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FINDING THE INTERSECTION POINT OF A NONPARAMETRIC SURFACE AND A LINE IN R^3

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ABSTRACT. We suggest Bisection method, Fixed point method and Newton's method for finding the intersection point of a nonparametric surface and a line in R^3 and apply ray-tracing in Color Picture Tube or Color Display Tube.

1. INTRODUCTION

In one dimensional equation $f(x) = 0$, the root finding methods are well developed [1,2,3,4,5]. Bisection method, Fixed point method and Newton's method are usually applied for finding solutions of the equation $f(x) = 0$. These methods are also applied to find the intersection point of a surface and a line.

2. BISECTION METHOD

The following recurrence is preferred for its numerical stability and efficiency in computation[6].

$$(2.1)\quad M(t|t_{i-k},\cdots,t_i) = \frac{t_i - t}{t_i - t_{i-k}} * M(t|t_{i-k+1},\cdots,t_i) + \frac{t - t_{i-k}}{t_i - t_{i-k}} * M(t|t_{i-k},\cdots,t_{i-1})$$

where the recurrence relation is started with

$$M(t|t_{i-1},t_i) = \begin{cases} \dfrac{1}{t_i - t_{i-1}}, & t_{i-1} \le t < t_i \\ 0, & \text{otherwise} . \end{cases}$$

Let $B_i(t) = M(t|t_{i-4},\cdots,t_i)$. Then we find the bicubic B-spline surface

$$(2.2)\qquad S(x,y) = \sum_{i=1}^{m}\sum_{j=1}^{n} a_{ij} B_i(x) B_j(y).$$

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The nonparametric surface $z = f(x, y)$ have polynomial surfaces $z = \sum_i \sum_j a_{ij} x^i y^j$, B-spline surface $z = \sum_i \sum_j b_{ij} B_i(x) B_j(y)$ which is of the form (2.2) and other type of surfaces. The equation of a line through (x_s, y_s, z_s) with direction vector (drx, dry, drz) is given by

$$\frac{x - x_s}{drx} = \frac{y - y_s}{dry} = \frac{z - z_s}{drz} = t. \tag{2.3}$$

The arbitrary point (x, y, z) of the equation (2.3) is

$$\begin{aligned} x &= x_s + tdrx \\ y &= y_s + tdry \\ z &= z_s + tdrz. \end{aligned} \tag{2.4}$$

Substituting these values into $z = f(x, y)$, we can change the problem into one dimensional problem for t. Let $g(t) = z_s + tdrz - f(x_s + tdrx, y_s + tdry)$. Then we can apply the Bisection method to the equation $g(t) = 0$.

3. Fixed point method

Substituting the equation (2.4) into the equation $z = f(x, y)$ gives

$$z_s + tdrz = f(x_s + tdrx, y_s + tdry).$$

Solving for t, we get

$$t = \frac{f(x_s + tdrx, y_s + tdry) - z_s}{drz}.$$

Let $g(t) = \frac{f(x_s+tdrx, y_s+tdry) - z_s}{drz}$ and define $t_{n+1} = g(t_n)$, with $t_0 = 0$.

Iterating this process, we can find t^* such that $t^* = g(t^*)$.
Differentiating with respect to t, we get

$$\begin{aligned} g'(t) &= \left(\tfrac{df}{dx}\tfrac{dx}{dt} + \tfrac{df}{dy}\tfrac{dy}{dt}\right)/drz \\ &= \left(\tfrac{df}{dx} drx + \tfrac{df}{dy} dry\right)/drz. \end{aligned}$$

If $|g'(t)| < 1$ then g converges.

4. Newton's method

Given a starting point (x_0, y_0) of the surface $z_0 = f(x_0, y_0)$, we can find the tangent plane and get the intersection point of that plane and a line. That is easy since the tangent plane equation at the point (x_0, y_0, z_0) is

$$z - z_0 = f_x(x_0, y_0)(x - x_0) + f_y(x_0, y_0)(y - y_0) \tag{4.1}$$

and the line equation is

$$\frac{x - x_s}{drx} = \frac{y - y_s}{dry} = \frac{z - z_s}{drz} = t. \tag{4.2}$$

Substituting the equation (4.2) into the equation (4.1), we obtain

$$z_s + tdrz - f(x_0, y_0) = f_x(x_0, y_0)(x_s + tdrx - x_0) + f_y(x_0, y_0)(y_s + tdry - y_0).$$

Solving for t, we get

$$t_1 = \frac{(x_s - x_0)f_x(x_0, y_0) + (y_s - y_0)f_y(x_0, y_0) - z_s + f(x_0, y_0)}{drz - f_x(x_0, y_0)drx - f_y(x_0, y_0)dry}.$$

Starting with point (x_0, y_0, z_0), we generate a sequence

$$\begin{aligned} x_1 &= x_s + t_1 drx \\ y_1 &= y_s + t_1 dry \\ z_1 &= z_s + t_1 drz. \end{aligned}$$

In general,

$$\begin{aligned} s_n &= (x_s - x_{(n-1)})f_x(x_{(n-1)}, y_{(n-1)}) + (y_s - y_{(n-1)})f_y(x_{(n-1)}, y_{(n-1)}) \\ t_n &= \frac{s_n - z_s + f(x_{(n-1)}, y_{(n-1)})}{drz - f_x(x_{(n-1)}, y_{(n-1)})drx - f_y(x_{(n-1)}, y_{(n-1)})dry} \\ x_n &= x_s + t_n drx \\ y_n &= y_s + t_n dry \\ z_n &= z_s + t_n drz \end{aligned}$$

where $n = 2, 3, 4, \cdots$.

The stopping-technique in Newton's method is that select a tolerance $\epsilon > 0$ and constructs $x_1, x_2, x_3, \cdots, x_n$ until

$$|z_{(n)} - f(x_{(n-1)}, y_{(n-1)})| < \epsilon.$$

5. Numerical Experiments

In Color Picture Tube(for TV usage) and Color Display Tube((for Monitor usage), the ray-tracing occurs in Exposure Process. The mechanism consists of Lamphouse, Correction Lens, Filter, Shadow Mask, Panel as in Figure 1.

The Lamphouse is the source of light. The surface type for correction lens is polynomial or B-spline surface, that of Filter is flat plane, that of Shadow mask is a polynomial and that of the panel is a polynomial.

Table 1. Computing time according to ray-tracing points

number of points	Bisection Method	Newton's Method
81	2 minites	0.6 minites
221	6 minites	2 minites
289	7.2 minites	2.5 minites

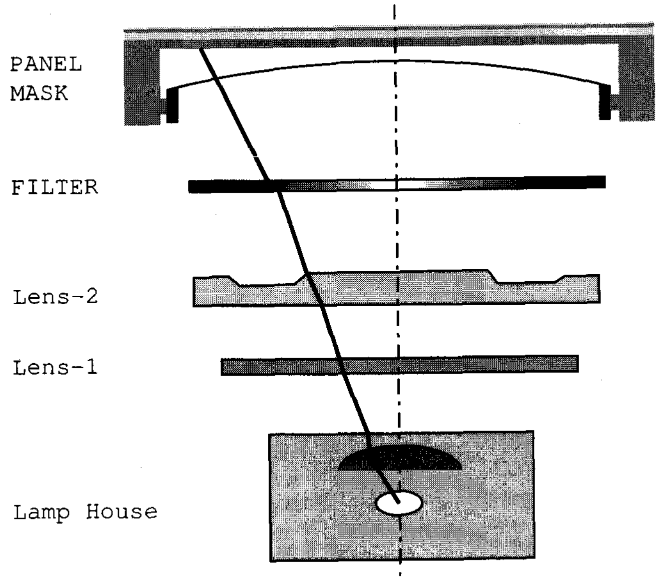

FIGURE 1. Exposure Process

In ray-tracing of landing error corrections, the intersection of a line and a surface(including plane) is found by a Bisection method up to now. In SAMSUNG SDI, ray-tracing program was developed by physicists and not updated because its accuracy was important and the root finding problem was not seriously suggested. In this paper, we study that problem and test. Measurement points of landing errors are 81 points, for example. As in Table 1, it requires much time for computation. Application of Newton's method reduces the computing time by approximately 35% .

6. Conclusion

We apply the root finding methods to find the intersection point of a nonparametric surface and a line, and to ray-tracing in Color Picture Tube or Color Display Tube.

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