지연귀환을 통한 비선형 섭동이 존재하는 불확실 시간지연 시스템의 성능보장 제어

Guaranteed Cost Control for Uncertain Time-Delay Systems with nonlinear Perturbations via Delayed Feedback

권 오 민*, 박 주 현
(Oh-Min Kwon and Ju-Hyun Park)

Abstract : In this paper, we propose a delayed feedback guaranteed cost controller design method for linear time-delay systems with norm-bounded parameter uncertainties and nonlinear perturbations. A quadratic cost function is considered as the performance measure for the given system. Based on the Lyapunov method, an LMI optimization problem is formulated to design a controller such that the closed-loop cost function value is not more than a specified upper bound for all admissible system uncertainties and nonlinear perturbations. Numerical example show the effectiveness of the proposed method.

Keywords : delay, guaranteed cost control, LMI, Lyapunov method, nonlinear perturbations, uncertain time-delay systems

I. Introduction

Since the guaranteed cost control was first introduced by Cheng and Peng[15], many researchers have presented the robust controller design method for uncertain time-delay systems to improve the system performance. These methods can be classified into two categories: delay-independent approach[5,9,10,13] and delay-dependent ones[2,18,23-28]. In general, delay dependent method is less conservative than delay independent method especially when the size of the delays is small[16]. The structure of the controllers in [2,18,24-26,28] is memoryless state-feedback ones. These have merits that it is simple and easy to implement. However, these memoryless state-feedback controllers have some limits to improve system performance for time-delay systems because the controller uses only the current states. Thus, if we design a delayed feedback controller, we may provide a better performance. This property have been shown in the literature[14,17,19,27].

In real world, we can encounter the systems with nonlinear perturbations[20,29]. These lead the system to an unexpectedly complicated situations, thereby leading to very complex dynamic behaviors. In design of a controller for such a complex system, it is important to ensure that the system be stable with respect to these nonlinear perturbations. However, to the best of our knowledge, few results have been reported in the literature concerning the methods of designing a controller for time-delay systems having both parameter uncertainties and nonlinear perturbations.

In this paper, we study the problems of a guaranteed cost controller design for linear time-delay systems with norm-bounded parameter uncertainties and nonlinear perturbations. The perturbations are a nonlinear function of time, current state and delayed state. By using the neutral model transformation[8] and the Lyapunov function method, an LMI optimization approach problem is formulated to design a controllerm, which stabilizes given uncertain linear systems with time-delay and minimizes the upper bound value of the cost function. This controller has feedback provisions o the current state and the retarded state integral. We also include numerical examples that show our results are less conservative than those of the existing methods.

Notations: $\lambda_{\min}(X)$ and $\lambda_{\max}(X)$ are the minimum and maximum eigenvalues of X. $\|\cdot\|$ refers to the Euclidean vector norm of the induced matrix two-norm.. R^n is the n-dimensional Euclidean space, $R^{m\times n}$ denotes the set of $m\times n$ real matrix. $diag\{\cdots\}$ denotes the block diagonal matrix. $L_2[a,b]$ is the space of the square integral function on the interval $[a,b]$. $C([0\ \infty), R^n)$ denotes the Banach space of continuous vector functions from $[0\ \infty)$ to R^n. (*) means the elements below the main diagonal of a symmetric block matrix.

II. Problem Statements

Consider the following uncertain time-delay system with norm-bounded parameter uncertainties and nonlinear perturbations:

$$\begin{aligned}\dot{x}(t) &= (A+\Delta A(t))x(t)+(A_1+\Delta A_1(t))x(t-h)\\ &\quad +(B+\Delta B(t))u(t)+f(t,x(t),x(t-h))\\ x(s) &= \phi(s),\ s\in[-h,\ 0]\end{aligned} \tag{1}$$

where $x(t)\in R^n$ is the state, $u(t)\in R^m$ is the control input, A, A_1, and B are known real parameter matrices of appropriate dimensions, $\Delta A(t), \Delta A_1(t)$, and $\Delta B(t)$ are norm-bounded time-varying uncertainties, $f(t,x(t),x(t-h))$ is nonlinear parameter perturbation with respect to the current state $x(t)$ and the delayed state $x(t-h)$, h is a known constant delay, and $\phi(s)\in L_2[-h,0]$ is a given continuous vector valued initial function. The parameter uncertainties $\Delta A(t), \Delta A_1(t)$, and $\Delta B(t)$ have the following form:

---

* 책임저자(Corresponding Author)
논문접수 : 2006. 6. 20., 채택확정 : 2007. 2. 23.
권오민 : 충북대학교 전기공학과(madwind@chungbuk.ac.kr)
박주현 : 영남대학교 전기공학과(jessie@ynu.ac.kr)
※ 이 논문은 2006학년도 충북대학교 학술연구지원사업에 의하여 연구되었음.

$$\Delta A(t) = D_1F_1(t)E_1,\ \Delta A_1(t) = D_2F_2(t)E_2,\ \Delta B(t) = D_3F_3(t)E_3$$

where D_i, $E_i\,(i=1,2,3)$ are known real constant matrices of appropriate dimensions, and $F_i(t) \in R^{k_i \times l_i}$ are unknown matrices,

which satisfy

$$F_i^T(t)F_i(T) \le I,\ (i=1,2,3).$$

Also, the nonlinear uncertainty $f(t,x(t),x(t-h))$ is assumed to be bounded in magnitude:

$$\|f(t,x(t),x(t-h))\| \le \beta_1\|x(t)\| + \beta_2\|x(t-h)\|, \tag{2}$$

where β_1 and β_2 are nonnegative constant scalar values.

We assume that the pair $(A+A_1, B)$ is controllable, and the measurement of the state $x(t)$ and the size of time-delay h are always available. In order to consider system performance, we define the following integral quadratic cost function

$$J = \int_0^\infty [x^T(t)W_1x(t) + u^T(t)W_2u(t)]dt, \tag{3}$$

where W_1 and W_2 are given state and control weighting matrices.

The objective of this paper is to design a controller

$$u(t) = Kz(t), \tag{4}$$

which makes the system (1) stable and minimizes the upper bound of the cost function. Here, $z(t)$ is defined as

$$z(t) = x(t) + \int_{t-h}^t A_1x(s)ds. \tag{5}$$

This is the neural model transformation [8]. Differentiating $z(t)$ with respect to leads to

$$\begin{aligned}\dot z(t) &= \dot x(t) + A_1x(t) - A_1x(t-h)\\ &= (A_0+\Delta A)x(t) + \Delta A_1x(t-h) + (B+\Delta B)u(t)\\ &\quad + f(t,x(t),x(t-h))\end{aligned} \tag{6}$$

where $A_0 = A + A_1$. Substituting controller (4) to system (6), we have

$$\begin{aligned}\dot z(t) &= ((A_0+\Delta A) + (B+\Delta B)K)z(t) + \Delta A_1x(t-h)\\ &\quad + f(t,x(t),x(t-h)) - (A_0+\Delta A)\int_{t-h}^t A_1x(s)ds.\end{aligned} \tag{7}$$

We will need the following definition and lemmas to obtain the main results.

Definition 1: For system (1) and cost function (3), if there exist a control law $u^*(t)$ and a positive scalar J^*, such that for all admissible uncertainties, the closed-loop system is asymptotically stable, and the closed-loop value of the cost function satisfies $J \le J^*$, then $u^*(t)$ is said to be a guaranteed cost control law for system (1), and J^* is said to be a guaranteed cost.

Fact 1: (Schur complement) Given constant symmetric matrices $\Sigma_1, \Sigma_2, \Sigma_3$, where $\Sigma_1 = \Sigma_1^T$ and $0 < \Sigma_2 = \Sigma_2^T$, then $\Sigma_1 + \Sigma_3^T\Sigma_2^{-1}\Sigma_3 < 0$ if and only if

$$\begin{bmatrix}\Sigma_1 & \Sigma_3^T\\ \Sigma_3 & -\Sigma_2\end{bmatrix} < 0,\ \text{or}\ \begin{bmatrix}-\Sigma_2 & \Sigma_3\\ \Sigma_3^T & \Sigma_1\end{bmatrix} < 0.$$

Fact 2: Let D, E, and Δ be real matrices of appropriate dimensions with $\Delta = diag(\Delta_1,\dots,\Delta_r), \Delta_i^T\Delta_i \le I_{n_i}$, $i=1,\dots,r$. Then, for any real matrix $\Lambda = diag(\lambda_1 I,\dots,\lambda_n I) > 0$, the following inequality

$$D\Delta E + E^T\Delta^T E \le D\Lambda D^T + E^T\Lambda^{-1}E \tag{8}$$

is always satisfied.

Lemma 1: [3] For a given positive scalar $\hat h > 0$ and α, where $0 < \alpha < 1$, if there exists a positive definite M, such that the LMI

$$\begin{bmatrix}-\alpha M & \hat h A_1^T M\\ \hat h M A_1 & -M\end{bmatrix} < 0$$

holds, then $z(t)$ is a stable operator for any $h \in [0, \hat h]$.

Lemma 2: For any matrix $Q > 0$, F, and scalar $h \ge 0$, the following inequality holds:

$$\begin{aligned}-\int_{t-h}^t x^T(s)Qx(s)ds \le& \begin{bmatrix} z(t)\\ \int_{t-h}^t x(s)ds\end{bmatrix}^T \begin{bmatrix}0 & 0\\ * & F+F^T\end{bmatrix}\begin{bmatrix} z(t)\\ \int_{t-h}^t x(s)ds\end{bmatrix}\\ &+ h\begin{bmatrix} z(t)\\ \int_{t-h}^t x(s)ds\end{bmatrix}^T \begin{bmatrix}0\\ F\end{bmatrix} Q^{-1}\begin{bmatrix}0 & F^T\end{bmatrix}\begin{bmatrix} z(t)\\ \int_{t-h}^t x(s)ds\end{bmatrix}.\end{aligned}$$

Proof: Utilizing Fact 2, we have

$$\begin{aligned}&-\int_{t-h}^t x^T(\gamma)Qx(\gamma)d\gamma\\ &\le 2\int_{t-h}^t x^T(\gamma)\begin{bmatrix}0 & F^T\end{bmatrix}\begin{bmatrix} z(t)\\ \int_{t-h}^t x(s)ds\end{bmatrix}d\gamma\\ &\quad + \int_{t-h}^t \begin{bmatrix} z(t)\\ \int_{t-h}^t x(s)ds\end{bmatrix}^T\begin{bmatrix}0\\ F\end{bmatrix}Q^{-1}\begin{bmatrix}0 & F^T\end{bmatrix}\begin{bmatrix} z(t)\\ \int_{t-h}^t x(s)ds\end{bmatrix}d\gamma.\\ &= 2\begin{bmatrix} z(t)\\ \int_{t-h}^t x(s)ds\end{bmatrix}^T\begin{bmatrix}0\\ I\end{bmatrix}\begin{bmatrix}0 & F^T\end{bmatrix}\begin{bmatrix} z(t)\\ \int_{t-h}^t x(s)ds\end{bmatrix}\\ &\quad + h\begin{bmatrix} z(t)\\ \int_{t-h}^t x(s)ds\end{bmatrix}^T\begin{bmatrix}0\\ F\end{bmatrix}Q^{-1}\begin{bmatrix}0 & F^T\end{bmatrix}\begin{bmatrix} z(t)\\ \int_{t-h}^t x(s)ds\end{bmatrix}.\end{aligned}$$

Remark 1: Lemma 2 is inspired by the integral-inequality approach [22]. In the previous results[11,12], to obtain the upper bound of the integral term, the bounding methods in [6] is utilized, which is more conservative than the proposed one in Lemma 2.

III. Main results

In this section, we propose the method of designing a delayed feedback guaranteed cost controller for system (1). For simplicity, we define

$$
\begin{aligned}
\Sigma \; &= A_0X + XA_0^T + BY + Y^TB^T + D_1(\Lambda_1+\Lambda_4)D_1^T \\
&\quad + D_2\Lambda_2D_2^T + D_3\Lambda_3D_3^T + \varepsilon I, \\
NN^T &= \int_{-h}^{0}\phi(s)\phi^T(s)ds, \\
N_dN_d^T &= \int_{-h}^{0}\int_{s}^{0}\phi(u)\phi^T(u)duds,
\end{aligned} \tag{9}
$$

where X and $\Lambda_i(i=1,\ldots,4)$ are positive definite matrices, Y is a matrix with an appropriate dimension, and ε are positive scalar values. Now, we give our main results.

Theorem 1: Consider the system (1) with the cost function (3). For a given constant delay h, the following inequalities (10)-(12) has a solution $X>0,\, R>0,\, G>0, M>0,\, \Lambda_i>0(i=1,\ldots,$ 4), matrix Y and L with appropriate dimensions, and a positive scalar value ε,

$$
\begin{bmatrix}
\Sigma & -A_0A_1R & 0 & 0 & hX & X & X & \sqrt{2}\beta_1X & XE_1^T & Y^TE_3^T & Y^T \\
* & L+L^T & RA_1^TE_1^T & L & -hRA_1^T & -RA_1^T & -RA_1^T & -\sqrt{2}\beta_1RA_1^T & 0 & 0 & 0 \\
* & * & -\Lambda_4 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
* & * & * & -R & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
* & * & * & * & -R & 0 & 0 & 0 & 0 & 0 & 0 \\
* & * & * & * & * & -W_1^{-1} & 0 & 0 & 0 & 0 & 0 \\
* & * & * & * & * & * & -G & 0 & 0 & 0 & 0 \\
* & * & * & * & * & * & * & -\varepsilon I & 0 & 0 & 0 \\
* & * & * & * & * & * & * & * & -\Lambda_1 & 0 & 0 \\
* & * & * & * & * & * & * & * & * & -\Lambda_3 & 0 \\
* & * & * & * & * & * & * & * & * & * & -R_2^{-1}
\end{bmatrix} < 0, \tag{10}
$$

$$
\begin{bmatrix} -M & hA_1^TM \\ * & -M \end{bmatrix} < 0, \tag{11}
$$

$$
\begin{bmatrix} -G & GE_2^T & \sqrt{2}\beta_2G \\ * & -\Lambda_2 & 0 \\ * & * & -\varepsilon I \end{bmatrix} < 0, \tag{12}
$$

then, controller $u(t)=YX^{-1}z(t)$ is the guaranteed cost controller for system (1) and the upper bound of the cost function (3) is

$$
J \le J^* = z^T(0)Pz(0) + \int_{-h}^{0}\int_{s}^{0}\phi^T(u)Q\phi(u)duds \tag{13}
$$

$$
+\int_{-h}^{0}\phi^T(s)T\phi(s)ds.
$$

Proof: Consider the Lyapunov function candidate as

$$
V(x_t) = z^T(t)Pz(t) + \int_{t-h}^{t}\int_{s}^{t}x^T(u)Qx(u)duds + \int_{t-h}^{t}x^T(s)Tx(s)ds, \tag{14}
$$

where the matrices $P, Q,$ and T are positive matrices and $x_t = x(t+s), s\in[-h,0]$.

Define

$$
\begin{aligned}
V_1(x_t) &= 2z^T(t)P(A_0+\Delta A+(B+\Delta B)K)z(t) \\
&\quad - 2z^T(t)P(A_0+\Delta A)\int_{t-h}^{t}A_1x(s)ds \\
&\quad + 2z^T(t)Pf(t,x(t),x(t-h)) \\
&\quad + 2z^T(t)P\Delta A_1x(t-h) + hx^T(t)Qx(t) \\
&\quad - \int_{t-h}^{t}x^T(s)Qx(s)ds + x^T(t)Tx(t) \\
&\quad - x^T(t-h)Tx(t-h) + x^T(t)W_1x(t) \\
&\quad - z^T(t)K^TW_2Kz(t).
\end{aligned} \tag{15}
$$

Taking the time-derivative of $V(x_t)$ leads to

$$
\dot V(x_t) = V_1(x_t) - x^T(t)W_1x(t) - z^T(t)K^TW_2Kz(t). \tag{16}
$$

By using Fact 2, we obtain

$$
\begin{aligned}
&2z^T(t)PD_1F_1(t)E_1z(t) \\
&\le z^T(t)PD_1\Lambda_1D_1^TPz(t) + z^T(t)E_1^T\Lambda_1^{-1}E_1z(t),
\end{aligned} \tag{17}
$$

$$
\begin{aligned}
&2z^T(t)PD_2F_2(t)E_2z(t) \le \\
&z^T(t)PD_2\Lambda_2D_2^TPz(t) + z^T(t-h)E_2^T\Lambda_2^{-1}E_2z(t-h),
\end{aligned} \tag{18}
$$

$$
\begin{aligned}
&2z^T(t)PD_3F_3(t)E_3Kz(t) \\
&\le z^T(t)PD_3\Lambda_3D_3^TPz(t) + z^T(t)K^TE_3^T\Lambda_3^{-1}E_3Kz(t),
\end{aligned} \tag{19}
$$

$$
\begin{aligned}
&-2z^T(t)PD_1F_1(t)E_1\int_{t-h}^{t}A_1x(s)ds \\
&\le z^T(t)PD_1\Lambda_4D_1^TPz(t) \\
&\quad + \left(\int_{t-h}^{t}x(s)ds\right)^T A_1^TE_1^T\Lambda_4^{-1}E_1A_1\left(\int_{t-h}^{t}x(s)ds\right),
\end{aligned} \tag{20}
$$

$$
\begin{aligned}
&2z^T(t)Pf(t,x(t),x(t-h)) \\
&\le z^T(t)\varepsilon PPz(t) + \varepsilon^{-1}f^T(t,x(t),x(t-h))f(t,x(t),x(t-h)) \\
&\le z^T(t)\varepsilon PPz(t) + \varepsilon^{-1}2(\beta_1^2x^T(t)x(t) + \beta_2^2x^T(t-h)x(t-h)).
\end{aligned} \tag{21}
$$

Let us define $T_1 = hQ + W_1 + T + 2\varepsilon^{-1}\beta_1^2I$, and then we have

$$
\begin{aligned}
x^T(t)T_1x(t) &= z^T(t)T_1z(t) - 2z^T(t)T_1\int_{t-h}^{t}A_1x(s)ds \\
&\quad + \left(\int_{t-h}^{t}x(s)ds\right)^T A_1^TT_1A_1\left(\int_{t-h}^{t}x(s)ds\right).
\end{aligned} \tag{22}
$$

Substituting Eq. (17)-(21) into (15), $V_1(x_t)$ has the new upper bound as follows

$$
\begin{aligned}
V_1(x_t) &\le z^T(t)(PA_0 + A_0^TP + PBK + K^TB^TP + \varepsilon PP)z(t) \\
&\quad + z^T(t)(PD_1\Lambda_4D_1^TP)z(t) \\
&\quad + \left(\int_{t-h}^{t}x(s)ds\right)^T (A_1^TT_1A_1 + A_1^TE_1^T\Lambda_4^{-1}E_1A_1)\left(\int_{t-h}^{t}x(s)ds\right)
\end{aligned}
$$

$$
\begin{aligned}
&-2z^T(t)(PA_0+T_1)\int_{t-h}^{t}A_1x(s)ds \\
&+2z^T(t)PD_2\Lambda_2D_2^TPz(t) \\
&+x^T(t-h)(-T+E_2^T\Lambda_2^{-1}E_2+2\varepsilon^{-1}\beta_2^2I)x(t-h) \\
&+z^T(t)T_1z(t)+z^T(t)PD_3\Lambda_3D_3^TPz(t) \\
&+z^T(t)K^TE_3^TE_3Kz(t)+z^T(t)K^TW_2Kz(t) \\
&+\begin{bmatrix} z(t) \\ \int_{t-h}^{t}x(s)ds \end{bmatrix}^T \begin{bmatrix} 0 & 0 \\ * & F+F^T \end{bmatrix} \begin{bmatrix} z(t) \\ \int_{t-h}^{t}x(s)ds \end{bmatrix} \\
&+h\begin{bmatrix} z(t) \\ \int_{t-h}^{t}x(s)ds \end{bmatrix}^T \begin{bmatrix} 0 \\ F \end{bmatrix} Q^{-1}\begin{bmatrix} 0 & F^T \end{bmatrix} \begin{bmatrix} z(t) \\ \int_{t-h}^{t}x(s)ds \end{bmatrix},
\end{aligned} \tag{23}
$$

where Lemma 2 is utilized in obtaining the upper bound of $-\int_{t-h}^{t}x^T(s)Qx(s)ds$. If

$$-T+E_2^T\Lambda_2^{-1}E_2+2\varepsilon^{-1}\beta_2^2I<0, \tag{24}$$

then, we have the following inequality

$$V(x_t)\le \begin{bmatrix} z(t) \\ \int_{t-h}^{t}x(s)ds \end{bmatrix}^T \Sigma \begin{bmatrix} z(t) \\ \int_{t-h}^{t}x(s)ds \end{bmatrix}, \tag{25}$$

where

$$\Sigma=\left[\begin{array}{c|c} \Sigma_1 & -PA_0A_1-T_1A_1 \\ \hline * & \begin{matrix} F+F^T+A_1^TT_1A_1 \\ +A_1^TE_1^T\Lambda_4^{-1}E_1A_1 \end{matrix} \end{array}\right]+h\begin{bmatrix} 0 \\ F \end{bmatrix}Q^{-1}\begin{bmatrix} 0 & F^T \end{bmatrix}, \tag{26}$$

and

$$
\begin{aligned}
\Sigma_1 = {} & PA_0+A_0^TP+PBK+K^TB^TP+PD_1(\Lambda_1+\Lambda_4)D_1^TP \\
& +PD_2\Lambda_2D_2^TP+PD_3\Lambda_3D_3^TP+E_1^T\Lambda_1^{-1}E_1+K^TE_3^T\Lambda_3^{-1}E_3K \\
& +\varepsilon PP+K^TW_2K.
\end{aligned} \tag{27}
$$

If $\Sigma<0$, then a positive scalar exists which satisfies

$$\dot V(x_t)<-\lambda\|z(t)\|^2. \tag{28}$$

Also, if the inequality (11) holds, then we can prove that a positive scalar δ which is less than one exists such that

$$\begin{bmatrix} -\delta M & hA_1^TM \\ * & -M \end{bmatrix}<0 \tag{29}$$

according to matrix theory. From Lemma 1, if LMI (29) holds, then the operator $z(t)$ is stable. According to Theorem 9.8.1 in [4], we can conclude that if $\Sigma<0$, and LMI (11) hold, then system (6) is asymptotically stable. From (26), $\Sigma<0$ can be represented as

$$
\begin{aligned}
&\left[\begin{array}{c|c} \Sigma_1 & -PA_0A_1 \\ \hline * & \begin{matrix} F+F^T+ \\ A_1^TE_1^T\Lambda_4^{-1}E_1A_1 \end{matrix} \end{array}\right]+\begin{bmatrix} I \\ -A_1^T \end{bmatrix}T_1\begin{bmatrix} I & -A_1 \end{bmatrix} \\
&\qquad +h\begin{bmatrix} 0 \\ F \end{bmatrix}Q^{-1}\begin{bmatrix} I & F^T \end{bmatrix}<0.
\end{aligned} \tag{30}
$$

By Schur complements, the above inequality is equivalent to

$$
\begin{bmatrix}
\Sigma_1 & -PA_0A_1 & 0 & hI & I & I & \sqrt{2}\beta_1I \\
* & \begin{pmatrix} F+F^T+ \\ A_1^TE_1^T\Lambda_4^{-1}E_1A_1 \end{pmatrix} & F & -hA_1^T & -A_1^T & -A_1^T & -\sqrt{2}\beta_1A_1^T \\
* & * & -h^{-1}Q & 0 & 0 & 0 & 0 \\
* & * & * & -hQ^{-1} & 0 & 0 & 0 \\
* & * & * & * & -W_1^{-1} & 0 & 0 \\
* & * & * & * & * & -T^{-1} & 0 \\
* & * & * & * & * & * & -\varepsilon I
\end{bmatrix}<0. \tag{31}
$$

Letting

$$X=P^{-1},\ G=T^{-1},\ R=hQ^{-1},\ Y=KX,\ L=RFR, \tag{32}$$

and pre-and post-multiplying both sides of (31) by $diag\{X,R,I,I,I,I,I\}$ leads to

$$
\begin{bmatrix}
\Sigma_2 & -A_0A_1R & 0 & hX & X & X & \sqrt{2}\beta_1X \\
* & \begin{pmatrix} L+L^T+ \\ RA_1^TE_1^T\Lambda_4^{-1}E_1A_1R \end{pmatrix} & L & -hRA_1^T & -RA_1^T & -RA_1^T & -\sqrt{2}\beta_1RA_1^T \\
* & * & -R & 0 & 0 & 0 & 0 \\
* & * & * & -R & 0 & 0 & 0 \\
* & * & * & * & -W_1^{-1} & 0 & 0 \\
* & * & * & * & * & -G & 0 \\
* & * & * & * & * & * & -\varepsilon I
\end{bmatrix}<0, \tag{33}
$$

where

$$
\begin{aligned}
\Sigma_2 = {} & A_0X+XA_0^T+BY+Y^TB^T+D_1(\Lambda_1+\Lambda_4)D_1^T \\
& +D_2\Lambda_2D_2^T+D_3\Lambda_3D_3^T+XE_1^T\Lambda_1^{-1}E_1X+Y^TE_3^T\Lambda_3^{-1}E_3Y \\
& +\varepsilon I+Y^TW_2Y.
\end{aligned} \tag{34}
$$

By Schur complements, (33) is equivalent to inequality (10). Also, pre- and post-multiplying both sides of (24) by G leads to the inequality (12). Therefore, system (1) under controller (4) is asymptotically stable if (10)-(12) hold.

If inequality (10)-(12) hold, then

$$\dot V(x_t)<-(x^T(t)W_1x(t)+u^T(t)W_2u(t))<0. \tag{35}$$

Integrating both sides of (35) from 0 to t_f, we obtain

$$\begin{aligned}&z^T(t_f)Pz(t_f)+\int_{t_f-h}^{t_f}\int_s^{t_f}x^T(u)Qx(u)duds\\&+\int_{t_f-h}^{t_f}x^T(s)Tx(s)ds-z^T(0)Pz(0)\\&-\int_{-h}^{0}\int_s^{0}\phi^T(u)Q\phi(u)duds-\int_{-h}^{0}\phi^T(s)T\phi(s)ds\\&<-\int_0^{t_f}(x^T(t)W_1x(t)+u^T(t)W_2u(t))dt.\end{aligned} \tag{36}$$

Since we already established the asymptotic stability of the closed-loop system (7), when $t_f\to\infty$,

$$z^T(t_f)Pz(t_f)\to 0, \tag{37}$$

$$\int_{t_f-h}^{t_f}\int_s^{t_f}x^T(u)Qx(u)duds\to 0, \tag{38}$$

$$\int_{t_f-h}^{t_f}x^T(s)Tx(s)ds\to 0. \tag{39}$$

Therefore, we obtain the upper bound of cost function (3) as

$$\begin{aligned}J\le\ &z^T(0)Pz(0)+\int_{-h}^{0}\int_s^{0}\phi^T(u)Q\phi(u)duds\\&+\int_{-h}^{0}\phi^T(s)T\phi(s)ds.\end{aligned} \tag{40}$$

■

From Theorem 1, we construct a controller which makes the closed-loop system (6) asymptotically stable by a solution set. The following Theorem 2 presents the method of choosing a controller which minimizes the upper bound of the cost function (3).

Theorem 2: Consider the system (6) with the cost function (3). For a given $h>0$, if the following minimization problem

$$\min\{\alpha+Trace(M_1)+Trace(M_2)\}$$

subject to

$$(i)\ inequalities\ (10)-(12), \tag{41}$$

$$(ii)\begin{bmatrix}-\alpha & z^T(0)\\ * & -X\end{bmatrix}<0, \tag{42}$$

$$(iii)\begin{bmatrix}-M & hN_d^T\\ * & -hR\end{bmatrix}<0, \tag{43}$$

$$(iv)\begin{bmatrix}-M_2 & N^T\\ * & -G\end{bmatrix}<0, \tag{44}$$

has a solution $X>0,\ R>0,\ G>0,\ M>0,\ M_1>0,\ M_2>0,$ $\Lambda_i>0(i=1,\dots,4)$, matrix Y and L with appropriate dimensions, and positive scalar values α and ε, then the obtained controller $u(t)=YX^{-1}z(t)$ is a guaranteed cost controller which minimizes the upper bound of the cost function (3) and makes the closed-loop system (6) stable for all admissible norm-bounded parameter uncertainties and nonlinear perturbations. The guaranteed cost J^* is obtained as $\alpha+Trace(M_1)+Trace(M_2)$.

Proof: If the LMIs (42), (43), (44) in Theorem 2 hold, then the following inequality

$$\begin{aligned}&\alpha+Trace(M_1)+Trace(M_1)\\&>z^T(0)X^{-1}z(0)+Trace(N_d^T(hR^{-1})N_d)+Trace(N^TG^{-1}N)\end{aligned} \tag{45}$$

holds by applying Schur Complements to the LMIs (42), (43) and (44) and adding each term. Since

$$P=X^{-1}, \tag{46}$$

$$\begin{aligned}\int_{-h}^{0}\int_s^{0}\phi^T(u)Q\phi(u)duds&=Trace(N_dN_d^TQ)\\&=Trace(N_d^TQN_d)\\&=Trace(N_d^ThR^{-1}N_d),\end{aligned} \tag{47}$$

$$\begin{aligned}\int_{-h}^{0}\phi^T(s)T\phi(s)ds&=\int_{-h}^{0}\phi^T(s)G^{-1}\phi(s)ds\\&=Trace(NN^TG^{-1})\\&=Trace(N^TG^{-1}N),\end{aligned} \tag{48}$$

we obtain

$$\begin{aligned}&z^T(0)X^{-1}z(0)+Trace(N_d^T(hR^{-1})N_d)\\&+Trace(N^TG^{-1}N)\\&=z^T(0)Pz(0)+\int_{-h}^{0}\int_s^{0}\phi^T(u)Q\phi(u)duds\\&+\int_{-h}^{0}\phi^T(s)T\phi(s)ds.\end{aligned} \tag{49}$$

From (40), (45), and (49), we can know that $\alpha+Trace(M_1)$ $+Trace(M_2)$ is the upper bound of the cost function (3). Therefore, the controller $u(t)=YX^{-1}z(t)$ constructed from Theorem 2 is a guaranteed cost controller which minimizes the upper bound value of the cost function (3) and $\alpha+$ $Trace(M_1)+Trace(M_2)$ is a guaranteed cost. This completes our proof. ■

Remark 2: Since the LMIs (10)-(14) in Theorem 1 can be easily solved by various efficient convex algorithms. In this paper, we utilize Matlab's LMI control Toolbox [21] which implements interior-point algorithms. These algorithms are significantly faster than classical convex optimization algorithms [1].

Remark 3: In [27], the delayed feedback observer-based control method was presented. However, two coupled LMI should be solved to obtain the desired controller. Moreover, the system performance had not considered in [27]. To the best of author's knowledge, delay-dependent observer-based guaranteed cost control has not been fully investigated. In future works, we will study the design problem for delay-dependent observer-based control for uncertain time-delay system with considering system performance by utilizing delayed feedback.

IV. Numerical Example

Example 1: Consider the uncertain time-delay system with norm-bounded parameter uncertainty and nonlinear perturbations:

$$\begin{aligned}\dot{x}(t)=&(A+\Delta A)x(t)+(A_1+\Delta A_1)x(t-1)\\&+(B+\Delta B)u(t)+f(t,x(t),x(t-1))\end{aligned} \tag{50}$$

$$\phi(s)=\begin{bmatrix}0.5e^{\frac{s}{2}}\\ -e^{\frac{s}{2}}\end{bmatrix},\ s\in[-1,\ 0]$$

where system matrices are

$$A=\begin{bmatrix}0 & 1\\ 1 & -2\end{bmatrix},\ A_1=\begin{bmatrix}0 & 0\\ 0.1 & 0.1\end{bmatrix},\ B=\begin{bmatrix}0\\ 1\end{bmatrix}, \tag{51}$$

표 1. 비선형 불확실성에 따른 성능 비용 및 제어 이득 (예제 1).

Table 1. The guaranteed cost and controller gain matrices with respect to nonlinear uncertainties(Example 1).

Nonlinear Uncertainty	Cost, J^*	Controller gain,
$\beta_1 = 0,\ \beta_2 = 0$	1.2244	[-10.5125 -4.3406]
$\beta_1 = 0.1,\ \beta_2 = 0$	2.1935	[-13.4451 -5.3839]
$\beta_1 = 0,\ \beta_2 = 0.1$	2.8597	[-15.2916 -6.0329]
$\beta_1 = 0.1,\ \beta_2 = 0.1$	3.4387	[-16.2086 -6.3034]

표 2. $\beta_1 = \beta_2 = 0$ 인경우 성능 비용 비교(예제 1).

Table 2. Comparision of the obtained guaranteed cost for $\beta_1 = \beta_2 = 0$ (Example 1).

Method	Cost, J^*
Method of [2]	3.5073
Method of [27]	1.3162
Our result	1.2244

and parameter uncertainties are

$$D_1 = \begin{bmatrix} 0.1 & 0.1 \\ 0 & 0 \end{bmatrix},\ D_2 = D_1,\ D_3 = \begin{bmatrix} 0 & 0 \\ 0 & 0.1 \end{bmatrix},$$
$$E_1 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix},\ E_2 = E_1,\ E_3 = \begin{bmatrix} 0 \\ 1 \end{bmatrix}. \tag{52}$$

Let's choose the weighting matrices

$$W_1 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix},\ W_2 = 1. \tag{53}$$

Table 1 shows the results of cost and corresponding controller gain matrices with respect to the nonlinear uncertainty bounds by applying Theorem 1. And table 2 compares the obtained guaranteed cost with recent results. From table 2, we can see the proposed controller gives less upper bound of the cost function by utilizing the proposed Lemma 2. If we increase the nonlinear uncertainty bounds, the guaranteed cost and controller gain become large, which means the stabilization condition becomes conservative due to the nonlinear perturbations.

Example 2:

$$\dot{x}(t) = (A + \Delta A)x(t) + (A_1 + \Delta A_1)x(t-h) + Bu(t) + f(t, x(t), x(t-h)) \tag{54}$$
$$\phi(s) = \begin{bmatrix} e^{s+1} \\ 0 \end{bmatrix},\ s \in [-h, 0]$$

where system matrices are

$$A = \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix},\ A_1 = \begin{bmatrix} -2 & -0.5 \\ 0 & -1 \end{bmatrix},\ B = \begin{bmatrix} 0 \\ 1 \end{bmatrix}, \tag{55}$$

and parameter uncertainties are

$$D_1 = \begin{bmatrix} 0.2 & 0 \\ 0 & 0.2 \end{bmatrix},\ D_2 = D_1, \tag{56}$$

표 3. 비선형 불확실성에 따른 성능 비용 및 제어 이득 (예제 2).

Table 3. The guaranteed cost and controller gain matrices with respect to nonlinear uncertainties (Example 2).

Nonlinear Uncertainty	Cost, J^*	Controller gain,
$\beta_1 = 0,\ \beta_2 = 0$	2.8712	[-0.0825 -95.2545]
$\beta_1 = 0.1,\ \beta_2 = 0$	3.2101	[-0.0514 -1406.6}
$\beta_1 = 0,\ \beta_2 = 0.1$	3.3268	[-0.039 -2147.4]
$\beta_1 = 0.1,\ \beta_2 = 0.1$	3.5057	[-0.0227 -3217.0]

표 4. $\beta_1 = \beta_2 = 0$ 인경우 성능 비용 비교(예제 2).

Table 4. Comparision of the obtained guaranteed cost for $\beta_1 = \beta_2 = 0$ (Example 2).

Method	Cost, J^*
Method of [28]	4.2
Our result	2.8712

$$E_1 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix},\ E_2 = E_1.$$

Let's choose the weighting matrices

$$W_1 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix},\ W_2 = 1. \tag{57}$$

It is noted that the system is not delay-indepedently stabilizable. For this system, we assume $h = 0.37$ which is the same value in [28]. Table 3 and table 4 show the same items represented in example 1. From table 3 and 4, we can see the proposed method gives less guaranteed cost in spite of the consideration of nonlinear perturbations. However, the obtained controller becomes large as nonlinear perturbation bound increase.

V. Conclusion

In this paper, a delayed feedback guaranteed cost controller design method for uncertain time-delay systems with norm-bounded parameter uncertainties and nonlinear perturbations has been proposed. An LMI optimization problem, which can be solved effectively by optimization algorithms, is expressed in terms of LMIs to design a controller with feedback of the current and the past history of the state. This controller stabilizes the closed-loop system and minimizes a better performance than other results in spite of nonlinear perturbations.

References

[1] S. Boyd, L. El Ghaoui, E. Feron, and V. balakrishnan, *Linear Matrix Inequalities in System and Control Theory*, SIAM, Philadelphia, 1994.

[2] C. Gouding, Y. Maying, L. Yu, and C. Jian, "Delay dependent guaranteed cost control for linear uncertain time-delay systems," *Proceedings of the 3rd World Congress on Intelligent Control and Automation*, Hefei, P.R. China, pp. 3363-3367, 2000.

[3] D. Yue and S. Won, "Delay-dependent robust stability of stochastic systems with time delay and nonlinear uncertainties,"

Electronics Letters, vol. 37, pp. 992-993, 2001.

[4] J. Hale and S. M. V. Lunel, *Introduction to Functional Differential Equations*, Springer-Verlag, New York, 1993.

[5] J. H. Kim, "Robust guaranteed cost control of discrete-time uncertain systems," *IEICE Trans. Fundamentals*, vol. E84-A, no. 8, pp. 2065-2069, 2001.

[6] K. Gu, "An integral inequality in the stability of time-delay Systems," *IEEE Conference on Decision and Control, Sydney, Australia*, 2000.

[7] K. Gu, "A further refinement of discretized Lyapunov functional method for the stability of time-delay systems," *International Journal of Control*, vol. 74, no. 10, pp. 967-976, 2001.

[8] K. Gu and S.-I. Niculescu, "Further remarks on additional dynamics in various model transformations of linear delay systems," *IEEE Trans. On Automatic Control*, vol. 46, no. 3, pp. 497-500, 2001.

[9] L. Yu and F. Gao, "Optimal guaranteed cost control of discrete-time uncertain systems with both state and input delays," *Journal of the Franklin Institute*, vol. 338, pp. 101-110, 2001.

[10] L. Yu and J. Chu, "An LMI approach to guaranteed cost control of linear uncertain time-delay systems," *Automatica*, vol. 35, pp. 1155-1159, 1999.

[11] O. M. Kwon and J. H. Park, "An improved delay-dependent robust control for uncertain time-delay systems," *IEEE Trans. On Automatic Control*, vol. 49, no. 11, pp. 1991-1995, 2004.

[12] O. M. Kwon and J. H. Park, "Robust stabilization of uncertain systems with delays in control input," *Applied Mathematics and Computation*, vol. 172, pp. 1067-1077, 2006.

[13] S. H. Esfahani and I. R. Peterson, "An LMI approach to output-feedback guaranteed cost control for uncertain time-delay systems," *International Journal of Robust and Nonlinear Control*, vol. 10, pp. 157-174, 2000.

[14] S. Oucherich, "Global stabilization of a class of linear continuous time-delay systems with saturating controls," *IEEE Trans. On Circuits and Systems-I : Fundamental Theory and Applications*, vol. 43, no. 12, pp. 1012-1015, 1996.

[15] S. S. L. Chang and T. K. C. Peng, "Adaptive guaranteed cost control of systems with uncertain parameters," *IEEE Trans. On Automatic Control*, vol. 17, no. 4, pp. 474-483, 1972.

[16] T. Mori, "Criteria for asymptotic stability of linear time-delay systems," *IEEE Trans. On Automatic Control*, vol. 30, no. 2, pp. 158-161, 1985.

[17] Y. H. Roh and J. H. Oh, "Robust stabilization of uncertain input-delay systems by sliding mode control with delay compensation," *Automatica*, vol. 35, pp. 1861-1865, 1999.

[18] Y. S. Lee, Y. S. Moon, and W. H. Kwon, "Delay-dependent guaranteed cost control for uncertain state-delayed systems," *Proceedings of the American Control Conference*, pp. 3376-3381, 2001.

[19] Y. S. Moon, P. Park, and W. H. Kwon, "Robust stabilization of uncertain input-delayed systems using reduction method," *Automatica*, vol. 37, pp. 307-312, 2001.

[20] Z. H. Gua, C. W. Chan, Andrew Y. T. Leung, and G. Chen, "Robust stabilization of singular-impulsive-delayed systems with nonlinear perturbations," *IEEE Trans. On Circuits and Systems-I: Fundamental Theory and Applications*, vol. 48, no. 8, pp. 1011-1019, 2001.

[21] P. Gahinet, A. Nemirovskii, A. Laub, and M. Chilali, *LMI Control Toolbox*, Math Works, Natick, Massachusetts, 1995.

[22] X. M. Zhang, M. Wu, J. H. She, and Y. He, "Delay-dependent stabilization of linear systems with time-varying state and input delays," *Automatica*, vol. 41, pp. 1405-1412, 2005.

[23] O. M. Kwon, J. H. Park, S. M. Lee, and S. C. Won, "LMI optimization approach to observer-based controller design of uncertain time-delay systems via delayed feedback," *Journal of Optimization Theory and Applications*, vol. 128, no. 1, pp. 103-117, 2006.

[24] W. H. Chen, Z. H. Guan, and X. M. Lu, "Delay-dependent guaranteed cost control for uncertain discrete-time systems with delay," *IEE Proceedings-Control Theory and Applications*, vol. 150, no. 4, pp. 412-416, 2003.

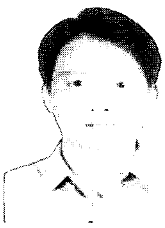
[25] W. H. Chen, Z. H. Guan, and X. M. Lu, "Delay-dependent output feedback guaranteed cost control for uncertain time-delay systems," *Automatica*, vol. 40, no. 7, pp. 1263-1268, 2004.

[26] W. H. Chen, Z. H. Guan, and X. M. Lu, "Delay-dependent guaranteed cost control for uncertain discrete-time systems with both state and input delays," *Journal of the Franklin Institute*, vol. 341, no. 5, pp. 419-430, 2004.

[27] O. Kwon, S. Won, and D. Yue, "Delayed feedback guaranteed cost controller design for uncertain time-delay systems," *IEICE Trans. Fundamentals*, vol. E86-A, no. 9, 2413-2417, 2003.

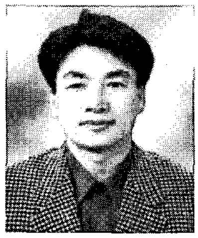
[28] Y. S. Lee, O. K. Kwon, and W. H. Kwon, "Delay-dependent guaranteed cost control for uncertain state-delayed systems," *International Journal of Control, Automation, and Systems*, vol. 3, no. 4, pp. 524-532, 2005.

[29] Z. Zuo and Y. Wang, "New stability criterion for a class of linear systems with time-varying delay and nonlinear perturbations," *IEE Proceedings-Control Theory and Applications*, vol. 153, no. 5, pp. 623-626, 2006.



Ohmin Kwon

was born in Dae-Gu, Korea in 1974. He received the B.S. degree in the department of electrical engineering, Kyungbuk National Univerity in February 1997, and M.S. and Ph. D degree in the Department of Electrical and Electronics Engineering, POSTECH. From February 2004, he joined Intelligent Control Part, Mechatronics Center, Samsung Heavy Industries, Co. Ltd., as Senior Researcher. Since March 2006, he is a full-time faculty member in School of Electrical & Engineering, Chungbuk Nationak University. He is a Managing Editor of Journal of Mathematical Control Science and Applications. His research interests includes stability analysis and controller synthesis of time-delay system, PID control, robust control, guaranteed cost control, non-fragile control, large-scale system, chaos system, robust filtering, looper control in hot rolling process, anti-windup compensator design, system identification, neural networks, embedded control system based on RTOS linux and system biology.



Juhyun Park

Ju H. Park received his Ph.D. degree from POSTECH in 1997. From May 1997 to February 2000, he was a Research Fellow at ERC-ARC, POSTECH. Then, He joined the Department of Electrical Engineering at Yeungnam University in March 2000 as full faculty member. He is an Editor-in-Chief of Journal of Mathematical Control Science and Appliations. He also served editorial positions in 10 international journals. His current main research interests include control theories in linear and nonlinear systems, and applied mathematics.