

## FIXED POINTS OF GENERALIZED KANNAN TYPE MAPPINGS IN GENERALIZED MENGER SPACES

BINAYAK S. CHOUDHURY AND KRISHNAPADA DAS

ABSTRACT. Generalized Menger space introduced by the present authors is a generalization of Menger space as well as a probabilistic generalization of generalized metric space introduced by Branciari [Publ. Math. Debrecen **57** (2000), no. 1-2, 31–37]. In this paper we prove a Kannan type fixed point theorem in generalized Menger spaces. We also support our result by an example.

### 1. Introduction

Branciari [1] introduced the concept of generalized metric space. He replaced triangular inequality by a quadrangular inequality. The definition of generalized metric space is as follows:

**Definition 1.1** ([1]). Let  $X$  be a nonempty set,  $\mathbb{R}^+$  be the set of all positive real numbers and  $d : X \times X \rightarrow \mathbb{R}^+$  be a mapping such that for all  $x, y \in X$  and for all points  $\xi, \eta \in X$ , each of them different from  $x$  and  $y$ , one has

- (i)  $d(x, y) = 0 \Leftrightarrow x = y$ ,
- (ii)  $d(x, y) = d(y, x)$  and
- (iii)  $d(x, y) \leq d(x, \xi) + d(\xi, \eta) + d(\eta, y)$ .

Also in the same work Banach contraction mapping theorem in generalized metric space was established. He also gave an example to show that there exist generalized metric spaces which are not metric spaces. In [5], [6], [14] and [16] some other fixed point results were established in generalized metric spaces.

The following is the definition of Kannan type mappings. These mappings are considered very important mapping in fixed point theory.

**Definition 1.2** ([9, 10]). Let  $(X, d)$  be a metric space and  $f$  be a mapping on  $X$ . The mapping  $f$  is called a Kannan type mapping if there exists  $0 \leq \alpha < \frac{1}{2}$  such that

$$(1.1) \quad d(fx, fy) \leq \alpha[d(x, fx) + d(y, fy)] \text{ for all } x, y \in X.$$

---

Received February 18, 2009; Revised April 28, 2009.

2000 *Mathematics Subject Classification.* 47H10, 54H25.

*Key words and phrases.* generalized Menger space, fixed point, Kannan type mapping,  $\Psi$ -function.

It is well known that every contraction and every Kannan type mapping on a complete metric space have unique fixed points. Now contraction mappings are always continuous but Kannan type mappings are not necessarily continuous. This is a big difference between the two types of mappings. Again it also may be noted that Banach contraction principle does not characterize metric completeness. In [21] it has been proved that every metric space  $X$  is complete if and only if every Kannan type mapping has a fixed point. Similarity between contractions and Kannan type mappings may be seen in [11] and [12]. It may also be noted that Kannan's fixed point result is not an extension of Banach contraction mapping principle. The above are the some of the reasons why the Kannan type and their generalizations have been considered as constituting an important class of mappings in fixed point theory.

A probabilistic generalization of the contraction mapping principle was proved by Sehgal and Bharucha-Reid in probabilistic metric spaces [18]. These spaces are probabilistic generalizations of metric spaces. Several aspects of this structure have been described by Schweizer and Sklar in their book [17]. Subsequently fixed point theory has developed to a large extent in probabilistic metric spaces. A comprehensive description of this development is given in the book of Hadzic and Pap [8]. Some other recent works on this topic are noted in [2], [3], [7], [13], [15] and [20].

**Definition 1.3 (n-th order t-norm, [19]).** A mapping  $T : \prod_{i=1}^n [0, 1] \rightarrow [0, 1]$  is called a n-th order t-norm if the following conditions are satisfied:

- (i)  $T(0, 0, \dots, 0) = 0$ ,  $T(a, 1, 1, \dots, 1) = a$  for all  $a \in [0, 1]$ ,
- (ii)  $T(a_1, a_2, a_3, \dots, a_n) = T(a_2, a_1, a_3, \dots, a_n) = T(a_2, a_3, a_1, \dots, a_n) = \dots = T(a_2, a_3, a_4, \dots, a_n, a_1)$ ,
- (iii)  $a_i \geq b_i$ ,  $i = 1, 2, 3, \dots, n$  implies  $T(a_1, a_2, a_3, \dots, a_n) \geq T(b_1, b_2, b_3, \dots, b_n)$ ,
- (iv)  $T(T(a_1, a_2, a_3, \dots, a_n), b_2, b_3, \dots, b_n) = T(a_1, T(a_2, a_3, \dots, a_n, b_2), b_3, \dots, b_n) = T(a_1, a_2, T(a_3, a_4, \dots, a_n, b_2, b_3), b_4, \dots, b_n) = \dots = T(a_1, a_2, \dots, a_{n-1}, T(a_n, b_2, b_3, \dots, b_n))$ .

When  $n = 2$ , we have binary t-norm, which is ordinarily known as t-norm.

**Definition 1.4 (Merger space, [8, 17]).** A Menger space is a triplet  $(S, F, T)$ , where  $S$  is a non-empty set,  $F$  is a function defined on  $S \times S$  to the set of distribution functions and  $T$  is a t-norm such that the following are satisfied:

- (i)  $F_{x,y}(0) = 0$  for all  $x, y \in S$ ,
- (ii)  $F_{x,y}(s) = 1$  for all  $s > 0$  if and only if  $x = y$ ,
- (iii)  $F_{x,y}(s) = F_{y,x}(s)$  for all  $x, y \in S$  and  $s > 0$ ,
- (iv)  $F_{x,y}(u+v) \geq T(F_{x,z}(u), F_{z,y}(v))$  for all  $u, v \geq 0$  and  $x, y, z \in S$ .

Menger spaces are special types of probabilistic metric spaces in which a probabilistic version of triangular inequality has been obtained with the help

of a t-norm. In [4] we introduced the concept of generalized Menger spaces by replacing the probabilistic triangular inequality by a quadrangular inequality. The definition is as follows.

**Definition 1.5 (Generalized Menger space).** Let  $S$  be a non-empty set and  $F$  is a function from  $X \times X$  to the set of all distribution functions. Then  $(S, F, T)$  is said to be a generalized Menger space if for all  $x, y \in S$  and all distinct points  $z, w \in S$  each of them different from  $x$  and  $y$ , the following conditions are satisfied.

- (i)  $F_{x,y}(0) = 0$ ,
- (ii)  $F_{x,y}(t) = 1$  for all  $t > 0$  if and only if  $x = y$ ,
- (iii)  $F_{x,y}(t) = F_{y,x}(t)$  for all  $t > 0$  and for all  $x, y \in S$ ,
- (iv)  $F_{x,y}(t) \geq T(F_{x,z}(t_1), F_{z,w}(t_2), F_{w,y}(t_3))$ , where  $t_1 + t_2 + t_3 = t$  and  $T$  is a 3-rd order t-norm (Definition 1.3).

We now show that Definition 1.1 is special case of Definition 1.5. Let  $(S, d)$  be a generalized metric space. This spaces can be treated as a generalized Menger space if we put  $F_{x,y}(t) = H(t - d(x, y))$ , where  $x, y \in S$ ,  $H$  is defined as

$$H(s) = \begin{cases} 1, & \text{if } s > 0, \\ 0, & \text{if } s \leq 0, \end{cases}$$

and the t-norm  $T$  is taken as  $T_M$  which is defined as  $T_M(\alpha, \beta, \gamma) = \min\{\alpha, \beta, \gamma\}$ , that is  $T_M$  is the 3rd order minimum t-norm.

Conditions (i) and (ii) of Definition 1.1 trivially follow from conditions (ii) and (iii) of Definition 1.5, respectively. We now show that condition (iii) of Definition 1.1 follows from conditions (iv) of Definition 1.5. Let  $x, y \in S$  and  $z, w$  be two distinct points in  $S$  different from  $x$  and  $y$ . If possible, let

$$(1.2) \quad d(x, y) \leq d(x, z) + d(z, w) + d(w, y)$$

be true and for some  $t_1, t_2, t_3 > 0$  and  $t = t_1 + t_2 + t_3$ , the equation

$$(1.3) \quad F_{x,y}(t) \geq T_M(F_{x,z}(t_1), F_{z,w}(t_2), F_{w,y}(t_3))$$

be false.

We assumed that  $F_{x,y}(t) = H(t - d(x, y))$ . Inequality (1.3) will be false only if  $F_{x,y}(t) = 0$  and  $F_{x,z}(t_1) = 1, F_{z,w}(t_2) = 1, F_{w,y}(t_3) = 1$ .

Now  $F_{x,y}(t) = 0$  implies that  $t - d(x, y) \leq 0$ , that is,

$$(1.4) \quad t \leq d(x, y),$$

and  $F_{x,z}(t_1) = 1$  implies that  $t_1 - d(x, z) > 0$ , that is,

$$(1.5) \quad t_1 > d(x, z).$$

Similarly, we must have

$$(1.6) \quad t_2 > d(z, w)$$

and

$$(1.7) \quad t_3 > d(w, y).$$

Therefore, from (1.4)-(1.7) we have

$$(1.8) \quad d(x, y) \geq t = t_1 + t_2 + t_3 > d(x, z) + d(z, w) + d(w, y),$$

which contradicts inequality (1.2).

Thus we can say that in this case condition (iii) of Definition 1.1 follows from conditions (iv) of Definition 1.5. Hence generalized metric space (Definition 1.1) is a special case of generalized Menger space.

**Definition 1.6.** Let  $(S, F, T)$  be a generalized Menger space. A sequence  $\{x_n\} \subset S$  is said to converge to some point  $x \in S$  if given  $\epsilon > 0$ ,  $\lambda > 0$  we can find a positive integer  $N_{\epsilon, \lambda}$  such that for all  $n > N_{\epsilon, \lambda}$

$$F_{x_n, x}(\epsilon) > 1 - \lambda.$$

**Definition 1.7.** A sequence  $\{x_n\}$  is said to be a Cauchy sequence in  $S$  if given  $\epsilon > 0, \lambda > 0$  there exists a positive integer  $N_{\epsilon, \lambda}$  such that

$$F_{x_n, x_m}(\epsilon) > 1 - \lambda \text{ for all } m, n > N_{\epsilon, \lambda}.$$

**Definition 1.8.** A generalized Menger space  $(S, F, T)$  is said to be complete if every Cauchy sequence in it is convergent.

The purpose of this paper is to prove a fixed point result for a class of generalized Kannan type mappings in generalized Menger spaces. We also support our result by constructing an example.

We will use the following function in our theorem.

**Definition 1.9 ( $\Psi$ -function).** A function  $\psi : [0, 1] \times [0, 1] \rightarrow [0, 1]$  is said to be a  $\Psi$ -function if

- (i)  $\psi$  is monotone increasing and continuous,
- (ii)  $\psi(x, x) > x$  for all  $0 < x < 1$ ,
- (iii)  $\psi(1, 1) = 1, \psi(0, 0) = 0$ .

An example of  $\Psi$ -function:

$$\psi(x, y) = \frac{p\sqrt{x} + q\sqrt{y}}{p + q}, \quad p \text{ and } q \text{ are positive numbers.}$$

## 2. Main result

**Theorem 2.1.** Let  $(S, F, T_M)$  be a complete generalized Menger space, where  $T_M$  is the 3rd order minimum  $t$ -norm given by  $T_M(\alpha, \beta, \gamma) = \min\{\alpha, \beta, \gamma\}$  and the mapping  $f : S \rightarrow S$  be a self mapping which satisfies the following inequality for all  $x, y \in S$

$$(2.1) \quad F_{fx, fy}(t) \geq \psi \left( F_{x, fx} \left( \frac{t_1}{a} \right), F_{y, fy} \left( \frac{t_2}{b} \right) \right),$$

where  $t_1, t_2, t > 0$  with  $t = t_1 + t_2$ ,  $a, b > 0$  with  $0 < a + b < 1$  and  $\psi$  is a  $\Psi$ -function. Then  $f$  has a unique fixed point.

*Proof.* Let  $x_0 \in S$ . We now construct a sequence  $\{x_n\}$  as follows:  $x_n = fx_{n-1}, n \in \mathbb{N}$ , where  $\mathbb{N}$  is the set of all positive integers. Now we have for  $t, t_1, t_2 > 0$ , with  $t = t_1 + t_2$ ,

$$\begin{aligned}
 & F_{x_{n+1},x_n}(t) = F_{fx_n,fx_{n-1}}(t) \\
 & \geq \psi \left( F_{x_n,fx_n} \left( \frac{t_1}{a} \right), F_{x_{n-1},fx_{n-1}} \left( \frac{t_2}{b} \right) \right) \\
 (2.2) \quad & = \psi \left( F_{x_n,x_{n+1}} \left( \frac{t_1}{a} \right), F_{x_{n-1},x_n} \left( \frac{t_2}{b} \right) \right) \\
 & = \psi \left( F_{x_{n+1},x_n} \left( \frac{t_1}{a} \right), F_{x_n,x_{n-1}} \left( \frac{t_2}{b} \right) \right).
 \end{aligned}$$

Let  $t_1 = \frac{at}{a+b}, t_2 = \frac{bt}{a+b}$  and  $c = a + b$ . Then we have from (2.2),

$$(2.3) \quad F_{x_{n+1},x_n}(t) \geq \psi \left( F_{x_{n+1},x_n} \left( \frac{t}{c} \right), F_{x_n,x_{n-1}} \left( \frac{t}{c} \right) \right).$$

We now claim that for all  $t > 0$ ,

$$(2.4) \quad F_{x_{n+1},x_n} \left( \frac{t}{c} \right) \geq F_{x_n,x_{n-1}} \left( \frac{t}{c} \right).$$

If possible, let for some  $t > 0, F_{x_{n+1},x_n} \left( \frac{t}{c} \right) < F_{x_n,x_{n-1}} \left( \frac{t}{c} \right)$ , then we have

$$\begin{aligned}
 (2.5) \quad & F_{x_{n+1},x_n}(t) \geq \psi \left( F_{x_{n+1},x_n} \left( \frac{t}{c} \right), F_{x_{n+1},x_n} \left( \frac{t}{c} \right) \right) \\
 & > F_{x_{n+1},x_n} \left( \frac{t}{c} \right) \\
 & \geq F_{x_{n+1},x_n}(t), \text{ since } 0 < c < 1
 \end{aligned}$$

which is a contradiction.

Therefore for all  $t > 0, F_{x_{n+1},x_n} \left( \frac{t}{c} \right) \geq F_{x_n,x_{n-1}} \left( \frac{t}{c} \right)$ .

Hence using (2.4) we have from (2.3),

$$\begin{aligned}
 F_{x_{n+1},x_n}(t) & \geq \psi \left( F_{x_{n+1},x_n} \left( \frac{t}{c} \right), F_{x_n,x_{n-1}} \left( \frac{t}{c} \right) \right) \\
 & \geq \psi \left( F_{x_n,x_{n-1}} \left( \frac{t}{c} \right), F_{x_n,x_{n-1}} \left( \frac{t}{c} \right) \right) \\
 & \geq F_{x_n,x_{n-1}} \left( \frac{t}{c} \right) \\
 & \geq F_{x_{n-1},x_{n-2}} \left( \frac{t}{c^2} \right) \\
 & \quad \dots \quad \dots \quad \dots \\
 & \geq F_{x_1,x_0} \left( \frac{t}{c^n} \right)
 \end{aligned}$$

that is

$$(2.6) \quad F_{x_{n+1}, x_n}(t) \geq F_{x_1, x_0} \left( \frac{t}{c^n} \right).$$

Therefore,

$$(2.7) \quad \lim_{n \rightarrow \infty} F_{x_{n+1}, x_n}(t) = 1 \quad \text{for all } t > 0.$$

Next we show that sequence  $\{x_n\}$  is a Cauchy sequence. If possible, let  $\{x_n\}$  be not a Cauchy sequence. Then there exist  $\varepsilon > 0$  and  $\lambda > 0$  for which we can find subsequences  $\{x_{m(k)}\}$  and  $\{x_{n(k)}\}$  of  $\{x_n\}$  with  $n(k) > m(k) > k$  for all positive integer  $k$  such that

$$(2.8) \quad F_{x_{m(k)}, x_{n(k)}}(\varepsilon) \leq 1 - \lambda.$$

$$\begin{aligned} \text{Now } 1 - \lambda &\geq F_{x_{m(k)}, x_{n(k)}}(\varepsilon) = F_{f x_{m(k)-1}, f x_{n(k)-1}}(\varepsilon) \\ &\geq \psi \left( F_{x_{m(k)-1}, f x_{m(k)-1}} \left( \frac{\varepsilon_1}{a} \right), F_{x_{n(k)-1}, f x_{n(k)-1}} \left( \frac{\varepsilon_2}{b} \right) \right) \text{ where } \varepsilon = \varepsilon_1 + \varepsilon_2 \\ &\geq \psi \left( F_{x_{m(k)-1}, x_{m(k)}} \left( \frac{\varepsilon_1}{a} \right), F_{x_{n(k)-1}, x_{n(k)}} \left( \frac{\varepsilon_2}{b} \right) \right). \end{aligned}$$

Therefore,

$$(2.9) \quad 1 - \lambda \geq \psi \left( F_{x_{m(k)-1}, x_{m(k)}} \left( \frac{\varepsilon_1}{a} \right), F_{x_{n(k)-1}, x_{n(k)}} \left( \frac{\varepsilon_2}{b} \right) \right).$$

By (2.7) we can choose  $k$  large enough such that

$$F_{x_{m(k)-1}, x_{m(k)}} \left( \frac{\varepsilon_1}{a} \right) > 1 - \lambda \quad \text{and} \quad F_{x_{n(k)-1}, x_{n(k)}} \left( \frac{\varepsilon_2}{b} \right) > 1 - \lambda.$$

Therefore,

$$(2.10) \quad 1 - \lambda \geq \psi(1 - \lambda, 1 - \lambda) > 1 - \lambda,$$

which is a contradiction.

Hence  $\{x_n\}$  is a Cauchy sequence.

As  $(S, F, T_M)$  is a complete generalized Menger space we have  $\{x_n\}$  is convergent in  $S$ .

Let

$$(2.11) \quad \lim_{n \rightarrow \infty} x_n = z.$$

We now show that  $fz = z$ . If possible, let  $0 < F_{z, fz}(t) < 1$  for some  $t > 0$ . Since  $0 < b < 1$ , we can choose  $\eta_1, \eta_2, t_1, t_2 > 0$  such that

$$(2.12) \quad t = \eta_1 + \eta_2 + t_1 + t_2 \quad \text{and} \quad \frac{t_2}{b} > t.$$

Then we have,

$$\begin{aligned}
 (2.13) \quad F_{z,fz}(t) &\geq T_M\{F_{z,x_n}(\eta_1), F_{x_n,x_{n+1}}(\eta_2), F_{x_{n+1},fz}(t_1 + t_2)\} \\
 &\geq T_M\left\{F_{z,x_n}(\eta_1), F_{x_n,x_{n+1}}(\eta_2), \psi\left(F_{x_n,x_{n+1}}\left(\frac{t_1}{a}\right), F_{z,fz}\left(\frac{t_2}{b}\right)\right)\right\} \\
 &\geq T_M\left\{F_{z,x_n}(\eta_1), F_{x_n,x_{n+1}}(\eta_2), \psi\left(F_{x_n,x_{n+1}}\left(\frac{t_1}{a}\right), F_{z,fz}(t)\right)\right\}.
 \end{aligned}$$

By (2.7) and (2.11), there exists a positive integer  $N_1$  such that

$$F_{z,x_n}(\eta_1), F_{x_n,x_{n+1}}(\eta_2), F_{x_n,x_{n+1}}\left(\frac{t_1}{a}\right) > F_{z,fz}(t) \text{ for all } n > N_1.$$

Then we have from (2.13)  $F_{z,fz}(t) > F_{z,fz}(t)$ , which is a contradiction. Hence  $F_{z,fz}(t) = 1$  for all  $t > 0$ , that is  $z = fz$ .

For uniqueness, let  $z$  and  $u$  be two fixed points.

Therefore, for all  $t > 0$ ,

$$\begin{aligned}
 F_{z,u}(t) &= F_{fz,fu}(t) \\
 &\geq \psi\left(F_{z,fz}\left(\frac{t_1}{a}\right), F_{u,fu}\left(\frac{t_2}{b}\right)\right) \text{ for } t_1, t_2 > 0 \text{ and } t_1 + t_2 = t \\
 &= \psi\left(F_{z,z}\left(\frac{t_1}{a}\right), F_{u,u}\left(\frac{t_2}{b}\right)\right) \\
 &= \psi(1, 1) = 1.
 \end{aligned}$$

Therefore  $z = u$ . □

**Example 2.2.** Let  $S = \{x_1, x_2, x_3, x_4\}$ ,  $T_M(\alpha, \beta, \gamma) = \min\{\alpha, \beta, \gamma\}$  that is  $T_M$  is the 3rd order minimum t-norm and  $F_{x,y}(t)$  be defined as

$$F_{x_1, x_2}(t) = F_{x_2, x_1}(t) = \begin{cases} 0, & \text{if } t \leq 0, \\ 0.70, & \text{if } 0 < t < 6, \\ 1, & \text{if } t \geq 6. \end{cases}$$

$$F_{x_1, x_3}(t) = F_{x_3, x_1}(t) = \begin{cases} 0, & \text{if } t \leq 0, \\ 0.90, & \text{if } 0 < t \leq 3, \\ 1, & \text{if } t > 3. \end{cases}$$

$$F_{x_1, x_4}(t) = F_{x_4, x_1}(t) = \begin{cases} 0, & \text{if } t \leq 0, \\ 0.80, & \text{if } 0 < t \leq 4, \\ 1, & \text{if } t > 4. \end{cases}$$

$$F_{x_2, x_3}(t) = F_{x_3, x_2}(t) = \begin{cases} 0, & \text{if } t \leq 0, \\ 0.95, & \text{if } 0 < t \leq 3, \\ 1, & \text{if } t > 3. \end{cases}$$

$$F_{x_2, x_4}(t) = F_{x_4, x_2}(t) = \begin{cases} 0, & \text{if } t \leq 0, \\ 0.80, & \text{if } 0 < t \leq 4, \\ 1, & \text{if } t > 4. \end{cases}$$

$$F_{x_3, x_4}(t) = F_{x_4, x_3}(t) = \begin{cases} 0, & \text{if } t \leq 0, \\ 0.70, & \text{if } 0 < t < 6, \\ 1, & \text{if } t \geq 6. \end{cases}$$

Then  $(S, F, T_M)$  is a complete generalized Menger space.

Let  $f : S \rightarrow S$  be given by  $fx_1 = fx_2 = fx_3 = x_3$  and  $fx_4 = x_1$ . If we take  $\psi(x, y) = \frac{\sqrt{x} + \sqrt{y}}{2}$  and  $a = 0.2$ ,  $b = 0.75$ , then  $f$  satisfies all the conditions of Theorem 2.1 and  $x_3$  is the unique fixed point of  $f$ .

In this example  $(S, F, T_M)$  is not a Menger space as can be seen from the fact that

$$F_{x_3, x_4}(5) \not\geq T_M\{F_{x_3, x_2}(1), F_{x_2, x_4}(4)\}.$$

This shows that generalized Menger spaces are effective generalization of generalized metric spaces.

### References

- [1] A. Branciari, *A fixed point theorem of Banach-Caccioppoli type on a class of generalized metric spaces*, Publ. Math. Debrecen **57** (2000), no. 1-2, 31–37.
- [2] B. S. Choudhury, *A unique common fixed point theorem for a sequence of self-mappings in Menger Spaces*, Bull. Korean Math. Soc. **37** (2000), no. 3, 569–573.
- [3] B. S. Choudhury and K. P. Das, *A new contraction principle in Menger spaces*, Acta Mathematica Sinica, English Series **24** (2008), 1379–1386.
- [4] ———, *Banach contraction mapping principle in generalized Menger spaces*, (Communicated).
- [5] P. Das, *A fixed point theorem on a class of generalized metric space*, Korean J. Math. Soc. **9** (2002), no. 1, 29–33.
- [6] P. Das and L. K. Dey, *A fixed point theorem in a generalized metric space*, Soochow J. Math. **33** (2007), no. 1, 33–39.
- [7] J. Fang and Y. Gao, *Common fixed point theorems under strict contractive conditions in Menger spaces*, Nonlinear Anal. **70** (2009), no. 1, 184–193.
- [8] O. Hadzic and E. Pap, *Fixed Point Theory in Probabilistic Metric Spaces*, Kluwer Academic Publishers, 2001.
- [9] R. Kannan, *Some results on fixed point*, Bull. Cal. Math. Soc. **60** (1968), 71–76.
- [10] ———, *Some results on fixed point II*, Amer. Math. Monthly **76** (1969), 405–408.
- [11] M. Kikkawa and T. Suzuki, *Some similarity between contractions and Kannan mappings* Fixed Point Theory and Applications **2008** (2008), Article ID 649749.
- [12] ———, *Some similarity between contractions and Kannan mappings II*, Bull. Kyushu Inst. Tech. Pure Appl. Math. (2008), no. 55, 1–13.
- [13] I. Kubiacyk and S. Sharma, *Some common fixed point theorems in Menger space under strict contractive conditions*, Southeast Asian Bull. Math. **32** (2008), 117–124.
- [14] B. K. Lahiri and P. Das, *Fixed point of a Ljubomir Ćirić's quasi-contraction mapping in a generalized metric space*, Publ. Math. Debrecen **61** (2002), no. 3-4, 589–594.
- [15] A. Razani and K. Fouladgar, *Extension of contractive maps in the Menger probabilistic metric space*, Chaos, Solitons and Fractals **34** (2007), 1724–1731.



- [16] P. K. Saha and R. Tiwari, *An alternative proof of Kannan's fixed point theorem in generalized metric space*, News Bull. Cal. Math. Soc. **31** (2008), 15–18.
- [17] B. Schweizer and A. Sklar, *Probabilistic Metric Space*, North-Holland, Amsterdam, 1983.
- [18] V. M. Sehgal and A. T. Bharucha-Reid, *Fixed points of contraction mappings on PM space*, Math. Sys. Theory **6** (1972), no. 2, 97–100.
- [19] Y. Shi, L. Ren, and X. Wang, *The extension of fixed point theorems for set valued mapping*, J. Appl. Math. & Computing **13** (2003), no. 1-2, 277–286.
- [20] B. Sing and S. Jain, *A fixed point theorem in Menger spaces through weak compatibility*, J. Math. Anal. Appl. **301** (2005), no. 2, 439–448.
- [21] P. V. Subrahmanyam, *Completeness and fixed points*, Monatsh. Math. **80** (1975), 325–330.

BINAYAK S. CHOUDHURY  
DEPARTMENT OF MATHEMATICS  
BENGAL ENGINEERING AND SCIENCE UNIVERSITY  
SHIBPUR, P.O. : B. GARDEN, SHIBPUR  
HOWRAH - 711103, WEST BENGAL, INDIA  
*E-mail address:* binayak12@yahoo.co.in

KRISHNAPADA DAS  
DEPARTMENT OF MATHEMATICS  
BENGAL ENGINEERING AND SCIENCE UNIVERSITY  
SHIBPUR, P.O. : B. GARDEN, SHIBPUR  
HOWRAH - 711103, WEST BENGAL, INDIA  
*E-mail address:* kestapm@yahoo.co.in