Agri. & Biosys. Eng. Vol. 7(1):13-17(2006)

Analysis of Planocentric Gear

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Abstract: The planocentric gear, known as wobble mechanism, has been used for speed reducing mechanism as an ingenious mechanism. The modern application can be found in the backrest adjusting mechanism of a vehicle reclinable seat, fluid pumps and aircraft hoist and winches. Higher speed reduction ratios, high load capacity, lower weight, and compactness are the main advantages of this gear. This paper presents velocity and static force analysis to investigate the friction lock of the planocentric gear. The rectilinear tooth profile is used to maximize the speed reduction ratio. The equivalent linkage system is used for the analysis of instantaneous motion. As the results, the transmission efficiency of the planocentric gear is found and the friction lock of the system is determined for the friction coefficients of journals. A numerical example that illustrates the developed analysis is presented.
Keywords: Planocentric Gear, Wobble Mechanism, Friction, Static Analysis

Introduction

A simple construction of the planocentric gear is illustrated in Figure 1. This is the simplest epicyclic gear train and consists of three main elements; an internal gear, an externally toothed pinion which has one tooth less than the internal gear, and eccentric shaft.

The pinion is mounted on an eccentric. A sliding bearing separates the pinion from the eccentric and its drive shaft. The pinion is thus under no constraint to rotate at the speed of the input shaft. The pinion wobbles instead of truly rotating. It does develop a rotation, superimposed on the wobble. The rotation of the plate welded or riveted on the pinion becomes the output speed.

If the number of teeth of the pinion is p, then the angular speed ratio between the pinion and the eccentric shaft is $\omega_2/\omega_1 = -1/p$ which maximize the reduction ratio. This gear train can develop as much as 100:1 ratio [1]. In order to avoid the interference between the internal gear and the pinion, the special rectilinear tooth profile is used.

The earliest applications of this kind mechanism are found in instrument counters, ship ladder actuators, and sewing machines. Modern uses for derivatives of this mechanism are recognized in circumstances where higher speed reduction ratios, lower weight, and compactness show an advantage over simple gear trains. Examples of three modern designs follow with application to fluid pumps, road vehicle seats and aircraft hoists and winches. In one unusual application, introduced in the late 1960's by Keiper AG, the gear train forms the backrest adjusting mechanism of vehicle reclinable seat [2]. The mass production of the Keiper mechanism was possible by the fine blanking press technology to produce the gear teeth.

Self-locking of this speed reduction mechanism is required in many applications. In order to investigate the self-locking of the gear train, a static force analysis including friction is performed. As the results, the transmission efficiency of the two-gear epicyclic gear train is found and the friction lock of the system is determined.

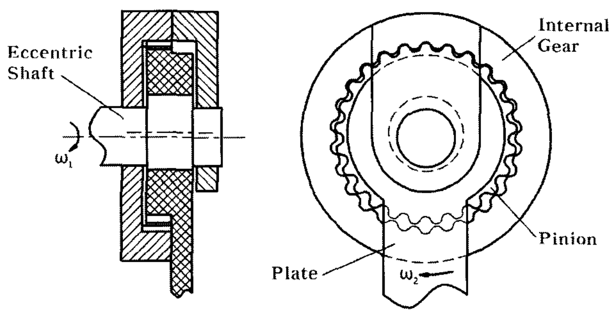

Figure 1 Two-gear epicyclic gear train.

Rectilinear tooth profile

Involute internal gears have interference problem if the difference in numbers of pinion and gear teeth is less than 5 [3]. In order to maximize the reduction ratio and avoid the teeth interference, a difference in numbers of pinion and gear teeth is to be 1 and the tooth shape is to be

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special type. The rectilinear tooth profile in this study as shown in Figure 2 is constructed by two lines with tooth angle, outside or inside circle, root circle and tooth edge radius at the intersection of the circles and the lines.

The power is transmitted by the arc part of the tooth of pinion slides along the linear side of the tooth of internal gear. These gearing dose not follow the general gear theorem, it tells that the rectilinear tooth profile gearing may have noise and torque ripple problems in the application that high speed is required [5].

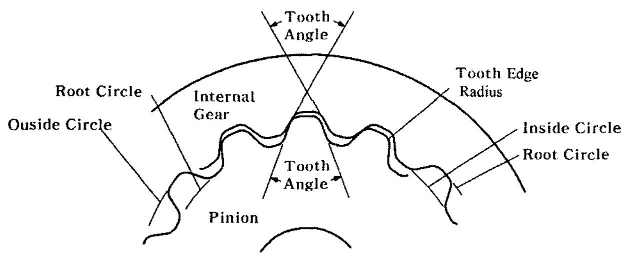

Figure 2 Rectilinear tooth profile.

Displacement analysis

Schematic representation for the kinematic analysis of the planocentric gear is shown in Figure 3. The internal gear is fixed and its center is O_1. The eccentric length of the eccentric shaft is crank r_1, One shaft of the eccentric shaft is revolute joint of O_1 and the other is O_2. The pinion is represented by its center O_2, the length r_2 between the center of the pinion and center A of the gear tooth arc, and the radius r_3 of the gear tooth arc. And point P is the point of contact between the arc portion of pinion gear tooth and the linear portion of internal gear tooth. The equation of the arc is

$$(x - r_1\cos\theta_1 - r_2\cos\theta_2)^2 + (y - r_1\sin\theta_1 - r_2\sin\theta_2)^2 = r_3^2 \tag{1}$$

And the equation of the linear portion of internal gear tooth can be written as $y+ax+b$. From the tangent condition

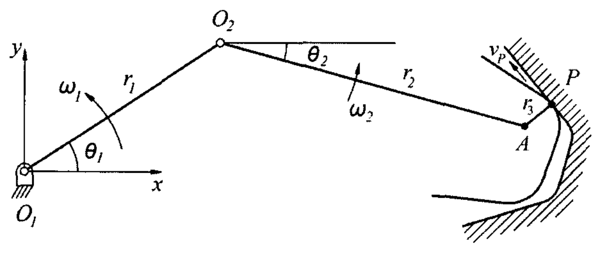

Figure 3 Equivalent-linkage for the two-gear epicyclic gear train.

between the arc of pinion gear tooth and the line of internal gear tooth,

$$\begin{aligned}&(r_1\cos\theta_1 + r_2\cos\theta_2 + r_1\sin\theta_1 + r_2\sin\theta_2 - b)^2 \\ &-(1+a^2)\left((r_1\cos\theta_1 + r_2\cos\theta_2)^2 + (r_1\sin\theta_1 + r_2\sin\theta_2 - b)^2 - r_3^2\right) = 0\end{aligned} \tag{2}$$

From the above equation, θ_2 is decided with regards to θ_1. Then the point P becomes

$$P_x = r_1\cos\theta_1 + r_2\cos\theta_2 + r_3\cos\left(\tan^{-1}(-1/a)\right) \tag{3}$$

$$P_x = r_1\sin\theta_1 + r_2\sin\theta_2 + r_3\sin\left(\tan^{-1}(-1/a)\right) \tag{4}$$

Velocity analysis

Contact point P on the arc of gear tooth of the pinion slides along the linear portion of the internal gear tooth. By letting the sliding velocity be v_P as shown in Figure 3, we have

$$\varpi_1 \times \overrightarrow{O_1O_2} + \varpi_2 \times \overrightarrow{O_2P} = v_P\left(-\cos\left(\tan^{-1}(a)\right)i - \sin\left(\tan^{-1}(a)\right)j\right) \tag{5}$$

From the above equation, we obtain

$$\varpi_2 = -\frac{r_1\varpi_1}{\overline{O_2P}}\left(\frac{\cos\theta_1 + a\sin\theta_1}{\cos\theta_2 + a\cos\theta_2}\right) \tag{6}$$

$$v_P = \frac{r_1\varpi_1\sin\theta_1 + \overline{O_2P}\cdot\varpi_2\sin\theta_2}{\cos\left(\tan^{-1}(a)\right)} \tag{7}$$

Friction in a journal bearing

In order to include the friction force in static analysis of the mechanism, the following equation for the friction couple at joint is used [4] :

$$C_{ji} = \frac{\mu R}{\sqrt{1+\mu^2}}\sqrt{F_{jx}^2 + F_{jy}^2}\, sign(\varpi_j - \varpi_i) \tag{8}$$

where C_{ji} is friction couple, μ is friction coefficient, R is journal radius, F is force exerted on the journal, and the sign function is defined as follows:

$$sign = \begin{cases} -1, & if(\omega_j - \omega_i)\langle 0 \\ 0, & if(\omega_j - \omega_i) = 0 \\ +1, & if(\omega_j - \omega_i))0 \end{cases} \tag{9}$$

Where ω_i and ω_j are the angular velocities of links i and j, respectively. The difference in the angular velocities ω_i-ω_j, is an indication of the motion of link i relative to link j.

As the sign of the friction couple is changed when the power is transmitted forward or backward, the equilibrium equations for the mechanism have to be derived for each motion.

Forward static analysis

Free bodies of the moving links are shown in Figure 4. The crank is driven by the input torque T and the external pinion gear produces output torque Q. The normal force F is exerted by the fixed internal gear on pinion gear. The angular displacement of link O_2P is expressed by θ'_2 and the length r'_2 of the link O_2P is obtained from the above displacement analysis. From static equilibriums of the bodies, the following equations are obtained.

$$F_{01x} + F_{21x} = 0 \tag{10}$$

$$F_{01y} + F_{21y} = 0 \tag{11}$$

$$T + C_{01} + C_{21} + F_{21y} r_1 \cos\theta_1 - F_{21x} r_1 \sin\theta_1 = 0 \tag{12}$$

$$F_{12x} - F\sin\alpha - \mu F\cos\alpha = 0 \tag{13}$$

$$F_{12y} + F\cos\alpha - \mu F\sin\alpha = 0 \tag{14}$$

$$C_{12} + Q + \mu F r_2' \sin(\theta_2' - \alpha) + r_2' F\cos(\theta_2' - \alpha) = 0 \tag{15}$$

In this case, ω_0, and when $\omega_1>0$, $\omega_2<0$. If R_1 and R_2 are the journal radiuses, μ_1 and μ_2 are the coefficients of friction of the journals of joint O_1 and O_2 respectively, then

$$C_{01} = -\frac{R_1\mu_1}{\sqrt{1+\mu_1^2}}\sqrt{F_{01x}^2 + F_{01y}^2} \tag{16}$$

$$C_{21} = -\frac{R_2\mu_2}{\sqrt{1+\mu_2^2}}\sqrt{F_{21x}^2 + F_{21y}^2} \tag{17}$$

From the above equations,

$$Q/T = -\frac{R_2\mu_2\sqrt{1+\mu^2}/\sqrt{1+\mu_2^2} + \mu r_2'\sin(\theta_2' - \alpha) + r_2'\cos(\theta_2' - \alpha)}{\sqrt{1+\mu^2}\left(R_1\mu_1/\sqrt{1+\mu_1^2} + R_2\mu_2/\sqrt{1+\mu_2^2}\right) - \mu r_1\sin(\theta_1 - \alpha) - r_1\cos(\theta_1 - \alpha)} \tag{18}$$

Output torque Q of the pinion is determined for the input torque T of the eccentric shaft. And the instantaneous efficiency becomes

$$\eta = \left(Q/T\right)\frac{r_1\cos(\theta_2 - \alpha)}{r_2'\cos(\theta_2' - \alpha)} \tag{19}$$

Backward static analysis

Free body diagrams for the backward static analysis requires the change of directions of ω_1, ω_2 and μF in Figure 4. Torque T of the crank is required for a backward input torque Q for the equilibrium. From the static equilibriums of the bodies, the following equations are obtained.

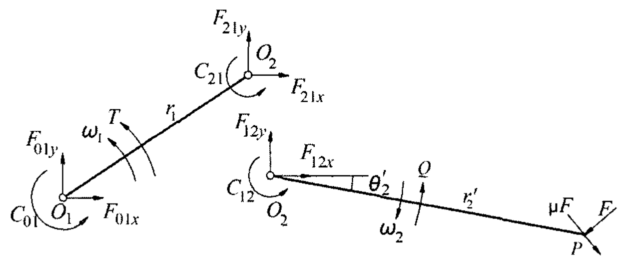

Figure 4 Free-body diagram of the moving members.

$$F_{01x} + F_{21x} = 0 \tag{20}$$

$$F_{01y} + F_{21y} = 0 \tag{21}$$

$$T + C_{01} + C_{21} + F_{21y} r_1 \cos\theta_1 - F_{21x} r_1 \sin\theta_1 = 0 \tag{22}$$

$$F_{12x} - F\sin\alpha + \mu F\cos\alpha = 0 \tag{23}$$

$$F_{12y} + F\cos\alpha + \mu F\sin\alpha = 0 \tag{24}$$

$$C_{12} + Q - \mu F r_2' \sin(\theta_2' - \alpha) + r_2' F\cos(\theta_2' - \alpha) = 0 \tag{25}$$

In this backward case, $\omega_0=0$, and when $\omega_2>0$, $\omega_1<0$. Then the friction couples become

$$C_{01} = \frac{R_1\mu_1}{\sqrt{1+\mu_1^2}}\sqrt{F_{01x}^2 + F_{01y}^2} \tag{26}$$

$$C_{21} = \frac{R_2\mu_2}{\sqrt{1+\mu_2^2}}\sqrt{F_{21x}^2 + F_{21y}^2} \tag{27}$$

From the above equations,

$$T/Q = -\frac{\sqrt{1+\mu^2}\left(R_1\mu_1/\sqrt{1+\mu_1^2} + R_2\mu_2/\sqrt{1+\mu_2^2}\right) - \mu r_1\sin(\theta_1 - \alpha) + r_1\cos(\theta_1 - \alpha)}{R_2\mu_2\sqrt{1+\mu^2}/\sqrt{1+\mu_2^2} + \mu r_2'\sin(\theta_2' - \alpha) - r_2'\cos(\theta_2' - \alpha)} \tag{28}$$

Output torque T of the eccentric shaft is determined for the input torque Q of the pinion. And if $T/Q<0$, then the planocentric gear system is in the condition of friction lock.

Numerical example

Table 1 shows the dimensions of a planocentric gear for the numerical example.

Table 1 Dimensions of the two-gear epicyclic gear train for the numerical example

	Pinion	Internal Gear
Number of teeth	25	26
Whole depth	2 mm	1.5 mm
Diameter of outside/inside circle	43.8 mm	43 mm
Diameter of root rircle	39.8 mm	46 mm
Tooth angle	40°	60°
Radius of tooth edge	1 mm	1 mm
Journal radius	10 mm	10 mm
Crank length	1 mm	

Figure 5 is the plot of the data which resulted from the velocity analysis. The initial angle -7.045° of the eccentric shaft is the condition which arc of pinion tooth is starting to contact on the line of internal gear and the ending angle 6.801° is the condition which the arc of the pinion is leaving the contact from the line of internal gear. The interval is 360°/26 where 26 is the number of internal gear teeth. The reduction ratio varies periodically with the eccentric shaft angle and the number of cycles over the intervals of the shaft angle from 0° to 360° is equal to the number of internal gear teeth. This tells that the epicyclic gear train has an inherent noise source and torque ripple with a frequency proportional to the input shaft frequency.s

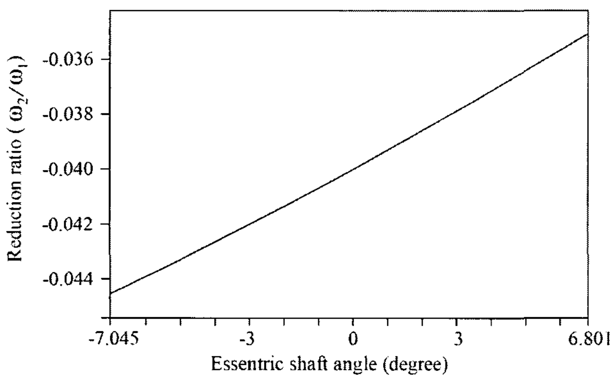

Figure 5 Analysis of reduction ratio.

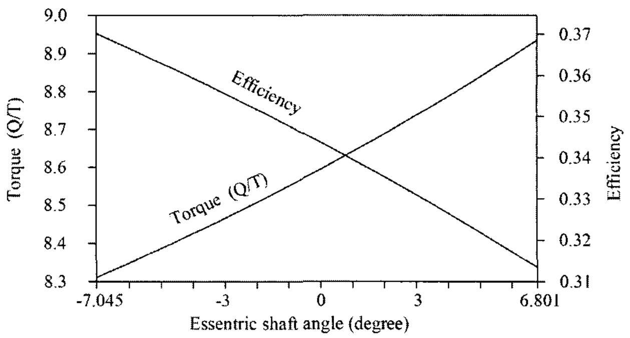

Figure 6 Analysis of torque and efficiency.

Figure 6 is the plot of the data which resulted from the forward static analysis. The torque and efficiency vary same pattern as the above reduction ratio.

Figure 7 is the plot of the data which resulted from the backward static analysis. The friction coefficients μ_1 and μ_2 are determined form the condition T/Q=0 from equation (28). If the system requires the lock, the coefficients of the journals are to be chosen in the area of friction lock in Figure 7.

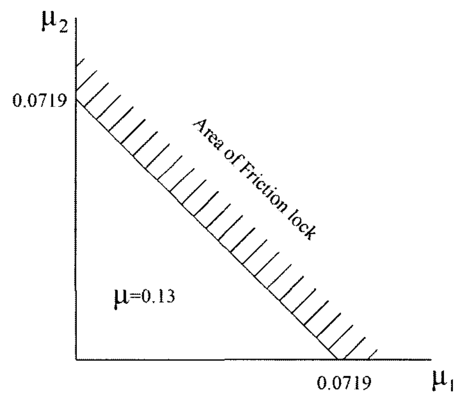

Figure 7 Analysis of friction lock.

Conclusions

The planocentric gear with rectilinear tooth profile is investigated. This gearing system with the simple tooth profile has high speed reduction ratio with simple structure and is used to the application of self-locking required. And this paper presents the static analysis for the friction locking conditions and the transmission efficiency of the planocentric gear with the rectilinear tooth profile. The numerical example in this work finds friction lock and efficiency for the various friction conditions.

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