Probabilistic Structural Integrity Assessment of a Reactor Vessel Under Pressurized Thermal Shock

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Abstract

A probabilistic integrity analysis method is presented for a reactor vessel under pressurized thermal shock (PTS) based on Monte Carlo simulation. This method can be applied to the structural integrity assessment of a reactor vessel subjected to pressurized thermal shock where the coolant temperature transient cannot be expressed explicitly as a time function. An axially or circumferentially oriented infinite length surface crack is assumed to be in the beltline weld region of the reactor vessel's inside surface. The random variables are the initial crack depth, neutron fluence on the vessel's inside surface, the copper and nickel content of the vessel materials, $R_{n_{cr}}$, $K_{ic}$, and $K_a$. The reliability of a sample reactor vessel under PTS is assessed quantitatively and the influence of the amount of neutron fluence is also examined by applying the present method.

Key Words: probability, fracture, pressurized thermal shock, monte carlo simulation

1. Introduction

When the Emergency Core Cooling System (ECCS) is actuated due to an emergency such as a Loss of Coolant Accident (LOCA), cold water is injected into the Reactor Coolant System and flows through the irradiated beltline region of the reactor vessel. This causes the unexpectedly severe cooldown of the reactor vessel wall and may result in a crack growth in the vessel wall provided that an unexpected overpressure condition exists; such an event is called Pressurized Thermal Shock (PTS) [1]. The PTS event causes not only a significant reduction in the material fracture toughness but also severe thermal gradient between the inner and the outer side of reactor vessel wall. Additionally, the irradiation embrittlement and high pressure in the reactor vessel may lead to a crack initiation and a reactor vessel failure. Probabilistic structural integrity analysis methodology is a tool to assess the reliability of a reactor vessel under PTS. The uncertainties can be considered explicitly by using probabilistic analysis in which the probabilities of crack initiation or vessel failure are calculated. Jackson et al. [2] calculated the failure probability of a reactor vessel subjected to pressurized thermal shock for the case that the time history of the coolant temperature could be expressed explicitly.

In this paper a probabilistic integrity analysis methodology is presented for a reactor vessel
under pressurized thermal shock in which the coolant temperature transient at the beltline region cannot be expressed explicitly as a time function. Since the limit state equation is implicitly expressed as a function of probabilistic parameters, it is very difficult to obtain an analytical solution of failure probability. The present method is based on Monte Carlo simulation and the failure probability is calculated by repeating deterministic fracture analysis with random values selected for various probabilistic parameters. This method is applied to a sample reactor and the failure probability is calculated quantitatively. The effect of fluence is also evaluated by performing an independent probabilistic calculation at each fluence level.

2. Probabilistic Fracture Analysis Procedure

The probabilistic fracture analysis consists of 3 main analysis steps such as stress analysis, fracture analysis, and probability analysis. The fracture analysis is included in the iterating process of probability analysis, but the stress analysis module is extracted from the iteration process to reduce the calculation time.

2.1. Stress Analysis

To perform probabilistic fracture analysis, the time history of stress distribution in the vessel wall due to the temperature and pressure transient should be estimated. If the stress calculation at each time step is carried out in the Monte Carlo simulation process, the time consumed would be excessive. To avoid this, the stress analysis is carried out before Monte Carlo simulation starts and the stress distribution along the vessel wall at each time step is approximated to a 3rd order polynomial equation, as follows:

\[
s(x) = C_0 + C_1x + C_2x^2 + C_3x^3 \quad (1)
\]

where \(s(x)\) is the stress at distance \(x\) from vessel's inside surface. The coefficients \(C_i\) of the

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Table 2. Nondimensional Stress Intensity Factors for the Circumferentially Oriented Continuous Surface Crack (Ri/Ro=0.91)

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polynomial are stored and used when the stress intensity factor is calculated at that time step. This enables the Monte Carlo simulation process to occur much faster.

2.2. Fracture Analysis

In this paper, infinite cracks such as an axially oriented infinite length surface crack or a circumferentially oriented continuous 360 degree surface crack are assumed and analyzed based on linear elastic fracture mechanics. The stress intensity factor is calculated using the formula given by

\[ K_I = (f_0 C_0 + f_1 C_1 + f_2 C_2 + f_3 C_3) \sqrt{\pi a} \]  (2)

where \( a \) is the crack depth and the coefficients \( C_i \) are from the polynomial equation (1) of stress distribution. The nondimensional stress intensity factors, \( f_i \), for the axial crack were interpolated based on the factors in Table 1 which were extrapolated using those for \( R_c/R_e = 1.25, 1.5, \) and \( 1.75 \) from reference [3], where \( R_e \) and \( R_c \) are the reactor’s inner and outer radius, respectively. The \( f_i \) for the circumferential crack were also interpolated based on the factors in Table 2, which were extrapolated using those for \( R_c/R_e = 0.7, 0.8, \) and \( 0.9 \).

Fracture initiation toughness, \( K_{IC} \), is calculated using \( (K_{IC})_{ASME} \), which is \( K_{IC} \) defined in ASME code[4] and given by

\[ K_{IC} = ERKIC \cdot (K_{IC})_{mean} \]

\[ (K_{IC})_{mean} = 1.43 \times (K_{IC})_{ASME} \]

\[ (K_{IC})_{ASME} = 36.5 + 3.087 \exp(0.036(T-RT_{NDT}+56)) \text{MPa}\sqrt{m} \]

where \( ERKIC \) is the probabilistic parameter of \( K_{IC} \) and sampled from a Gaussian distribution. \( T \) is the time dependent temperature(°C) at the crack tip and \( RT_{NDT} \) is the adjusted reference temperature(°C).

Fracture Arrest Toughness, \( K_{IA} \), is

\[ K_{IA} = ERKIA \cdot (K_{IA})_{mean} \]

\[ (K_{IA})_{mean} = 1.25 \times (K_{IA})_{ASME} \]

\[ (K_{IA})_{ASME} = 29.48 + 1.345 \exp(0.0261(T-RT_{NDT}+83)) \text{MPa}\sqrt{m} \]

where \( ERKIA \) is the probabilistic parameter of \( K_{IA} \).
ERKIC and ERKIA are simulated at each new crack tip position. The factor of 1.43 or 1.25 is multiplied because the ASME values are from the lower bound curve of experimental data[5].

The adjusted reference temperature, RT_{NDT}, is obtained based on Regulatory Guide 1.99, Revision 2[6], and given by

\[ RT_{NDT} = \frac{RT_{NDT0} + \Delta RT_{NDT} + ERRTN(\sigma(RT_{NDT0}) + \sigma(\Delta RT_{NDT}))}{5} \]

where RT_{NDT0} : Initial RT_{NDT}
\( \Delta RT_{NDT} \): RT_{NDT} due to irradiation-induced embrittlement
\( \sigma(RT_{NDT0}) \): 1σ uncertainty for mean value of RT_{NDT0}
- for weld regions = -8.3°C
- for plate regions = -5.3°C
\( \sigma(\Delta RT_{NDT}) \): 1σ uncertainty for the correlation used to predict RT_{NDT}
- for weld regions = -2.2°C
- for plate regions = -8.3°C
ERRTN is probabilistic parameter and sampled from a Gaussian distribution. ERRTN is simulated once per vessel.

2.3. Probability Analysis

The following parameters are probabilistically simulated in this paper.
- initial crack depth
- neutron fluence at the vessel inside surface
- copper contents
- nickel contents
- RT_{NDT}
- K_{IC}
- K_{IO}

Limit state function, Z, which is dependent on the failure criteria is defined in terms of following two conditions:

2.3.1. Crack Initiation

This is the case when the initial crack begins to propagate. The limit state function can be expressed as

\[ Z(x) = K_{IC}(x) - K_{I}(x) \] (6)

where \( K_{IC} \) is fracture initiation toughness and \( K_{I} \) is stress intensity factor at the crack tip. x is the probabilistic parameter and A(x) means that A is a function of the probabilistic parameter(s). The failure probability in this case is

\[ p_f = P(Z < 0) = P(K_{IC} - K_{I} < 0) \] (7)

where P(x) is the probability of outcome x.

2.3.2. Vessel Failure

This is the case when the initial crack propagates through the wall thickness. The limit state function is defined by crack depth as following:

\[ Z(x) = t - a(x) \] (8)

where t is the vessel wall thickness and a is the crack depth as a function of probabilistic parameters. The failure probability in this case is

\[ p_f = P(Z < 0) = P(t - a < 0) \] (9)

Since both the limit state equations are implicitly expressed as a function of probabilistic parameters it is very difficult to obtain an analytical solution of failure probability. In this paper, the Monte Carlo simulation method is used for probabilistic fracture analysis. The Monte Carlo simulation uses an appropriate random number generator to generate independent sample values of each
probabilistic parameter and determine the stress intensity factor $K_i$ and limit state value $Z$ using deterministic fracture mechanics analysis. By repeating this process many times it is possible to simulate the probability distribution of the limit state value $Z$. The flow diagram for the present probabilistic fracture analysis method is shown in Fig. 1.

Whenever a set of the random variables such as initial crack depth, neutron fluence at the vessel inside surface, copper and nickel contents of the vessel materials, $RT_{nor}$, and $ERKIC$ from their respective distributions is generated, a probabilistic reactor vessel model is created. The stress intensity factor at the crack tip which is calculated with the simulated initial crack and the stored stress distribution is compared at every time step with the fracture toughness value which is calculated with the simulated random variables. If the stress intensity factor is higher than the fracture toughness, the failure occur by the first failure criterion (crack initiation). To check the second failure criterion (crack propagation through the wall thickness), the crack is assumed to propagate by small amount of $\Delta a$ and the stress intensity factor is calculated again and compared with the crack arrest toughness that is calculated with simulated ERKIA. If arrest
occurs, the simulation moves to the next step and it is checked whether the re-propagation of the crack occurs or not. If arrest does not occur the crack is assumed to propagate by another $\Delta a$ and the same procedure is repeated.  If the propagated crack depth is equal to or more than the vessel wall thickness the reactor vessel is regarded failed by second failure criterion. The failure probability is calculated with the number of the failure divided by the total simulation number.

3. Sample Problem

Vessel geometry

The reactor vessel used for application of present probabilistic integrity analysis method has the following geometries and materials:
- Internal diameter = 4,394 mm
- Wall thickness of the base metal (SA 508 Class
3) 219 mm
- Cladding thickness(SUS 309L) = 4.8 mm

Loading Conditions

Following two loading conditions are considered as thermal shock transients. The convective heat transfer coefficient for both transients is constant and equal to 1700 W/m²K.

(1) Simplified stylized transient(ST), in which the thermal transient is characterized by a stylized exponentially decaying coolant temperature and given by

\[ T(t) = T_i + (T_e - T_i) \exp(-\beta t) \]  \hspace{1cm} (10)

where
- \( T(t) \): coolant temperature at time \( t \)
- \( T_i \): coolant temperature at \( t=0, (=288°C) \)
- \( T_e \): final coolant temperature, (=6°C)
- \( \beta \): reduction coefficient, (0.15 min⁻¹)

the reactor internal pressure is assumed to be uniform value of 6.9MPa during this transient.

(2) Small break loss of coolant transient(Transient T1)

The time histories of temperature and pressure for this transient are shown in Fig.2 and Fig.3.

Neutron Fluence

Independent analyses for various mean fluence such as followings at the inside surface of the
vessel were performed 
 0.3, 0.5, 1.0, 1.5, 2.0, 2.5, 3.0, 3.5×10^{19} n/cm^2

**Probabilistic Parameters**

The distribution models, mean values, and standard deviations of probabilistic parameters are summarized in Table 3[7].

4. **Analyses and Results**

ANSYS Code[8] was used to calculate the stress distribution along the vessel wall. Plane55 axisymmetric element and equivalent structural element were used for transient heat transfer analysis and thermal stress analysis. The finite element model consists of 14 elements and 30 nodes. The clad was included in the model and consists of 2 elements along the thickness. The transient was divided by 50 time steps for the simplified stylized transient and 1001 steps for the small break loss of coolant transient. Plasticity was taken into account.

Probabilistic integrity analyses for following 4 different cases were carried out. All the cracks are assumed to be in the beltl ine weld region of the reactor vessel’s inside surface.
(1) the axially oriented infinite length surface breaking crack subjected to simplified stylized transient 
(2) the circumferentially oriented continuous 360 degree surface breaking crack subjected to simplified stylized transient 
(3) the axially oriented infinite length surface breaking crack subjected to small break loss of coolant transient 
(4) the circumferentially oriented continuous 360 degree surface breaking crack subjected to small break loss of coolant transient

Probabilistic integrity analysis results based on the present method are shown on Fig.4 through Fig.7.

The analysis results show that the failure probability for the axial crack is higher than that for the circumferential crack for all cases. The failure probability was rapidly increased as the operating time goes by and the axial crack is more sensitive to the fluence than the circumferential crack.

5. **Conclusions**

A probabilistic integrity analysis method was presented based on Monte Carlo simulation for reactor vessel under pressurized thermal shock where the coolant temperature transient cannot be expressed explicitly as a time function. The calculating time could be reduced by extracting the stress analysis module from the Monte Carlo Simulation. The reliability of the reactor vessel under PTS could be evaluated quantitatively by applying this method on a sample reactor vessel. The results show that the failure probability for the axial crack is higher than that for the circumferential crack, and the neutron fluence is a very sensitive factor to failure probability.

**Acknowledgement**

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**References**

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