

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[19]. Many authors studied the concept of I-FM space and its applications [1, 2, 12, 20, 21, 22, 23].

In the present paper, we give some new fixed point theorems for contractive type mappings in I-FM spaces. We improve and generalize the well-known fixed point theorems of Banach [4] and Edelstein [8] in I-FM spaces. Our main results are intuitionistic fuzzy version of Fang's results [10]. Finally, we obtain some applications to validate our main results on the product of an I-FM space.

## 2. Intuitionistic fuzzy metric spaces

**Definition 1** ([24])**.** A binary operation $* : [0,1] \times [0,1] \to [0,1]$ is continuous t-norm if $*$ is satisfying the following conditions:

(i) $*$ is commutative and associative;
(ii) $*$ is continuous;
(iii) $a * 1 = a$ for all $a \in [0,1]$;
(iv) $a * b \leq c * d$ whenever $a \leq c$ and $b \leq d$ for all $a, b, c, d \in [0,1]$.

**Definition 2** ([24])**.** A binary operation $\Diamond : [0,1] \times [0,1] \to [0,1]$ is continuous t-conorm if $\Diamond$ is satisfying the following conditions:

(i) $\Diamond$ is commutative and associative;
(ii) $\Diamond$ is continuous;
(iii) $a \Diamond 0 = a$ for all $a \in [0,1]$;
(iv) $a \Diamond b \leq c \Diamond d$ whenever $a \leq c$ and $b \leq d$ for all $a, b, c, d \in [0,1]$.

The concepts of triangular norms (t-norms) and triangular conorms (t-conorms) are known as the axiomatic skeletons that we use for characterizing fuzzy intersections and unions, respectively. These concepts were originally introduced by Menger [17] in his study of statistical metric spaces.

The following definition and the fundamental properties of I-FM spaces due to Kramosil and Michalek [15] was given by Alaca et al. [2].

**Definition 3** ([2])**.** A 5-tuple $(X, M, N, *, \Diamond)$ is said to be an I-FM space if $X$ is an arbitrary set, $*$ is a continuous t-norm, $\Diamond$ is a continuous t-conorm and $M$, $N$ are fuzzy sets on $X^2 \times [0, \infty)$ satisfying the following conditions:

(i) $M(x, y, t) + N(x, y, t) \leq 1$ for all $x, y \in X$ and $t > 0$;
(ii) $M(x, y, 0) = 0$ for all $x, y \in X$;
(iii) $M(x, y, t) = 1$ for all $x, y \in X$ and $t > 0$ if and only if $x = y$;
(iv) $M(x, y, t) = M(y, x, t)$ for all $x, y \in X$ and $t > 0$;
(v) $M(x, y, t) * M(y, z, s) \leq M(x, z, t + s)$ for all $x, y, z \in X$ and $s, t > 0$;
(vi) for all $x, y \in X$, $M(x, y, \cdot) : [0, \infty) \to [0,1]$ is left continuous;
(vii) $\lim_{t \to \infty} M(x, y, t) = 1$ for all $x, y \in X$ and $t > 0$;
(viii) $N(x, y, 0) = 1$ for all $x, y \in X$;
(ix) $N(x, y, t) = 0$ for all $x, y \in X$ and $t > 0$ if and only if $x = y$;
(x) $N(x, y, t) = N(y, x, t)$ for all $x, y \in X$ and $t > 0$;
(xi) $N(x, y, t) \Diamond N(y, z, s) \geq N(x, z, t + s)$ for all $x, y, z \in X$ and $s, t > 0$;

(xii) for all $x, y \in X$, $N(x, y, \cdot) : [0, \infty) \to [0, 1]$ is right continuous;

(xiii) $\lim_{t\to\infty} N(x, y, t) = 0$ for all $x, y$ in $X$.

Then $(M, N)$ is called an intuitionistic fuzzy metric on $X$. The functions $M(x, y, t)$ and $N(x, y, t)$ denote the degree of nearness and the degree of non-nearness between $x$ and $y$ with respect to $t$, respectively.

*Remark* 1. Every fuzzy metric space $(X, M, *)$ is an I-FM space of the form $(X, M, 1 - M, *, \Diamond)$ such that t-norm $*$ and t-conorm $\Diamond$ are associated ([16]), i.e., $x\Diamond y = 1 - ((1 - x) * (1 - y))$ for all $x, y \in X$.

*Remark* 2. In I-FM space $X$, $M(x, y, \cdot)$ is non-decreasing and $N(x, y, \cdot)$ is non-increasing for all $x, y \in X$.

**Definition 4** ([2])**.** Let $(X, M, N, *, \Diamond)$ be an I-FM space. Then

(i) a sequence $\{x_n\}$ in $X$ is said to be Cauchy sequence if, for all $t > 0$ and $p > 0$,

$$\lim_{n\to\infty} M(x_{n+p}, x_n, t) = 1, \quad \lim_{n\to\infty} N(x_{n+p}, x_n, t) = 0.$$

(ii) a sequence $\{x_n\}$ in $X$ is said to be convergent to a point $x \in X$ if, for all $t > 0$,

$$\lim_{n\to\infty} M(x_n, x, t) = 1, \quad \lim_{n\to\infty} N(x_n, x, t) = 0.$$

Since $*$ and $\Diamond$ are continuous, the limit is uniquely determined from (v) and (xi), respectively.

**Definition 5** ([2])**.** An I-FM space $(X, M, N, *, \Diamond)$ is said to be complete if and only if every Cauchy sequence in $X$ is convergent.

**Definition 6** ([2])**.** An I-FM space $(X, M, N, *, \Diamond)$ is said to be compact if every sequence in $X$ contains a convergent subsequence.

**Lemma 1** ([2])**.** (i) *If* $\lim_{n\to\infty} x_n = x$ *and* $\lim_{n\to\infty} y_n = y$, *then,*

(2.1)
$$M(x, y, t) \le \lim_{n\to\infty} \inf M(x_n, y_n, t) \text{ and } N(x, y, t) \ge \lim_{n\to\infty} \sup N(x_n, y_n, t)$$

*for all* $t > 0$.

(ii) *If* $\lim_{n\to\infty} x_n = x$ *and* $\lim_{n\to\infty} y_n = y$, *then,*

(2.2)
$$M(x, y, t) \ge \lim_{n\to\infty} \sup M(x_n, y_n, t) \quad \text{and} \quad N(x, y, t) \le \lim_{n\to\infty} \inf N(x_n, y_n, t)$$

*for all* $t > 0$.

Particularly, if $M(x, y, \cdot)$ is continuous at point $t$, then

$$\lim_{n\to\infty} M(x_n, y_n, t) = M(x, y, t) \text{ and } \lim_{n\to\infty} N(x_n, y_n, t) = N(x, y, t).$$

The following two theorems extend the well-known fixed point theorems of Banach [4] and Edelstein [8] to I-FM spaces in the sense of Kramosil and Michalek [15] was given by Alaca et al. [2].

**Theorem 1** ([2]). *Let $(X, M, N, *, \Diamond)$ be a complete I-FM space. Let $T : X \to X$ be a mapping satisfying*

$$M(Tx, Ty, kt) \geq M(x, y, t) \quad \text{and} \quad N(Tx, Ty, kt) \leq N(x, y, t) \tag{2.3}$$

*for all $x, y$ in $X$, $0 < k < 1$. Then $T$ has a unique fixed point.*

**Theorem 2** ([2]). *Let $(X, M, N, *, \Diamond)$ be a compact space. Let $T : X \to X$ be a mapping satisfying*

$$M(Tx, Ty, \cdot) > M(x, y, \cdot) \quad \text{and} \quad N(Tx, Ty, \cdot) < N(x, y, \cdot) \tag{2.4}$$

*for all $x \neq y$.*

$$\left( \begin{array}{c} \text{i.e., } M(Tx, Ty, \cdot) \geq M(x, y, \cdot) \text{ and } M(Tx, Ty, \cdot) \neq M(x, y, \cdot) \\ N(Tx, Ty, \cdot) \leq N(x, y, \cdot) \text{ and } N(Tx, Ty, \cdot) \neq N(x, y, \cdot) \text{ for all } x \neq y. \end{array} \right)$$

*Then $T$ has a unique fixed point.*

## 3. Main results

**Lemma 2.** *Let $*$ be a continuous t-norm and $\Diamond$ be a continuous t-conorm. Then for each $\lambda \in (0, 1)$, there is a sequence $\{\lambda_n\}$ in $(0, 1)$ such that*

$$(1 - \lambda_n) * (1 - \lambda_n) > 1 - \lambda_{n-1} \text{ and } \lambda_n \Diamond \lambda_n < \lambda_{n-1}, \quad n = 1, 2, \ldots \tag{3.1}$$

*where $\lambda_0 = \lambda$ (obviously, the sequence $\{\lambda_n\}$ satisfying condition (3.1) is decreasing).*

*Proof.* Since $*$ is continuous at point Definition 3 [(vii) and (xiii)] and $a * b \leq 1 * 1 = 1$ and $(1 - a) \Diamond (1 - a) \geq 0 \Diamond 0 = 0$ for all $a, b \in [0, 1]$, we get

$$\sup_{0<\mu<1} [(1 - \mu) * (1 - \mu)] = 1, \ \inf_{0<\mu<1} [\mu * \mu] = 0. \tag{3.2}$$

Hence, for each $\lambda \in (0, 1)$, there exists $\lambda_1 \in (0, 1)$ such that

$$(1 - \lambda_1) * (1 - \lambda_1) > 1 - \lambda \text{ and } \lambda_1 \Diamond \lambda_1 < \lambda.$$

Similarly, from (3.2) there exists $\lambda_2 \in (0, 1)$ such that

$$(1 - \lambda_2) * (1 - \lambda_2) > 1 - \lambda_1 \text{ and } \lambda_2 \Diamond \lambda_2 < \lambda_1.$$

Continuing this procedure we can obtain a sequence $\{\lambda_n\} \subset (0, 1)$ satisfying condition (3.1). This completes the proof. □

**Lemma 3** ([5]). *Let the function $\phi(t) : [0, \infty) \to [0, \infty)$ satisfying the following condition:*

($\phi_1$) *$\phi(t)$ is strictly increasing, $\phi(0) = 0$ and $\lim_{n\to\infty} \phi^n(t) = \infty$ for all $t > 0$, where $\phi^n(t)$ denotes the $n$-th iterative function of $\phi(t)$. Then $\phi(t) > t$, $\phi^n(t) > \phi^{n-1}(t)$, $\forall t > 0$, $n = 1, 2, \ldots$.*

**Lemma 4.** *Let $(X, M, N, *, \diamondsuit)$ be an I-FM space. Let $T : X \to X$ be a mapping satisfying*

$$M(Tx, Ty, t_1) > M(x, y, t_1) \text{ and } N(Tx, Ty, t_1) < N(x, y, t_1), \tag{3.3}$$

*where $t_1$ is a fixed positive number. Then there exists a continuity point $t_0$ of $M(x, y, \cdot)$ such that*

$$M(Tx, Ty, t_0) > M(x, y, t_0) \text{ and } N(Tx, Ty, t_0) < N(x, y, t_0). \tag{3.4}$$

*Proof.* Since $M(Tx, Ty, \cdot) - M(x, y, \cdot)$ and $N(Tx, Ty, \cdot) - N(x, y, \cdot)$ are left-continuous and right-continuous, respectively, at point $t_1$, by (3.3) there exists $0 < t_2 < t_1$ such that

$$M(Tx, Ty, t) > M(x, y, t) \text{ and } N(Tx, Ty, t) < N(x, y, t)$$

for all $t \in [t_2, t_1]$. Note that the set of discontinuous points of $M(x, y, \cdot)$ and $N(x, y, \cdot)$ are countable at most. Thus, there exists $t_0 \in [t_2, t_1]$ such that $M(x, y, \cdot)$ and $N(x, y, \cdot)$ are continuous at $t_0$. Thus (3.4) holds. This completes the proof. □

**Theorem 3.** *Let $(X, M, N, *, \diamondsuit)$ be a complete I-FM space. Let $T : X \to X$ be a mapping satisfying the following conditions*:

(i) *there exists $x_0 \in X$ such that*

$$\lim_{t\to\infty} M(x_0, T^i x_0, t) = 1 \text{ and } \lim_{t\to\infty} N(x_0, T^i x_0, t) = 0, \; i = 1, 2, \ldots; \tag{3.5}$$

(ii) *there exists a mapping $m : X \to \mathbb{N}$ such that for any $x, y$ in $X$,*

$$\begin{aligned} &M(T^{m(x)}x, T^{m(x)}y, t) \ge M(x, y, \phi(t)) \text{ and} \\ &N(T^{m(x)}x, T^{m(x)}y, t) \le N(x, y, \phi(t)), \end{aligned} \tag{3.6}$$

*where the function $\phi(t)$ satisfies condition $(\phi_1)$ and*

$(\phi_2)$ $\lim_{t\to\infty} [\phi(t) - t] = \infty$.

*Then $T$ has a unique fixed point $x_*$, and the quasi-iterative sequence $\{x_n : T^{m(x_{n-1})}x_{n-1}\}$ converges to $x_*$.*

*Proof.* First we prove that

$$\sup_{s>0} \inf_{x\in O_T(x_0)} M(x_0, x, s) = 1 \text{ and } \inf_{s>0} \sup_{x\in O_T(x_0)} N(x_0, x, s) = 0, \tag{3.7}$$

where $O_T(x_0) = \{x_0, Tx_0, T^2x_0, \ldots\}$ is called the orbit of $x_0$ for $T$. For any $n \in \mathbb{N}$ with $n > m(x_0)$, we can denote

$$n = km(x_0) + s, \text{ where } 0 \le s < m(x_0).$$

Note that $\phi(t) > t$ for all $t > 0$ and $\lim_{t\to\infty} [\phi(t) - t] = \infty$. By (3.5), we have

$$\lim_{t\to\infty} M(x_0, T^i x_0, \phi(t)) = 1 \text{ and } \lim_{t\to\infty} N(x_0, T^i x_0, \phi(t)) = 0 \tag{3.8}$$

for $i = 1, 2, \ldots, m(x_0)$ and

$$(3.9)\quad \lim_{t\to\infty} M(x_0, T^{m(x_0)}x_0, \phi(t)-t) = 1 \text{ and } \lim_{t\to\infty} N(x_0, T^{m(x_0)}x_0, \phi(t)-t) = 0.$$

Moreover, from Lemma 2, for any $\lambda \in (0,1)$, there exists a sequence $\{\lambda_n\}$ in $(0,1)$ such that

$$(1-\lambda_n) * (1-\lambda_n) > 1-\lambda_{n-1} \text{ and } \lambda_n \Diamond \lambda_n < \lambda_{n-1},\ \ (\lambda_0 = \lambda)\ n = 1, 2, \ldots.$$

Thus, it follows from (3.8) and (3.9) that for given $\lambda_k$ there exists $t_0 > 0$ such that

$$\min_{1\le i\le m(x_0)} M(x_0, T^i x_0, \phi(t)) > 1-\lambda_k \text{ and } \max_{1\le i\le m(x_0)} N(x_0, T^i x_0, \phi(t)) < \lambda_k,$$

and

$$M(x_0, T^{m(x_0)}x_0, \phi(t)-t) > 1-\lambda_k \text{ and } N(x_0, T^{m(x_0)}x_0, \phi(t)-t) < \lambda_k,\ \forall t > t_0.$$

Thus, from (3.6), we get

$$\begin{aligned}
& M(x_0, T^n x_0, \phi(t)) \\
= \ & M(x_0, T^{km(x_0)+s}x_0, \phi(t)) \\
\ge \ & M(x_0, T^{m(x_0)}x_0, \phi(t)-t) * M(T^{m(x_0)}x_0, T^{km(x_0)+s}x_0, t) \\
\ge \ & M(x_0, T^{m(x_0)}x_0, \phi(t)-t) * M(x_0, T^{(k-1)m(x_0)+s}x_0, \phi(t)) \ge \cdots \\
\ge \ & M(x_0, T^{m(x_0)}x_0, \phi(t)-t) * \overset{(k)}{\cdots} * M(x_0, T^{m(x_0)}x_0, \phi(t)-t) \\
& * M(x_0, T^s x_0, \phi(t)) \\
> \ & (1-\lambda_k) \overset{(k+1)}{* \cdots *} (1-\lambda_k) > (1-\lambda_{k-1}) * \overset{(k)}{\cdots} * (1-\lambda_{k-1}) \\
> \ & \cdots > (1-\lambda_1) * (1-\lambda_1) > 1-\lambda,\ \forall t > t_0,
\end{aligned}$$

and

$$\begin{aligned}
& N(x_0, T^n x_0, \phi(t)) \\
= \ & N(x_0, T^{km(x_0)+s}x_0, \phi(t)) \\
\le \ & N(x_0, T^{m(x_0)}x_0, \phi(t)-t) \Diamond N(T^{m(x_0)}x_0, T^{km(x_0)+s}x_0, t) \\
\le \ & N(x_0, T^{m(x_0)}x_0, \phi(t)-t) \Diamond N(x_0, T^{(k-1)m(x_0)+s}x_0, \phi(t)) \le \cdots \\
\le \ & N(x_0, T^{m(x_0)}x_0, \phi(t)-t) \Diamond \overset{(k)}{\cdots} \Diamond N(x_0, T^{m(x_0)}x_0, \phi(t)-t) \\
& \Diamond N(x_0, T^s x_0, \phi(t)) \\
< \ & \lambda_k \overset{(k+1)}{\Diamond \cdots \Diamond} \lambda_k < \lambda_{k-1} \Diamond \overset{(k)}{\cdots} \Diamond \lambda_{k-1} \\
< \ & \cdots < \lambda_1 \Diamond \lambda_1 < \lambda,\ \forall t > t_0.
\end{aligned}$$

Therefore

$$\inf_{x\in O_T(x_0)} M(x_0, x, \phi(t)) \ge 1-\lambda \text{ and } \sup_{x\in O_T(x_0)} N(x_0, x, \phi(t)) \le \lambda,\ \forall t > t_0.$$

Hence

$$\sup_{s>0}\inf_{x\in O_T(x_0)} M(x_0,x,s)\geq 1-\lambda \text{ and } \inf_{s>0}\sup_{x\in O_T(x_0)} N(x_0,x,s)\leq \lambda.$$

By the arbitrariness of $\lambda$, we have

$$\sup_{s>0}\inf_{x\in O_T(x_0)} M(x_0,x,s)= 1 \text{ and } \inf_{s>0}\sup_{x\in O_T(x_0)} N(x_0,x,s)= 0.$$

Next, we prove that the quasi-iterative sequence $\left\{x_n = T^{m(x_{n-1})}x_{n-1}\right\}_{n=1}^{\infty}$ is a Cauchy sequence. For convenience, put $m_i = m(x_i)$, $i = 0, 1, \dots$. Then by (3.5),

$$\begin{aligned}
M(x_n,x_{n+p},t) &= M(T^{m_{n-1}}x_{n-1}, T^{m_{n+p-1}+m_{n+p-2}+\cdots+m_{n-1}}x_{n-1},t)\\
&\geq M(x_{n-1}, T^{m_{n+p-1}+m_{n+p-2}+\cdots+m_{n}}x_{n-1},\phi(t))\\
&\geq M(x_{n-2}, T^{m_{n+p-1}+m_{n+p-2}+\cdots+m_{n}}x_{n-2},\phi^2(t))\\
&\geq \cdots \geq M(x_0, T^{m_{n+p-1}+m_{n+p-2}+\cdots+m_{n}}x_0,\phi^n(t))\\
&\geq \inf_{x\in O_T(x_0)} M(x_0,x,\phi^n(t))\\
&\geq \sup_{0<s>\phi^n(t)}\inf_{x\in O_T(x_0)} M(x_0,x,s), \quad \forall t>0,
\end{aligned}$$

and

$$\begin{aligned}
N(x_n,x_{n+p},t) &= N(T^{m_{n-1}}x_{n-1}, T^{m_{n+p-1}+m_{n+p-2}+\cdots+m_{n-1}}x_{n-1},t)\\
&\leq N(x_{n-1}, T^{m_{n+p-1}+m_{n+p-2}+\cdots+m_{n}}x_{n-1},\phi(t))\\
&\leq N(x_{n-2}, T^{m_{n+p-1}+m_{n+p-2}+\cdots+m_{n}}x_{n-2},\phi^2(t))\\
&\leq \cdots \leq N(x_0, T^{m_{n+p-1}+m_{n+p-2}+\cdots+m_{n}}x_0,\phi^n(t))\\
&\leq \sup_{x\in O_T(x_0)} N(x_0,x,\phi^n(t))\\
&\leq \inf_{0<s>\phi^n(t)}\sup_{x\in O_T(x_0)} N(x_0,x,s), \quad \forall t>0.
\end{aligned}$$

Then by condition $(\phi_1)$ and (3.7) we have

$$\lim_{n\to\infty} M(x_{n+p},x_n,t)=1, \quad \lim_{n\to\infty} N(x_{n+p},x_n,t)=0, \quad \forall t>0.$$

This means that $\{x_n\}$ is a Cauchy sequence in $X$. By the completeness of $X$, there exists $\lim\limits_{n\to\infty} x_n = x_* \in X$.

Now we prove that $x_*$ is the unique fixed point of $T^{m_*}$, where $m_* = m(x_*)$. By Definition 3 [(v) and (xi)] and (3.6), we have

$$M(x_*,T^{m_*}x_*,t)\geq M\left(x_*,T^{m_*}x_n,\frac{t}{2}\right) * M\left(T^{m_*}x_n,T^{m_*}x_*,\frac{t}{2}\right)$$

and

$$N(x_*,T^{m_*}x_*,t)\leq N\left(x_*,T^{m_*}x_n,\frac{t}{2}\right) \lozenge N\left(T^{m_*}x_n,T^{m_*}x_*,\frac{t}{2}\right).$$

Then

$$\begin{aligned} M(x_*, T^{m_*}x_*, t) &\geq M\left(x_*, T^{m_*}x_n, \frac{t}{2}\right) * M\left(x_n, x_*, \phi\left(\frac{t}{2}\right)\right), \\ N(x_*, T^{m_*}x_*, t) &\leq N\left(x_*, T^{m_*}x_n, \frac{t}{2}\right) \Diamond N\left(x_n, x_*, \phi\left(\frac{t}{2}\right)\right). \end{aligned} \tag{3.10}$$

It is easy to prove that

$$\lim_{n\to\infty} M(x_*, T^{m_*}x_n, u) = 1 \text{ and } \lim_{n\to\infty} N(x_*, T^{m_*}x_n, u) = 0,\ \forall u > 0.$$

In fact,

$$\begin{aligned} M(x_*, T^{m_*}x_n, u) &\geq M\left(x_*, x_n, \frac{1}{2}u\right) * M\left(x_n, T^{m_*}x_n, \frac{1}{2}u\right) \\ &= M\left(x_*, x_n, \frac{1}{2}u\right) * M\left(T^{m_{n-1}}x_{n-1}, T^{m_{n-1+m_*}}x_{n-1}, \frac{1}{2}u\right) \\ &\geq M\left(x_*, x_n, \frac{1}{2}u\right) * M\left(x_{n-1}, T^{m_*}x_{n-1}, \phi\left(\frac{1}{2}u\right)\right) \geq \cdots \\ &\geq M\left(x_*, x_n, \frac{1}{2}u\right) * M\left(x_{n-1}, T^{m_*}x_{n-1}, \phi^n\left(\frac{1}{2}u\right)\right) \to 1 \end{aligned}$$

and

$$\begin{aligned} N(x_*, T^{m_*}x_n, u) &\leq N\left(x_*, x_n, \frac{1}{2}u\right) \Diamond N\left(x_n, T^{m_*}x_n, \frac{1}{2}u\right) \\ &= N\left(x_*, x_n, \frac{1}{2}u\right) \Diamond N\left(T^{m_{n-1}}x_{n-1}, T^{m_{n-1+m_*}}x_{n-1}, \frac{1}{2}u\right) \\ &\leq N\left(x_*, x_n, \frac{1}{2}u\right) \Diamond N\left(x_{n-1}, T^{m_*}x_{n-1}, \phi\left(\frac{1}{2}u\right)\right) \leq \cdots \\ &\leq N\left(x_*, x_n, \frac{1}{2}u\right) \Diamond N\left(x_{n-1}, T^{m_*}x_{n-1}, \phi^n\left(\frac{1}{2}u\right)\right) \to 0 \end{aligned}$$

for $n \to \infty$. Then, letting $n \to \infty$ on the right side of (3.10), and noting the continuity of $*$ and $\Diamond$ we have

$$M(x_*, T^{m_*}x_*, t) = 1 \text{ and } N(x_*, T^{m_*}x_*, t) = 0,\ \forall t > 0.$$

This implies that $T^{m_*}x_* = x_*$, i.e., $x_*$ is a fixed point of $T^{m(x_*)}$. To show uniqueness, assume that $T^{m(x_*)}y = y$ for $y \in X$. Then

$$M(x_*, y, t) = M(T^{m(x_*)}x_*, T^{m(x_*)}y, t) \geq M(x_*, y, \phi(t))$$

and

$$N(x_*, y, t) = N(T^{m(x_*)}x_*, T^{m(x_*)}y, t) \leq N(x_*, y, \phi(t)).$$

On the other hand, as $M(x_*, y, t)$ is non-decreasing and $N(x_*, y, t)$ is non-increasing, we have

$$M(x_*, y, t) \leq M(x_*, y, \phi(t)) \text{ and } N(x_*, y, t) \geq N(x_*, y, \phi(t)).$$

Hence

$$M(x_*, y, t) = M(x_*, y, \phi(t)) = M(x_*, y, \phi^n(t)),\ \forall t > 0,$$

and

$$N(x_*, y, t) = N(x_*, y, \phi(t)) = N(x_*, y, \phi^n(t)),\ \forall t > 0.$$

By the condition $(\phi_1)$,

$$M(x_*, y, t) = 1 \text{ and } N(x_*, y, t) = 0,\ \forall t > 0.$$

Then by Definition 3 [(iii) and (ix)] we have $x_* = y$.

Finally, we prove that $x_*$ is unique fixed point of $T$, too. In fact, since $T^{m(x_*)}x_* = x_*$, it follows that $Tx_* = T\left(T^{m(x_*)}x_*\right) = T^{m_*}(Tx_*)$. Hence, $Tx_* = x_*$.

Uniqueness is obvious. This completes the proof. □

**Corollary 1.** *Let $(X, M, N, *, \Diamond)$ be a complete I-FM space. Let $T : X \to X$ be a mapping satisfying the following conditions*:

(i) *there exists $x_0 \in X$ such that*

$$\lim_{t\to\infty} M(x_0, T^i x_0, t) = 1 \text{ and } \lim_{t\to\infty} N(x_0, T^i x_0, t) = 0,\ i = 1, 2, \ldots;$$

(ii) *there exists a mapping $m : X \to \mathbb{N}$ such that for any $x, y$ in $X$,*

$$M(T^{m(x)}x, T^{m(x)}y, t) \geq M(x, y, \frac{t}{k}) \text{ and } N(T^{m(x)}x, T^{m(x)}y, t) \leq N(x, y, \frac{t}{k}),$$

*where* $0 < k < 1$.

Then the conclusion of Theorem 3 remains true.

*Proof.* Taking $\phi(t) = \frac{t}{k}$. Obviously, $\phi(t)$ satisfies the conditions $(\phi_1)$ and $(\phi_2)$. Therefore the conclusion follows from Theorem 3 directly. □

**Corollary 2.** *Let $(X, M, N, *, \Diamond)$ be a complete I-FM space. Let $T : X \to X$ be a mapping. If there exists a mapping $m : X \to \mathbb{N}$ such that for any $x, y$ in $X$,*

$$M(T^{m(x)}x, T^{m(x)}y, t) \geq M(x, y, \phi(t)) \text{ and } N(T^{m(x)}x, T^{m(x)}y, t) \leq N(x, y, \phi(t)),$$

*where the function $\phi(t)$ satisfies conditions $(\phi_1)$ and $(\phi_2)$. Then $T$ has a unique fixed point $x_*$, and the iterative sequence $\{T^n x\}$ converges to $x_*$ for every $x \in X$.*

*Proof.* From Theorem 3, we need only to show that the iterative sequence $\{T^n x\}$ converges to $x_*$. For any $n \in \mathbb{N}$ with $n > m(x_*)$,

$$n = km(x_*) + s,\quad 0 \leq s < x_*.$$

Since

$$\begin{aligned} M(x_*, T^n x, t) &= M(T^{m(x_*)}x_*, T^{km(x_*)+s}x, t) \\ &\geq M(x_*, T^{(k-1)m(x_*)+s}x, \phi(t)) \\ &\geq \cdots \geq M(x_*, T^s x, \phi^k(t)) \to 1 \end{aligned}$$

and

$$\begin{aligned} N(x_*, T^n x, t) &= N(T^{m(x_*)}x_*, T^{km(x_*)+s}x, t) \\ &\leq N(x_*, T^{(k-1)m(x_*)+s}x, \phi(t)) \\ &\leq \cdots \leq N(x_*, T^s x, \phi^k(t)) \to 0 \end{aligned}$$

for $n \to \infty$. It follows that

$$\lim_{n\to\infty} M(x_*, T^n x, t) = 1 \text{ and } \lim_{n\to\infty} N(x_*, T^n x, t) = 0, \ \forall t > 0.$$

Then we get $\lim_{n\to\infty} T^n x = x_*$. This completes the proof. □

*Remark* 3. Taking $\phi(t) = \frac{t}{k}$ $(0 < k < 1)$ and $m(x) \equiv 1$ in Corollary 2, we at once obtain Theorem 1. Hence Theorem 1 is a special case of Corollary 2.

**Theorem 4.** *Let* $(X, M, N, *, \Diamond)$ *be a complete I-FM space with* $t * t \geq t$ *and* $(1-t)\Diamond(1-t) \leq (1-t)$ *for all* $t \in [0,1]$, *and* $T : X \to X$ *be a continuous mapping satisfying*

$$\begin{aligned} &M(Tx, Ty, \cdot) > M(x, Tx, \cdot) * M(y, Ty, \cdot) * M(x, y, \cdot), \\ &N(Tx, Ty, \cdot) < N(x, Tx, \cdot)\Diamond N(y, Ty, \cdot)\Diamond N(x, y, \cdot) \end{aligned} \tag{3.11}$$

*for all* $x \neq y$. *If there exists* $x_0 \in X$ *such that* $\{T^n x_0\}_{n=0}^{\infty}$ *has an accumulation point* $x_* \in X$, *and*

$$\begin{aligned} &M(T^{n-1}x_0, T^n x_0, t) \leq M(T^n x_0, T^{n+1}x_0, t), \\ &N(T^{n-1}x_0, T^n x_0, t) \geq N(T^n x_0, T^{n+1}x_0, t), \forall t > 0, \ n = 1, 2, \ldots, \end{aligned} \tag{3.12}$$

*then* $x_*$ *is the unique fixed point of* $T$, *and* $\lim_{n\to\infty} T^n x_0 = x_*$.

*Proof.* Assume $T^n x_0 \neq T^{n+1}x_0$ for each $n \in \mathbb{N}$. (If not, there is $n_0 \in \mathbb{N}$ such that $T^{n_0}x_0 \neq T^{n_0+1}x_0$. This means that $x_* = T^{n_0}x_0$ is a fixed point of $T$, and $\lim_{n\to\infty} T^n x_0 = x_*$). Since $\{T^n x_0\}_{n=0}^{\infty}$ has an accumulation point $x_* \in X$, there exists a subsequence $\{T^{n_i}x_0\}$, $\lim_i T^{n_i}x_0 = x_*$. $\left\{M(T^n x_0, T^{n+1}x_0, t)\right\}$ is non-decreasing and bounded and $\left\{N(T^n x_0, T^{n+1}x_0, t)\right\}$ is non-increasing and bounded. Thus, we have

$$\begin{aligned} &\left\{M(T^{n_i}x_0, T^{n_i+1}x_0, t)\right\} \text{ and } \left\{N(T^{n_i}x_0, T^{n_i+1}x_0, t)\right\}, \\ &\left\{M(T^{n_i+1}x_0, T^{n_i+2}x_0, t)\right\} \text{ and } \left\{N(T^{n_i+1}x_0, T^{n_i+2}x_0, t)\right\} \end{aligned}$$

are convergent to a common limit, i.e.,

$$\begin{aligned}\lim_i M(T^{n_i}x_0, T^{n_i+1}x_0, t) &= \lim_i M(T^{n_i+1}x_0, T^{n_i+2}x_0, t),\\ \lim_i N(T^{n_i}x_0, T^{n_i+1}x_0, t) &= \lim_i N(T^{n_i+1}x_0, T^{n_i+2}x_0, t),\ \forall t > 0.\end{aligned}$$

By the continuity of $T$, we have

$$\lim_i T^{n_i+1}x_0 = \lim_i T(T^{n_i}x_0) = Tx_*.$$

Suppose $Tx_* \neq x_*$. Putting $y = Tx$ in (3.11), we have

$$M(x, Tx, \cdot) < M(Tx, T^2x, \cdot) \text{ and } N(x, Tx, \cdot) > N(Tx, T^2x, \cdot)$$

for every $x \neq Tx$.

So by Lemma 4, there exists a continuous point $t_0$ of $M(x_*, Tx_*, \cdot)$ and $N(x_*, Tx_*, \cdot)$ such that $M(Tx_*, T^2x_*, \cdot) > M(x_*, Tx_*, t_0)$ and $N(Tx_*, T^2x_*, \cdot) < N(x_*, Tx_*, t_0)$. On the other hand, from Lemma 1,

$$\begin{aligned}M(x_*, Tx_*, t_0) &= \lim_i M(T^{n_i}x_0, T(T^{n_i}x_0), t_0)\\ &= \lim_i M(T^{n_i+1}x_0, T^{n_i+2}x_0, t_0)\\ &\geq M(Tx_*, T^2x_*, t_0)\end{aligned}$$

and

$$\begin{aligned}N(x_*, Tx_*, t_0) &= \lim_i N(T^{n_i}x_0, T(T^{n_i}x_0), t_0)\\ &= \lim_i N(T^{n_i+1}x_0, T^{n_i+2}x_0, t_0)\\ &\leq N(Tx_*, T^2x_*, t_0),\end{aligned}$$

a contradiction. Therefore $T\ x_* = x_*$, i.e., $x_*$ is a fixed point of $T$. Uniqueness follows at once from (3.11).

Finally, we prove that $\lim_{n\to\infty} T^n x_0 = x_*$. Since $\lim_i T^{n_i}x_0 = x_*$ and $\lim_i T^{n_i+1}x_0 = T\ x_* = x_*$, by Lemma 1,

$$\liminf_i M(T^{n_i}x_0, T^{n_i+1}x_0, t) \geq M(x_*, x_*, t) = 1$$

and

$$\limsup_i N(T^{n_i}x_0, T^{n_i+1}x_0, t) \leq N(x_*, x_*, t) = 0,\ \forall t > 0.$$

So $\lim_i M(T^{n_i}x_0, T^{n_i+1}x_0, t) = 1$ and $\lim_i N(T^{n_i}x_0, T^{n_i+1}x_0), t) = 0$, $\forall t > 0$. For any $n \in \mathbb{N}$ with $n > n_1$, there exists $n_i$ with $n_{i+1} \geq n > n_i$. From (3.11),

$$\begin{aligned}M(T^n x_0, x_*, t) &\geq M(T^{n-1}x_0, T^n x_0, t) * 1 * M(T^{n-1}x_0, x_*, t)\\ &\geq M(T^{n-1}x_0, T^n x_0, t) * M(T^{n-2}x_0, T^{n-1}x_0, t)\\ &\quad * M(T^{n-2}x_0, x_*, t)\\ &= M(T^{n-2}x_0, T^{n-1}x_0, t) * M(T^{n-2}x_0, x_*, t)\\ &\geq \cdots \geq M(T^{n_i}x_0, T^{n_i+1}x_0, t) * M(T^{n_i}x_0, x_*, t)\end{aligned}$$

and

$$\begin{aligned} N(T^n x_0, x_*, t) &\leq N(T^{n-1}x_0, T^n x_0, t)\Diamond 0 \Diamond N(T^{n-1}x_0, x_*, t) \\ &\leq N(T^{n-1}x_0, T^n x_0, t)\Diamond N(T^{n-2}x_0, T^{n-1}x_0, t) \\ &\quad \Diamond N(T^{n-2}x_0, x_*, t) \\ &= N(T^{n-2}x_0, T^{n-1}x_0, t)\Diamond N(T^{n-2}x_0, x_*, t) \\ &\leq \cdots \leq N(T^{n_i}x_0, T^{n_i+1}x_0, t)\Diamond N(T^{n_i}x_0, x_*, t). \end{aligned}$$

Letting $n \to \infty$ $(n_i \to \infty)$, we have

$$\lim_n M(T^n x_0, x_*, t) \geq 1 \text{ and } \lim_n N(T^n x_0, x_*, t) \leq 0, \; \forall t > 0.$$

Hence we get $\lim_n T^n x_0 = x_*$. This completes the proof. □

*Remark* 4. Theorem 2 (i.e., Theorem 1 of [2]) is the immediate consequence of Theorem 4.

## 4. Applications to product spaces

In this chapter, we apply Theorem 3, Corollary 1 and Corollary 2 to obtain fixed point type theorems on the product of an I-FM space.

**Theorem 5.** *Let $X$ be a complete I-FM space and $T : X \times X \to X$ such that be a mapping satisfying the following conditions*:

(i) *there exists $(x_0, y_0) \in X \times X$ such that*

$$\lim_{t\to\infty} M((x_0, y_0), T^i(x_0, y_0), t) = 1 \text{ and}$$
$$\lim_{t\to\infty} N((x_0, y_0), T^i(x_0, y_0), t) = 0, \; i = 1, 2, \ldots;$$

(ii) *there exists a mapping $m : X \times X \to \mathbb{N}$ such that for any $(x, y), (u, v)$ in $X \times X$,*

$$\begin{aligned} M(T^{m(x,y)}(x, y), T^{m(x,y)}(u, v), t) &\geq M((x, y), (u, v), \phi(t)), \\ N(T^{m(x,y)}(x, y), T^{m(x,y)}(u, v), t) &\leq N((x, y), (u, v), \phi(t)), \end{aligned}$$

*where the function $\phi(t)$ satisfies condition $(\phi_1)$ and*

$(\phi_2)$ $\lim_{t\to\infty}[\phi(t) - t] = \infty$.

*Then there exists exactly one point $q \in X$ such that $T^{m(q,y)}(q, y) = q$ for all $y \in X$ for each $m(q, y) \in \mathbb{N}$.*

*Proof.* For a fixed $x \in X$ and $y = v$, the inequality (ii) corresponds to the condition (ii) of Theorem 3. Therefore for each $x \in X$, there exists one and only one $x(y)$ in $X$ such that $T^{m(x(y),y)}(x(y), y) = x(y)$ and $T^{m(x(y),y)}(x(v), v) = x(v)$, $m(x(y), y) \in \mathbb{N}$.

Now, for every $y, v \in X$, from (ii) we get

$$\begin{aligned} M(x(y), x(v), t) &= M(T^{m(x(y),y)}(x(y), y), T^{m(x(y),y)}(x(v), v), t) \\ &\geq M(x(y), x(v), \phi(t)), \\ N(x(y), x(v), t) &= N(T^{m(x(y),y)}(x(y), y), T^{m(x(y),y)}(x(v), v), t) \\ &\leq N(x(y), x(v), \phi(t)). \end{aligned}$$

On the other hand, as $M(x(y), x(v), t)$ is non-decreasing and $N(x(y), x(v), t)$ is non-increasing, we have

$$M(x(y), x(v), t) \leq M(x(y), x(v), \phi(t)) \text{ and}$$
$$N(x(y), x(v), t) \geq N(x(y), x(v), \phi(t)).$$

Hence

$$M(x(y), x(v), t) = M(x(y), x(v), \phi(t)) = M(x(y), x(v), \phi^n(t)), \ \forall t > 0,$$

and

$$N(x(y), x(v), t) = N(x(y), x(v), \phi(t)) = N(x(y), x(v), \phi^n(t)), \ \forall t > 0.$$

By the condition $(\phi_1)$,

$$M(x(y), x(v), t) = 1 \text{ and } N(x(y), x(v), t) = 0, \ \forall t > 0.$$

Then by Definition 3 [(iii) and (ix)] we have $x(y) = x(v)$. So, $u(\cdot)$ is some constant $q \in X$ and conclusions of the theorem are obtained. □

If $\phi(t) = \frac{t}{k}$ in Theorem 5, we obtain an application on the product of an I-FM space of Corollary 1.

**Corollary 3.** *Let $X$ be a complete I-FM space and $T : X \times X \to X$ such that be a mapping satisfying the following conditions*:

(i) *there exists $(x_0, y_0) \in X \times X$ such that*

$$\lim_{t\to\infty} M((x_0, y_0), T^i(x_0, y_0), t) = 1 \textit{ and}$$
$$\lim_{t\to\infty} N((x_0, y_0), T^i(x_0, y_0), t) = 0, \ i = 1, 2, \ldots;$$

(ii) *there exists a mapping $m : X \times X \to \mathbb{N}$ such that for any $(x, y), (u, v)$ in $X \times X$,*

$$\begin{aligned} M(T^{m(x,y)}(x, y), T^{m(x,y)}(u, v), t) &\geq M((x, y), (u, v), \frac{t}{k}), \\ N(T^{m(x,y)}(x, y), T^{m(x,y)}(u, v), t) &\leq N((x, y), (u, v), \frac{t}{k}), \end{aligned}$$

*where $0 < k < 1$. Then the conclusion of Theorem 5 remains true.*

*Proof.* Taking $\phi(t) = \frac{t}{k}$. Obviously, $\phi(t)$ satisfies the conditions $(\phi_1)$ and $(\phi_2)$. Therefore the conclusion follows from Theorem 5 directly. □

**Corollary 4.** *Let $X$ be a complete I-FM space and $T : X \times X \to X$ be a mapping. If there exists a mapping $m : X \times X \to \mathbb{N}$ such that for any $(x, y)$ in $X$,*

$$\begin{aligned} M(T^{m(x,y)}(x,y), T^{m(x,y)}(u,v), t) &\geq M((x,y),(u,v),\phi(t)), \\ N(T^{m(x,y)}(x,y), T^{m(x,y)}(u,v), t) &\leq N((x,y),(u,v),\phi(t)), \end{aligned}$$

*where the function $\phi(t)$ satisfies conditions $(\phi_1)$ and $(\phi_2)$. Then there exists exactly one point $q \in X$ such that $T^{m(q,y)}(q,y) = q$ for all $y \in X$ for each $m(q,y) \in \mathbb{N}$.*

*Proof.* It is clear from proof of Theorem 5. □

*Remark* 5. Taking $\phi(t) = \frac{t}{k}$ $(0 < k < 1)$ and $m(x,y) \equiv 1$ in Corollary 4, we obtain an application on product space of Theorem 1.

**Conclusion.** Essentially, from (2.4) it is easy to see that $T$ is continuous and (3.11) hold for any $x_0 \in X$. In addition, from the compactness of $X$, $\{T^n x_0\}$ has an accumulation point. Hence Theorem 2 follows immediately from Theorem 4. Thus we improve and generalize the well-known fixed point theorems of Banach [4] and Edelstein [8] were given by Alaca et al. [2] in intuitionistic fuzzy metric spaces. Our main results are intuitionistic fuzzy version of Fang's results [10]. These fixed point theorems are applied to obtain solutions of fixed point type equations on product spaces.

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## References

[1] C. Alaca and H. Efe, *On Intuitionistic fuzzy Banach spaces*, Int. J. Pure Appl. Math. **32** (2006), 347–364.
[2] C. Alaca, D. Turkoglu, and C. Yildiz, *Fixed points in intuitionistic fuzzy metric spaces*, Chaos, Solitons & Fractals **29** (2006), 1073–1078.
[3] K. Atanassov, *Intuitionistic fuzzy sets*, Fuzzy Sets and Systems **20** (1986), 87–96.
[4] S. Banach, *Theorie les operations Lineaires*, Manograie Mathematyezne Warsaw Poland, 1932.
[5] S. S. Chang, *Fixed point theorem of mappings on probabilistic metric spaces with applications*, Sci. Sinica (Ser. A) **26** (1983), 1144–1155.
[6] D. Çoker, *An introduction to intuitionistic fuzzy topological spaces*, Fuzzy Sets and Systems **88** (1997), 81–89.
[7] Zi-Ke Deng, *Fuzzy pseudo-metric spaces*, J. Math. Anal. Appl. **86** (1982), 74–95.
[8] M. Edelstein, *On fixed and periodic points under contractive mappings*, J. London Math. Soc. **37** (1962), 74–79.
[9] M. A. Erceg, *Metric spaces in fuzzy set theory*, J. Math. Anal. Appl. **69** (1979), 205–230.
[10] Jin-Xuan Fang, *On fixed point theorems in fuzzy metric spaces*, Fuzzy Sets and Systems **46** (1992), 107–113.
[11] M. Grabiec, *Fixed points in fuzzy metric spaces*, Fuzzy Sets and Systems **27** (1988), 385–389.

[12] V. Gregori, S. Romaguera, and P. Veeramani, *A note on intuitionistic fuzzy metric spaces*, Chaos, Solitons & Fractals **28** (2006), 902–905.
[13] G. Jungck, *Commuting maps and fixed points*, Amer. Math. Monthly **83** (1976), 261–263.
[14] O. Kaleva and S. Seikkala, *On fuzzy metric spaces*, Fuzzy Sets and Systems **12** (1984), 225–229.
[15] O. Kramosil and J. Michalek, *Fuzzy metric and statistical metric spaces*, Kybernetica **11** (1975), 326–334.
[16] R. Lowen, *Fuzzy set theory*, Kluwer Academic Pub., Dordrecht, 1996.
[17] K. Menger, *Statistical metrics*, Proc. Nat. Acad. Sci. USA **28** (1942), 535–537.
[18] S. N. Mishra, S. L. Singh, and V. Chadha, *Coincidences and fixed points in fuzzy metric spaces*, J. Fuzzy Math. **6** (1998), 491–500.
[19] R. P. Pant, *Common fixed points of noncommuting mappings*, J. Math. Anal. Appl. **188** (1994), 436–440.
[20] J. H. Park, *Intuitionistic fuzzy metric spaces*, Chaos, Solitons & Fractals **22** (2004), 1039–1046.
[21] J. S. Park, Y. C. Kwun, and J. H. Park, *A fixed point theorem in the intuitionistic fuzzy metric spaces*, Far East J. Math. Sci. **16** (2005), no. 2, 137–149.
[22] A. Razani, *Existence of fixed point for the nonexpansive mappings in intuitionistic fuzzy metric spaces*, Chaos, Solitons & Fractals **30** (2006), 367–373.
[23] R. Saadati and J. H. Park, *On the intuitionistic topological spaces*, Chaos, Solitons & Fractals **27** (2006), 331–344.
[24] B. Schweizer and A. Sklar, *Statistical metric spaces*, Pacific J. Math. **10** (1960), 314–334.
[25] S. L. Singh and A. Tomar, *Fixed point theorems in FM-spaces*, J. Fuzzy Math. **12** (2004), 845–859.
[26] D. Turkoglu, C. Alaca, Y. J. Cho, and C. Yildiz, *Common fixed point theorems in intuitionistic fuzzy metric spaces*, J. Appl. Math. & Computing **22** (2006), 411–424.
[27] D. Turkoglu, C. Alaca, and C. Yildiz, *Compatible maps and compatible maps of types* ($\alpha$) *and* ($\beta$) *in intuitionistic fuzzy metric spaces*, Demonstratio Mathematica **39** (2006), 671–684.
[28] L. A. Zadeh, *Fuzzy sets*, Inform. and Control **8** (1965), 338–353.

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