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ON RECURRENT SPACE-LIKE COMPLEX HYPERSURFACES OF A SEMI-DEFINITE COMPLEX SPACE FORM

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ABSTRACT. The purpose of this paper is to study some properties of n-dimensional recurrent space-like complex hypersurfaces in an $(n+1)$-dimensional semi-definite complex space form $M^{n+1}_{0+t}(c)$, $t = 0$ or 1, $c \neq 0$.

1. Introduction

Theory of semi-definite complex submanifolds of a semi-definite complex space form is one of the most interesting topics in complex differential geometry and it has been investigated by many geometers from the various points of view ([1]-[5] and [9]).

Now, let us denote by M a semi-definite Kaehler manifold. We denote by R the Riemannian curvature tensor on M. Then M is said to be *semi-symmetric* if it satisfies the condition R(X,Y)R=0 for any vector field X and Y on M. Cartan introduced the notion of semi-symmetric Riemannian spaces and Szabó [12] studied systematically in detail the manifold structure.

On the other hand, in [7] Kobayashi and Nomizu have introdeced the notion of recurrent tensor field of type (r, s) on a manifold M with a linear connection. That is, a non-zero tensor field K of type (r, s) on M is said to be *recurrent* if there exists a 1-form α such that $\nabla K = K \otimes \alpha$, where ∇ denotes the Kaehler connection of M. Besides, in

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[11] Suh classified real hypersurfaces in a complex space form which has η-recurrent second fundamental tensor.

In their paper [5], Choi, the present author and Suh proved the following theorem.

THEOREM A. *Let $M = M_s^n$ be an n-dimensional semi-symmetric and semi-definite complex hypersurface of index $2s$ in semi-definite complex space form $M_{s+t}^{n+1}(c)$, $0 \leqq s \leqq n$, $t = 0$ or 1, $c \neq 0$. Then M is either totally geodesic with $r = n(n+1)c$ or Einstein with $r = n^2 c$, where r denotes the scalar curvature.*

On the other hand, Ryan [10] proved the following theorem.

THEOREM B. *Let M be an n-dimensional complex hypersurface of an $(n+1)$-dimensional complex space form $\overline{M} = M^{n+1}(c)$ $(c \neq 0)$. Then the following conditions are equivalent :*

(1) *RR=0 on M ;*
(2) *RS=0 on M ;*
(3) *M is Einstein ;*
(4) *S is parallel ;*
(5) *R is parallel ;*
(6) *M is totally geodesic or $c > 0$ and M is locally the complex quadric Q^n (globally, if M is complete and $\overline{M} = CP^{n+1}(c)$).*

The purpose of this paper is to investigate some properties of semi-definite complex submanifolds immersed in a semi-definite complex space form and determine recurrent space-like complex hypersurfaces of semi-definite complex space form.

By Theorem A, we prove the following theorem.

THEOREM 1. *Let M' be an $(n+1)$-dimensional semi-definite complex space form $M_{0+t}^{n+1}(c)$ of index $2t$, $t = 0$ or 1, $n \geqq 2$ and of constant holomorphic sectional curvature $c(\neq 0)$ and let M be a space-like complex hypersurface. If M is recurrent, then it is locally symmetric.*

2. Semi-definite Kaehler manifolds

Let M be a complex $m(\geqq 2)$-dimensional semi-definite Kaehler manifold equipped with semi-definite Kaehler metric tensor g and almost complex structure J. For the semi-definite Kaehler structure $\{g, J\}$, it follows that J is integrable and the index of g is even, say $2q$ $(0 \leqq q \leqq m)$. In such a case M can be denoted by M_q^m. The index q is contained in the range $0 < q < m$, M is called an *indefinite Kaehler manifold* and the structure $\{g, J\}$ is called an *indefinite Kaehler structure* and in particular, in the case where $q = 0$ or m, M is only called a *Kaehler manifold*, and then the structure $\{g, J\}$ is called a *Kaehler structure*. We can choose a local field $\{E_A, E_{A^*}\} = \{E_1, \dots, E_m, E_{1^*}, \dots, E_{m^*}\}$ of orthonormal frames on a neighborhood of M, where $E_{A^*} = JE_A$ and $A^* = m + A$. Here the indices $A, B, \dots$ run from 1 to m. We set

$$U_A = \frac{1}{\sqrt{2}}(E_A - iE_{A^*}), \quad \overline{U}_A = \frac{1}{\sqrt{2}}(E_A + iE_{A^*}),$$

where i denotes the imaginary unit. Then $\{U_A\}$ constitutes a local field of unitary frames on the neighborhood of M. This is a complex linear frame which is orthonormal with respect to the semi-definite Kaehler metric, that is, $g(U_A, \overline{U}_B) = \varepsilon_A \delta_{AB}$, where

$$\varepsilon_A = -1 \text{ or } 1, \text{ according as } 1 \leqq A \leqq q \text{ or } q + 1 \leqq A \leqq m.$$

Let $\{\omega_A\}$ be the dual coframe field with respect to the local field $\{U_A\}$ of unitary frames on the neighborhood of M. Then $\{\omega_A\} = \{\omega_1, \dots, \omega_m\}$ consists of complex-valued 1-forms of type $(1,0)$ on M such that $\omega_A(U_B) = \varepsilon_A \delta_{AB}$ and $\{\omega_A, \overline{\omega}_A\} = \{\omega_1, \dots, \omega_m, \overline{\omega}_1, \dots, \overline{\omega}_m\}$ are linearly independent. The semi-definite Kaehler metric g of M can be expressed as $g = 2\sum_A \varepsilon_A \omega_A \otimes \overline{\omega}_A$. Associated with the frame field $\{U_A\}$, there exist complex-valued forms ω_{AB}, which are usually called *connection forms* on M such that they satisfy the structure equations of M ;

$$\begin{aligned} &d\omega_A + \sum_B \varepsilon_B \omega_{AB} \wedge \omega_B = 0, \quad \omega_{AB} + \overline{\omega}_{BA} = 0, \\ &d\omega_{AB} + \sum_C \varepsilon_C \omega_{AC} \wedge \omega_{CB} = \Omega_{AB}, \\ &\Omega_{AB} = \sum_{C,D} \varepsilon_C \varepsilon_D R_{\bar{A}BC\bar{D}} \omega_C \wedge \overline{\omega}_D, \end{aligned} \tag{2.1}$$

where $\Omega = (\Omega_{AB})$ (resp. $R_{\bar{A}BC\bar{D}}$) denotes the curvature form (resp. the components of the semi-definite Riemannian curvature tensor R) of M. The equation (2.1) implies the skew-Hermitian symmetry of Ω_{AB}, which is equivalent to the symmetric condition $R_{\bar{A}BC\bar{D}} = \overline{R}_{\bar{B}AD\bar{C}}$. Moreover, the first Bianchi equation $\sum_B \varepsilon_B \Omega_{AB} \wedge \omega_B = 0$ is given by the exterior differential of the first and third equations of (2.1), which implies the further symmetric relations

$$R_{\bar{A}BC\bar{D}} = R_{\bar{A}CB\bar{D}} = R_{\bar{D}CB\bar{A}} = R_{\bar{D}BC\bar{A}}.$$

Now, relative to the frame field chosen above, the Ricci tensor S of M can be expressed as follows ;

$$S = \sum_{A,B} \varepsilon_A \varepsilon_B (S_{A\bar{B}} \omega_A \otimes \overline{\omega}_B + S_{\bar{A}B} \overline{\omega}_A \otimes \omega_B),$$

where $S_{A\bar{B}} = \sum_C \varepsilon_C R_{\bar{C}CA\bar{B}} = S_{\bar{B}A} = \overline{S}_{\bar{A}B}$. The scalar curvature r is also given by $r = 2\sum_A \varepsilon_A S_{A\bar{A}}$. An n-dimensional semi-definite Kaehler manifold M is said to be *Einstein* if the Ricci tensor S is given by

$$S_{A\bar{B}} = \frac{r}{2n} \varepsilon_A \delta_{AB}. \tag{2.2}$$

The components $R_{\bar{A}BC\bar{D}E}$ and $R_{\bar{A}BC\bar{D}\bar{E}}$ of the covariant derivative of the Riemannian curvature tensor R are defined by

$$\begin{aligned}\sum_E \varepsilon_E (R_{\bar{A}BC\bar{D}E} \omega_E + R_{\bar{A}BC\bar{D}\bar{E}} \overline{\omega}_E) = dR_{\bar{A}BC\bar{D}} \\ - \sum_E \varepsilon_E (R_{\bar{E}BC\bar{D}} \overline{\omega}_{EA} + R_{\bar{A}EC\bar{D}} \omega_{EB} + R_{\bar{A}BE\bar{D}} \omega_{EC} + R_{\bar{A}BC\bar{E}} \overline{\omega}_{ED}).\end{aligned}$$

The second Bianchi identity is given by

$$R_{\bar{A}BC\bar{D}E} = R_{\bar{A}BE\bar{D}C}. \tag{2.3}$$

Let M be an m-dimensional semi-definite Kaehler manifold of index $2q$ $(0 \leqq q \leqq m)$. A plane section P of the tangent space T_xM of M at any point x is said to be *non-degenerate* provided that $g_x|_P$ is non-degenerate. It is easily seen that P is non-degenerate if and only if it has a basis $\{X, Y\}$ such that

$$g(X,X)g(Y,Y) - g(X,Y)^2 \neq 0.$$

If the non-degenerate plane P is invariant by the complex structure J, it is said to be *holomorphic*. It is also trivial that the plane P is holomorphic if and only if it contains a vector X in P such that $g(X,X) \neq 0$.

For the non-degenerate plane P spanned by X and Y in P, the sectional curvature $K(P)$ is usually defined by

$$K(P) = K(X,Y) = \frac{g(R(X,Y)Y,X)}{g(X,X)g(Y,Y) - g(X,Y)^2} .$$

The sectional curvature $K(P)$ of the holomorphic plane P is called the *holomorphic sectional curvature*, which is denoted by $H(P)$. The semi-definite Kaehler manifold M is said to be *of constant holomorphic sectional curvature* if its holomorphic sectional curvatures $H(P)$ are constant for all holomorphic planes at all points of M. Then M is called a *semi-definite complex space form*, which is denoted by $M_q^m(c)$ provided that it is of constant holomorphic sectional curvature c, of complex dimension m and of index $2q(\geqq 0)$. It is seen in Barros and Romero [4] and Wolf [13] that the standard models of semi-definite complex space forms are the following three kinds : the semi-definite complex projective space CP_q^m, the semi-definite complex Euclidean space C_q^m or the semi-definite complex hyperbolic space CH_q^m, according as $c > 0$, $c = 0$ or $c < 0$. For any integer q $(0 \leqq q \leqq m)$ it is also seen by [13] that they are complete simply connected and connected semi-definite complex space forms of dimension m and of index $2q$. The Riemannian curvature tensor $R_{\bar{A}BC\bar{D}}$ of $M_q^m(c)$ is given by

$$R_{\bar{A}BC\bar{D}} = \frac{c}{2}\varepsilon_B\varepsilon_C(\delta_{AB}\delta_{CD} + \delta_{AC}\delta_{BD}).$$

3. Semi-definite complex submanifolds

Let M' be an $(n+p)$-dimensional connected semi-definite Kaehler manifold of index $2(s+t)$ $(0 \leqq s \leqq n,\ 0 \leqq t \leqq p)$ with semi-definite Kaehler structure (g', J'). Let M be an n-dimensional connected semi-definite complex submanifold of M' and let g be the induced semi-definite Kaehler metric tensor of index $2s$ on M from g'. We can choose a local field $\{U_A\} = \{U_j, U_x\} = \{U_1, \ldots, U_{n+p}\}$ of unitary frames on a neighborhood of M' in such a way that, restricted to M, $U_1, \ldots, U_n$ are tangent to M and the others are normal to M. Here and in the sequel, the following convention on the range of indices is used throughout this paper, unless otherwise stated ;

$$A, B, C, \cdots = 1, \ldots, n, n+1, \ldots, n+p \ ;$$
$$i, j, k, l, \cdots = 1, \ldots, n \ ; \quad x, y, z, \cdots = n+1, \ldots, n+p.$$

With respect to the frame field, let $\{\omega_A\} = \{\omega_j, \omega_y\}$ be its dual frame fields. Then the semi-definite Kaehler metric tensor g' of M' is given by $g' = 2\sum_A \varepsilon_A \omega_A \otimes \overline{\omega}_A$, where $\{\varepsilon_A\} = \{\varepsilon_j, \varepsilon_y\}$, $\varepsilon_A = \pm 1$. The connection forms on M' are denoted by $\{\omega_{AB}\}$. The canonical forms ω_A and the connection forms ω_{AB} of the ambient space M' satisfy (2.1). Restricting these forms to the submanifold M, we have

$$\omega_x = 0, \tag{3.1}$$

and the induced semi-definite Kaehler metric tensor g of index $2s$ of M is given by $g = 2\sum_j \varepsilon_j \omega_j \otimes \overline{\omega}_j$. Then $\{U_j\}$ is a local unitary frame field with respect to this metric and $\{\omega_j\}$ is a local dual frame field due to $\{U_j\}$, which consists of complex-valued 1-forms of type $(1,0)$ on M. Moreover $\omega_1, \dots, \omega_n, \overline{\omega}_1, \dots, \overline{\omega}_n$ are linearly independent, and $\{\omega_j\}$ is the canonical forms on M. It follows from (3.1) and Cartan's lemma that the exterior derivative of (3.1) give rise to

$$\omega_{xi} = \sum_j \varepsilon_j h_{ij}{}^x \omega_j, \quad h_{ij}{}^x = h_{ji}{}^x. \tag{3.2}$$

The quadratic form $\sigma = \sum_{i,j,x} \varepsilon_i \varepsilon_j \varepsilon_x h_{ij}{}^x \omega_i \otimes \omega_j \otimes U_x$ with values in the normal bundle NM on M in M' is called the *second fundamental form* of the submanifold M. From the structure equations for M are similarly given by

$$\begin{aligned} &d\omega_i + \sum_j \varepsilon_j \omega_{ij} \wedge \omega_j = 0, \quad \omega_{ij} + \overline{\omega}_{ji} = 0, \\ &d\omega_{ij} + \sum_k \varepsilon_k \omega_{ik} \wedge \omega_{kj} = \Omega_{ij}, \quad \Omega_{ij} = \sum_{k,l} \varepsilon_k \varepsilon_l R_{\bar{i}jk\bar{l}} \omega_k \wedge \overline{\omega}_l. \end{aligned} \tag{3.3}$$

Moreover the following relationships are obtained ;

$$d\omega_{xy} + \sum_z \varepsilon_z \omega_{xz} \wedge \omega_{zy} = \Omega_{xy}, \quad \Omega_{xy} = \sum_{k,l} \varepsilon_k \varepsilon_l R_{\bar{x}yk\bar{l}} \omega_k \wedge \overline{\omega}_l, \tag{3.4}$$

where Ω_{xy} is called the *normal curvature form* of M and $R_{\bar{x}yk\bar{l}}$ are the components of the normal curvature tensor of M. For the Riemannian curvature tensors R and R' of M and M' respectively, it follows from (2.1) and (3.1)-(3.4) that the Gauss and Ricci equations are given by

$$R_{\bar{i}jk\bar{l}} = R'_{\bar{i}jk\bar{l}} - \sum_x \varepsilon_x h_{jk}{}^x \overline{h}_{il}{}^x, \quad R_{\bar{x}yk\bar{l}} = R'_{\bar{x}yk\bar{l}} + \sum_r \varepsilon_r h_{kr}{}^x \overline{h}_{rl}{}^y.$$

The components $S_{i\bar{j}}$ of the Ricci tensor S and the scalar curvature r of M are given by

$$(3.5) \qquad S_{i\bar{j}} = \sum_k \varepsilon_k R'_{\bar{j}ik\bar{k}} - h_{i\bar{j}}{}^2, \quad r = 2(\sum_{k,j} \varepsilon_k \varepsilon_j R'_{\bar{k}kj\bar{j}} - h_2),$$

where we put $h_{i\bar{j}}{}^2 = h_{\bar{j}i}{}^2 = \sum_{x,r} \varepsilon_x \varepsilon_r h_{ir}{}^x \overline{h}_{rj}{}^x$ and $h_2 = \sum_j \varepsilon_j h_{j\bar{j}}{}^2$.

Now, the components $h_{ijk}{}^x$ and $h_{ij\bar{k}}{}^x$ of the covariant derivative of the second fundamental form on M are given by

$$(3.6) \qquad \sum_k \varepsilon_k (h_{ijk}{}^x \omega_k + h_{ij\bar{k}}{}^x \overline{\omega}_k)$$
$$= dh_{ij}{}^x - \sum_k \varepsilon_k (h_{kj}{}^x \omega_{ki} + h_{ik}{}^x \omega_{kj}) + \sum_y \varepsilon_y h_{ij}{}^y \omega_{xy}.$$

Then, substituting $dh_{ij}{}^x$ in this definition into the exterior derivative of (3.2) and using (2.1), (3.1)-(3.3) and (3.6), we have

$$(3.7) \qquad h_{ijk}{}^x = h_{ikj}{}^x, \quad h_{ij\bar{k}}{}^x = -R'_{\bar{x}ij\bar{k}}.$$

In particular, let the ambient space M' be an $(n+p)$-dimensional semi-definite complex space form $M^{n+p}_{s+t}(c)$ of constant holomorphic sectional curvature c and of index $2(s+t)$ $(0 \leqq s \leqq n,\ 0 \leqq t \leqq p)$. Then we get

$$(3.8) \qquad R_{\bar{i}jk\bar{l}} = \frac{c}{2} \varepsilon_j \varepsilon_k (\delta_{ij}\delta_{kl} + \delta_{ik}\delta_{jl}) - \sum_x \varepsilon_x h_{jk}{}^x \overline{h}_{il}{}^x,$$

$$(3.9) \qquad S_{i\bar{j}} = \frac{(n+1)c}{2} \varepsilon_i \delta_{ij} - h_{i\bar{j}}{}^2,$$

$$(3.10) \qquad r = n(n+1)c - 2h_2,$$

$$(3.11) \qquad h_{ij\bar{k}}{}^x = 0.$$

4. Proof of Theorem 1

As an application of semi-symmetric complex hypersurfaces in an semi-definite complex space form this section is devoted to the investigation of recurrent complex hypersurfaces. Let M' be an $(n+1)$-dimensional semi-definite complex space form $M^{n+1}_{0+t}(c)$ of index $2t$, $t = 0$

or 1 and of holomorphic sectional curvature $c(\neq 0)$ and let M be a space-like complex hypersurface. Assume that M is recurrent, namely, there exists a 1-form α such that $\nabla R = R \otimes \alpha$, where ∇ and R are the Kaehler connection and the Riemannian curvature tensor of M.

Let $R_{\bar{i}jk\bar{l}}$ and α_h be the components of R and α with respect to a local complex coordinate system (z^j), respectively. Then we have

$$(4.1) \qquad R_{\bar{i}jk\bar{l}h} = \alpha_h R_{\bar{i}jk\bar{l}}, \quad \text{or} \quad R_{\bar{i}jk\bar{l}\bar{h}} = \alpha_{\bar{h}} R_{\bar{i}jk\bar{l}}.$$

By the assumption (4.1) we see that the components $R_{\bar{i}jk\bar{l}h\bar{r}}$ of $\nabla^2 R$ are given by

$$(4.2) \qquad R_{\bar{i}jk\bar{l}h\bar{r}} = \alpha_h \alpha_{\bar{r}} R_{\bar{i}jk\bar{l}} + \alpha_{h\bar{r}} R_{\bar{i}jk\bar{l}} = (\alpha_h \alpha_{\bar{r}} + \alpha_{h\bar{r}}) R_{\bar{i}jk\bar{l}},$$

where $\alpha_{i\bar{j}}$ denote the components of $\nabla\alpha$. On the other hand, we can show that α is the differential of a differentiable function defined on M. It suffices to show in the case of non-flat. We define the analytic function f by $g(R, R)$. Let M_0 be a subset of M consisting of points x in M such that $f(x) = 0$. Suppose that the interior of M_0 is not empty. Then, by the property of analytic functions, M_0 coincides with whole M, which means that f vanishes identically on M. So M is flat, a contradiction to the assumption that M is not flat. Accordingly the interior of M_0 is empty, from which it follows that $M - M_0$ is dense in M. Because of $\nabla R = R \otimes \alpha$, we have

$$\nabla g(R, R) = 2g(R \otimes \alpha, R) = 2\alpha g(R, R)$$

and hence $df = 2\alpha f$. It implies that $2\alpha = d\, log|f|$ on M since $M - M_0$ is dense. From (4.2), we have

$$R_{\bar{i}jk\bar{l}h\bar{r}} = R_{\bar{i}jk\bar{l}\bar{r}h},$$

which is equivalent to the fact that M is semi-symmetric, that is,

$$R(X, Y)R = 0$$

for any vectors X and Y on M. According to Theorem A, if $c \neq 0$, then M is totally geodesic with $r = n(n+1)c$ or Einstein with $r = n^2 c$. Suppose that M is totally geodesic. Then it is easily seen that it is locally symmetric, because of (3.8). Next, suppose that it is not totally geodesic. By the second Bianchi identity (2.3) and (4.1) we have

$$R_{\bar{i}jk\bar{l}h} = R_{\bar{i}jh\bar{l}k} = \alpha_h R_{\bar{i}jk\bar{l}} = \alpha_k R_{\bar{i}jh\bar{l}}$$

and therefore we have $\alpha_h S_{k\bar{l}} = \alpha_k S_{h\bar{l}}$. Since M is Einstein, using (2.2) we have $r\alpha_h \delta_{kl} = r\alpha_k \delta_{hl}$, which implies that $(n-1)r\alpha_j = 0$. Because $r = n^2 c \neq 0$ and $n \geqq 2$, we obtain $\alpha_j = 0$ for any index j, which means that M is locally symmetric.

EXAMPLE 4.1 ([1], [9]). For any integer p ($\geqq 2$) an indefinite complex hypersurface $M(p, \lambda)$ of a $(2n+1)$-dimensional indefinite complex Euclidean space C_n^{2n+1} of index $2n$ defined as follows ; Let $(z^A) = (z^j, z^{j^*}, z^{2n+1}) = (z^1, \dots, z^{2n+1})$ be a complex coordinate of C_n^{2n+1} and let λ be a complex number such that $|\lambda| = 1$. Then $M(p, \lambda)$ is an indefinite complete complex hypersurface of index $2n$ defined by

$$z^{2n+1} = \sum_j f_j(z^j + \lambda z^{j^*}), \quad j^* = n + j, \quad f_j(z) = z^p.$$

By a simple calculation, we can easily see that the second fundamental form and its covariant derivatives of M are of the forms ;

$$\begin{gathered} h_{ij} = p(p-1)\delta_{ij}{\mu_i}^{p-2}, \quad h_{i^*j} = p(p-1)\lambda\delta_{ij}{\mu_i}^{p-2}, \\ h_{i^*j^*} = p(p-1)\lambda^2\delta_{ij}{\mu_i}^{p-2}, \\ h_{ijk} = p(p-1)(p-2)\delta_{ij}\delta_{ik}{\mu_i}^{p-3}, \\ h_{i^*jk} = p(p-1)(p-2)\lambda\delta_{ij}\delta_{ik}{\mu_i}^{p-3}, \\ h_{i^*j^*k} = p(p-1)(p-2)\lambda^2\delta_{ij}\delta_{ik}{\mu_i}^{p-3}, \\ h_{i^*j^*k^*} = p(p-1)(p-2)\lambda^3\delta_{ij}\delta_{ik}{\mu_i}^{p-3}, \end{gathered}$$

where $\mu_i = z^i + \lambda z^{i^*}$.

REMARK 4.1. In this example, by the property of the second fundamental form, we see that M is recurrent. Furthermore if $p \geqq 3$, then the second fundamental form is not parallel and M is Ricci flat, but not locally symmetric. Accordingly, if $c = 0$, Theorem 1 does not hold.

In the case where the ambient space is a complex projective space, as a direct consequence of Theorem 1, by Theorem B we can prove

THEOREM 4.1. *Let M be an n-dimensional complete complex hypersurface of an $(n+1)$-dimensional complex projective space CP^{n+1}. If M is recurrent, then M is congruent to a complex quadric Q^n or a complex projective space CP^n.*

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