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Effect of variable viscosity on combined forced and free convection boundary-layer flow over a horizontal plate with blowing or suction

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Abstract: The effects of variable viscosity, blowing or suction on mixed convection flow of a viscous incompressible fluid past a semi-infinite horizontal flat plate aligned parallel to a uniform free stream in the presence of the wall temperature distribution inversely proportional to the square root of the distance from the leading edge have been investigated. The equations governing the flow are transformed into a system of coupled non-linear ordinary differential equations by using similarity variables. The similarity equations have been solved numerically. The effect of the viscosity temperature parameter, the buoyancy parameter and the blowing or suction parameter on the velocity and temperature profiles as well as on the skin-friction coefficient and the Nusselt number are discussed.

Keywords: Mixed convection; variable viscosity; blowing or suction.

1. Introduction:

Combined forced and free convection or "mixed" convection arises in many transport processes in natural and engineering applications. Atmospheric-boundary layer flow, heat exchangers, solar collectors, nuclear reactors and electronic equipment are examples in which the effect of buoyancy force on forced flow is significant.

In contrast to the problem of mixed convective flow along a vertical flat plate, less attention has been given to studies of buoyancy force effects on laminar forced convection over a horizontal flate plate. Mori [1] and Sparrow and Minkowycz [2] were the first investigators to treat this problem. Since then, extensive studies which has been conducted by Schneider [3], Dey[4], Ramarchandran et.al.[5], Raju et.al.[6], De Hong et.al.[7], Afzal and Hussain [8], Merkin and Ingham [9], Risbeck et.al.[10], Schneider et.al. [11], Risbeck and Chen [12], Steinrück[13], Rudischer and Steinruck [14], Rudischer and Steinrück [14] and Magyari et.al. [15].

In all the above mentioned studies, the fluid viscosity was assumed uniform in the flow region. But it is known that these physical properties change significantly with temperature. To illustrate the need to include this viscosity temperature variation, we quote two examples of common fluids: the viscosity of carbon tetrachloride varies from 1.329 centipoise at $0^{o}C$ to 0.384 cp at $100^{o}C$; Olive oil has viscosities of 138 and 12.4 cp for respective temperature of $10^{o}C$ and $70^{o}C$ [16].

A survey of literature reveals that the combined effects of variable viscosity and buoyancy forces on mixed convention heat transfer over a semi-infinite horizontal plate in the presence of blowing or suction have not been studied yet.

In the present study it is proposed to investigate the mixed convection flow of a viscous incompressible fluid having viscosity depending on temperature from horizontal flat plate with a non-uniform temperature in the presence of blowing or suction.

2- Mathematical Formulation

Let us consider the steady two-dimensional laminar free-forced convective flow of a viscous incompressible fluid over a semi-infinite horizontal flat plate aligned parallel to a uniform free stream with velocity u_∞, density ρ_∞ and temperature T_∞ in the presence of the wall temperature distribution $T_w(\bar{x}) \sim \bar{x}^{-1/2}$. The $\bar{x}$- axis is measured from the leading edge along the plate and the $\bar{y}$-axis is normal to it. We assume that property variation with temperature are limited to viscosity, and density. The temperature dependent density is taken in the buoyancy force term in the momentum equation only. Neglecting the viscous dissipation, and under Boussinesq approximation, the two-dimensional boundary layer equations for the mixed convection flow of fluid past a semi-infinite horizontal plate may be written as [17].

$$\frac{\partial \bar{u}}{\partial \bar{x}} + \frac{\partial \bar{v}}{\partial \bar{y}} = 0 \;, \tag{1}$$

$$\bar{u}\frac{\partial \bar{u}}{\partial \bar{x}} + \bar{v}\frac{\partial \bar{u}}{\partial \bar{y}} = -\frac{1}{\rho_\infty}\frac{\partial \bar{p}}{\partial \bar{x}} + \frac{1}{\rho_\infty}\frac{\partial}{\partial \bar{y}}\left(\bar{\mu}\frac{\partial \bar{u}}{\partial \bar{y}}\right) \;, \tag{2-a}$$

$$-\frac{1}{\rho_\infty}\frac{\partial \bar{p}}{\partial \bar{y}} = g\beta\,(T - T_\infty) \;, \tag{2-b}$$

$$\bar{u}\frac{\partial T}{\partial \bar{x}} + \bar{v}\frac{\partial T}{\partial \bar{y}} = \frac{1}{\rho_\infty c_p}\frac{\partial}{\partial \bar{y}}\left(k\frac{\partial T}{\partial \bar{y}}\right) \;, \tag{3}$$

where $\bar{u}$ and $\bar{v}$ the velocity components in the $\bar{x}$ and $\bar{y}$ directions, respectively, $\bar{\mu}$ is the viscosity of the fluid, T is the temperature of the fluid in the boundary layer, g is the acceleration due to gravity, β is the coefficient of volumetric expansion, $\bar{p}$ is the pressure, k is the thermal conductivity and c_p is the specific heat at constant pressure.

The boundary conditions to be satisfied are given by

$$\bar{y} = o: \quad \bar{u} = 0, \bar{v} = v_w , \quad T = T_w(\bar{x}), \tag{4}$$

$$\bar{y} \to \infty : \quad \bar{u} \to u_\infty , \ \bar{p} \to 0, \quad T \to T_\infty .$$

For a viscous fluid, Ling and Dybbs [18] and Lai and Kulacki [19] suggest a viscosity $\bar{\mu}$ dependence on temperature T of the form

$$\bar{\mu} = \frac{\mu_\infty}{(1+\gamma(T - T_\infty))} \text{ or } \frac{1}{\bar{\mu}} = \alpha(T - T_r), \tag{5}$$

with $\alpha = \frac{\gamma}{\mu_\infty}, \ T_r = T_\infty - \frac{1}{\gamma},$ (6)

where α and T_r are constants and their values depend on the reference state of the fluid. In general, $\alpha < 0$ for gases and $\alpha > 0$ for liquids.

Introducing the following non-dimensional variables into equation (1) – (3):

$$x = \frac{\bar{x}}{\ell} , \ y = \frac{\sqrt{\text{Re}}}{\ell}\bar{y} , \ T_w(\bar{x}) = T_\infty + T^* / \sqrt{x}$$

$$\eta = \frac{y}{x^{1/2}} , \quad \psi = v_\infty \sqrt{R_e \, x} \, f(\eta), \quad v_\infty = \frac{\mu_\infty}{\rho_\infty} \tag{7}$$

$$T - T_\infty = T^* \frac{\theta(\eta)}{\sqrt{x}} , \ \bar{P} - P_\infty = \rho u_\infty^2 \, p(\eta) ,$$

where R_e is the Reynolds number, T^* represents a characteristic temperature difference between the plate and free stream and l is a reference length, we get

$$2\mu f''' + 2\mu' f'' + ff'' + \lambda \eta \theta = 0, \tag{8}$$

$$2\theta'' + p_r(f'\theta + f\theta') = 0, \tag{9}$$

where $\mu = \frac{\bar{\mu}}{\mu_\infty}$, μ_∞ is the viscosity of the ambient fluid, and the prime denotes differentiation with respect to η.

Introducing equation (5) into equations (8) and (9), we have

$$f''' - \frac{\theta'}{(\theta-\theta_r)} f'' - \frac{2(\theta-\theta_r)}{\theta_r}(ff'' + \lambda\eta\theta) = 0, \tag{10}$$

$$2\theta'' + p_r(f'\theta + f\theta') = 0, \tag{11}$$

where θ_r is a constant viscosity-temperature parameter given by

$$\theta_r = \frac{T_r - T_\infty}{T_w - T_\infty} = -\frac{1}{\gamma(T_w - T_\infty)}, \tag{12}$$

$P_r = \nu_\infty / k_\infty$ is the Prandtl number, and $\lambda = \frac{gl\beta T^*}{\sqrt{R_e}\, u_\infty^2}$ is the mixed convection parameter.

Equation (7) transform the boundary conditions (4) into

$$\begin{aligned} &\eta = 0 : \quad f = f_w, \; f' = 0, \quad \theta = 1, \\ &\eta \to \infty : \quad f' \to 1, \quad \theta = 0, \end{aligned} \tag{13}$$

where $f_w = -2\sqrt{x}v_w$ is the blowing (<0) or the suction (>0) parameter. Integrating equation (11) and using the boundary conditions (13) we obtain

$$\theta' + p_r f\theta = 0. \tag{14}$$

It is interesting to note that if $f_w = 0$, then $\theta'(0) = 0$ for all values of θ_r, p_r and λ. This means that no heat transferred, except for the singularity at the leading edge $x = 0$ of the plate [3].

The shearing stress at the plate is defined by

$$\tau_w = (\bar{\mu}\frac{\partial \bar{u}}{\partial y})_{\bar{y}=0}. \tag{15}$$

The local skin friction coefficient is defined by

$$C_\rho = \frac{2\tau_w}{\rho_\infty u_\infty^2} = 2\sqrt{R_{ex}}\frac{\theta_r}{(\theta_r - 1)} f''(0,\theta_r), \tag{15}$$

where R_{ex} is the local Reynolds number.

The local Nusselt number is defined by

$$N_{ux} = -\sqrt{R_{ex}}\,\theta'(0,\theta_r) \tag{16}$$

3. Solution and discussion

The coupled non-linear ordinary differential equations (10) and (11) along with the boundary conditions (13) are solved numerically by the fourth-order Runge-Kutta scheme with the Newton-Raphson iteration method. This numerical solution technique is similar to that described in ref. [20]. The accuracy of the numerical results can be verified by comparing our results taking $f_w = 0$ with that obtained by Pop and Gorla [21] where $n = 1$. It was found that the results are in good agreement as shown in table 1.

Samples of the resulting velocity and temperature profiles for $p_r = 0.72$ and different values of the mixed-convection parameter λ , the temperature parameter θ_r and the blowing or suction parameter f_w are presented in Figs.1-3. The effect of the mixed-convection parameter λ and the temperature parameter θ_r on the dimensionless velocity f' and the dimensionless temperature θ in the presence of suction, $f_w = 0.5$ is shown in Fig.1. This figure shows that the velocity gradient at the wall and the overshoot of the velocity decreases while it is accompanied by a further location of the peak from the wall as λ or θ_r decreases. But at a certain distance from the plate, $\eta = 2.12$ for $\lambda = 2$ and $\eta = 3.17$ for $\lambda = 0.5$ it is noticed that the velocity increases as θ_r decreases. Also, it is observed from Fig.1 that the temperature drops more quickly and the thermal boundary layer thickness becomes thinner when λ or θ_r is higher. This means that the mixed-convection parameter and the viscosity temperature parameter effects have a tendency to induce more flow near the plate at the expense of small reduction in temperature. Figure 2 displays the effects of λ and θ_r in the presence of blowing, $f_w = -0.5$. It is found from this figure that the dimensionless velocity increases near the plate as λ increases. But an opposite effect is noticed at a certain distance from the plate $\eta = 2.36$ for $\lambda = 2$ and $\eta = 3.31$ for $\lambda = 0.5$.

The effect of the viscosity temperature parameter θ_r on f' in the presence of injection is similar to the suction case. Also, we see that from Fig. 2 that there are sharp rises in the temperature profiles near the wall which yields an overshoot of fluid temperature beyond the wall temperature, especially for small values of λ and θ_r. These temperature distributions are quite different from those profiles shown in Fig.1. The influence of the blowing / suction parameter f_w on f' and θ is shown in Fig.3. It can be seen from this figure that the maximum velocities are decreased with increasing f_w. Also, we see that as f_w increases from negative to positive values, the temperature gradient at the wall decreases from positive to negative, as predicted by Eq. (14). From table 2, one sees that in the presence of blowing or suction the skin-friction coefficient increases as either λ or θ_r increases. For fixed values of λ and θ_r, the skin-friction coefficient increased with the increasing of the blowing parameter and decreased as the suction parameter was increased. Also, it is found from table2 that the Nusselt number increases as the suction parameter increases while it increases as the blowing parameter increases.

4. Conclusion

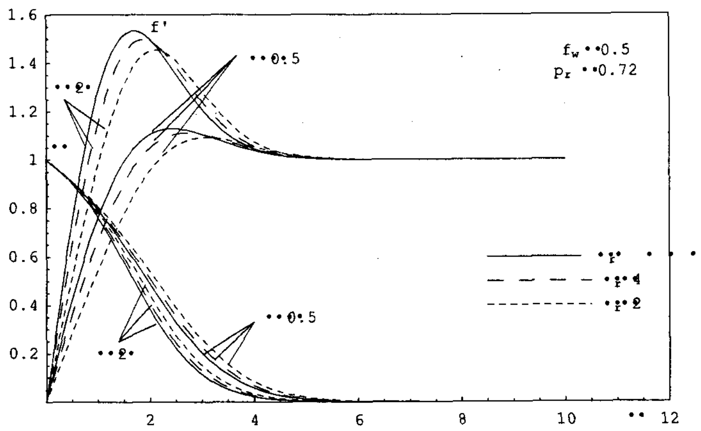
The problem of mixed convection in laminar boundary layer flow along horizontal flat plate with variable viscosity in the presence of blowing or suction is analyzed. The governing equations are transformed and solved numerically by means of the shooting technique. It was found that the velocity increases near the plate as λ or θ_r increases, while the thermal boundary layer thickness decreases as λ or θ_r increases. Also, it was found that in the presence of suction the velocity decreased as the suction parameter was increased and increased as the blowing parameter was increased. Furthermore it was found that the temperature increased as the suction parameter was increased. The opposite is true in the presence of blowing.

Table 1. Comparison of $f''(0)$ for various values of λ

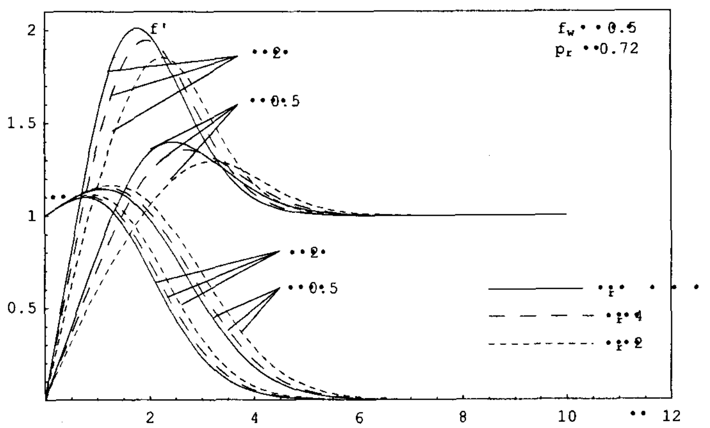
λ	Pop and Gorla	Present
0	0.3320	0.3320
0.2	0.5525	0.5516
0.5	0.7757	0.7740
1	1.0574	1.0545

Table 2. Values of $f''(0)$ and $-\theta'(0)$ for various values of θ_r, λ and f_w with $p_r = 0.72$

λ	f_w	θ_r	$f''(0)$	$-\theta'(0)$
0.2	0.2	2	0.40122	0.07199
0.5	0.2	2	0.5647	0.07199
2	0.2	2	1.0955	0.07199
0.2	-0.2	2	0.4018	-0.07199
0.5	-0.2	2	0.5936	-0.07199
1	-0.2	2	1.1825	-0.07199
0.5	-0.5	2	0.6406	-0.1800
0.5	-0.2	2	0.5936	-0.07199
0.5	0.2	2	0.5647	0.07199
0.5	0.5	2	0.5622	0.1800
0.5	0.5	2	0.5622	0.1800
0.5	0.5	4	0.7541	0.1800
0.5	0.5	6	0.8118	0.1800
0.5	-0.5	2	0.6405	-0.1800
0.5	-0.5	4	0.7046	-0.1800
0.5	-0.5	6	0.7221	-0.1800



Fig•. 1. Velocity and temperature distribution for various values of • •and • ;



Fig•. 2. Velocity and temperature distribution for various values of • •and • ;

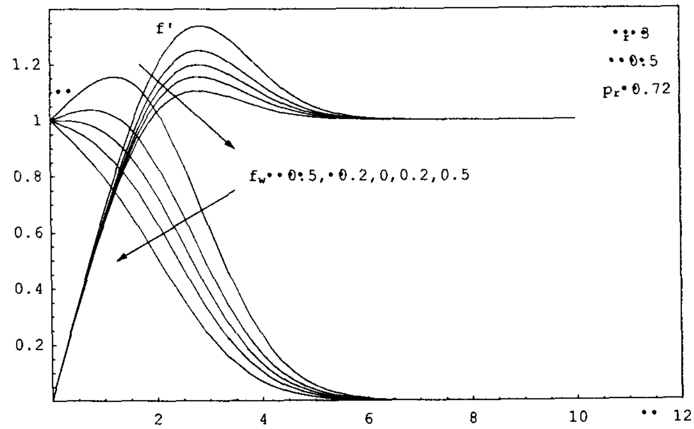


Fig. 3. Velocity and temperature distribution for various values of f_w

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