

## Three-dimensional odd ring dark spatial solitons

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The propagation properties of three-dimensional dark spatial solitons having odd ring formation is analyzed numerically in the frame of the (1+2)-dimensional nonlinear Schrödinger equation and compared with a pair of odd dark solitons. We discuss the experimental excitation condition of an odd ring dark soliton, which is superimposed on a finite-width background beam, with phase masks.

Spatial solitons have provided new methods of controlling light by light and thus become a promising candidate for all-optical processing because they establish self-induced waveguides through the compensation of diffraction by the nonlinear effect. Two kinds of solitons exist under specific boundary conditions, bright solitons under the condition that the field amplitude goes to zero as the transverse distance goes to infinity [1], and dark solitons under the condition that the field amplitude goes to a constant as the transverse distance goes to infinity [2]. It is well-known that bright solitons propagate stationary in self-focusing materials [3] and dark solitons do so in self-defocusing materials [4]. Earlier studies showed that dark solitons are more robust than bright solitons with respect to loss and noise [5], and interaction between adjacent solitons [6]. In addition, dark solitons are not constrained by the transverse spatial dimensions while bright solitons are constrained. So, dark solitons have been excited in the form of stripes and grids in bulk materials with the properties similar to those of (1+1)-dimensional solitons [4].

Considering two transverse dimensions in a bulk medium, circular symmetric solitons may exist even though exact soliton solutions are not known yet, for example, optical vortex solitons and ring dark solitons. Until now, most studies of three-dimensional dark solitons have been oriented to optical vortex solitons which can serve as a self-induced spatial fiber for a weak probe beam to be guided through a dark filament [7]. Recently, Kivshar and Yang showed that three-dimensional dark solitary waves with ring formation can exist in a self-defocusing medium and discussed their dynamic characteristics [8]. Even ring dark solitons were demonstrated with amplitude masks afterwards [9]. And it was shown that the ring dark solitons may be used to guide multiple signal beams simultaneously [10]. In this paper we examine the propagation

properties of odd ring dark solitons and discuss the experimental excitation condition of an odd ring dark soliton, which is superimposed on a finite-width background beam, with phase masks.

We consider the propagation of a monochromatic electric field through a nonlinear medium with an intensity-dependent refractive index,  $n = n_0 + \Delta n = n_0 + n_2|E|^2/2$ , where  $n_0$  is the linear index of refraction and  $n_2$  is the nonlinear refractive index coefficient. Here our concern is limited to materials with self-defocusing nonlinearity ( $n_2 < 0$ ). The propagation is described by the nonlinear Schrödinger equation when the refractive index change is small enough ( $n_0 \gg \Delta n$ ), which is normalized in the form:

$$i \frac{\partial A}{\partial Z} = \frac{1}{2} \nabla_{\perp}^2 A - |A|^2 A \quad (1)$$

where  $E = U \exp(ikz)$  is the electric field,  $A = U/ka(n_0/|n_2|)^{1/2}$  is the normalized field amplitude,  $k = 2\pi n_0/\lambda$  is the wave number,  $\lambda$  is the wavelength in free space,  $Z = z/ka^2$  is the normalized propagation distance, and the transverse coordinates  $(X, Y)$  or  $(\rho, \phi)$  are normalized by the normalization parameter  $a$  which is related to the soliton width. The transverse Laplacian  $\nabla_{\perp}^2 (= \partial^2/\partial X^2 + \partial^2/\partial Y^2 = \partial^2/\partial \rho^2 + \partial/\rho \partial \rho + \partial^2/\rho^2 \partial \phi^2)$  represents the diffraction, and the last term in Eq.(1) represents the nonlinear effect of the spatial-phase modulation which is dependent on the intensity distribution.

Even though the exact soliton solution is not known yet, on the analogy of the well-known (1+1)-dimensional dark soliton, a ring dark soliton with circular symmetry (independent of azimuthal angle  $\phi$ ) may be initiated in the form [8]:

$$A(Z=0, \rho) = A_0(\cos \theta \tanh \zeta + i \sin \theta), \\ \zeta = A_0(\rho - \rho_0) \cos \theta \quad (2)$$

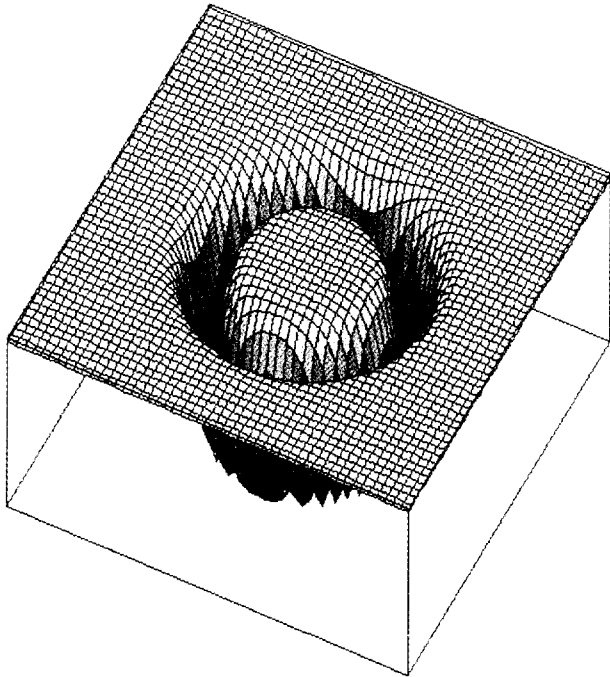


FIG. 1. Typical intensity profile of a ring dark soliton.

where  $A_0$  is a characteristic amplitude. Physically,  $\cos^2 \theta$  ( $|\theta| \leq \pi/2$ ) represents the blackness of the dark soliton through the relation,  $|A|^2 = A_0^2(1 - \cos^2 \theta / \cosh^2 \zeta)$  and  $\rho_0$  is the ring radius of the dark soliton. A typical intensity profile of a ring dark soliton is shown in Fig. 1. We will denote the ring dark solitons for  $\cos^2 \theta = 1$  (the soliton contrast is maximum) and the ring gray solitons for  $\cos^2 \theta < 1$  on the analogy of (1+1)-dimensional dark solitons.

We solve Eq.(1) numerically by using a (1+2)-dimensional beam propagation method. A transverse rectangular mesh with grid spacing of  $\Delta x = \Delta y = 5\mu\text{m}$  is used. We take  $45\mu\text{m}$  as the input soliton width (FWHM) where the change of the intensity-dependent refractive index corresponds to  $\Delta n = 5 \times 10^{-6}$ . Notice that we take here the linear refractive index  $n_0 = 1.0$  and the wavelength  $\lambda = 500\text{nm}$ .

The propagation of a ring dark soliton is shown in Fig. 2 up to the distance of  $z = 30\text{cm}$ . A ring black soliton ( $\theta = 0$ ) with the ring diameter of  $2\rho_0 = 250\mu\text{m}$  is launched (Fig. 2(a)). The evolution is presented as a gray-scale image (Fig. 2(b)) where the ring diameter increases and the soliton contrast decreases (the dark soliton becomes grayer). The intensity profile at the final stage is shown in Fig. 2(c), where a ring gray soliton with the blackness of about 0.3 is observed. We also plot the output intensity profile in a half scale when the nonlinear effect is ignored, that is, for linear diffraction (dotted curve). The diffraction modulates the background field and diffractive ringing of the intensity distribution is observed. The ring radius of the intensity minimum is much smaller than that of the ring dark soliton. So, we can say that the increase of

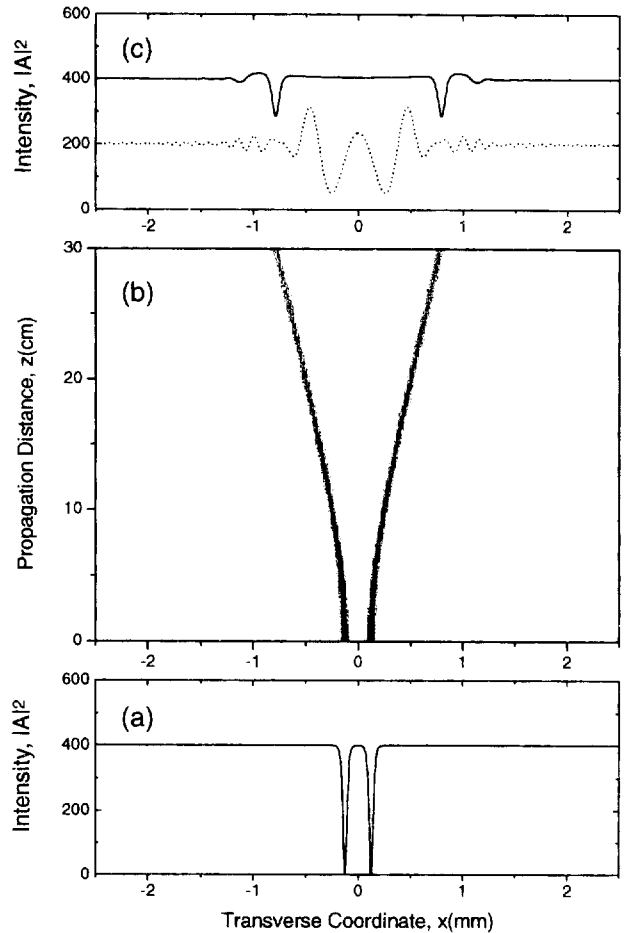


FIG. 2. The propagation of a ring dark soliton up to the distance of  $z = 30\text{cm}$  when the soliton width is  $45\mu\text{m}$  (FWHM) and the ring radius is  $\rho_0 = 125\mu\text{m}$ : (a) the intensity profile of input ring dark soliton, (b) the evolution of ring dark soliton presented as gray-scale image, and (c) the intensity profiles of output ring dark soliton. The change of the intensity-dependent refractive index corresponds to  $\Delta n = 5 \times 10^{-6}$ . Notice we take here the linear refractive index  $n_0 = 1.0$  and the wavelength  $\lambda = 500\text{nm}$ . We also plot the output intensity profile (dotted curve) when the nonlinear effect is ignored, that is, for linear diffraction.

the ring diameter is owing to the self-dynamics of the ring dark soliton. The change of the soliton blackness is explained by the conservation of dark soliton energy, which is a major difference between ring dark solitons and (1+1)-dimensional dark solitons. Fig. 3 shows the variation of the soliton radius (a) and the soliton blackness (b) for three different soliton angle with the same ring radius of  $\rho_0 = 125\mu\text{m}$ :  $\theta = -0.15\pi$  (dotted curves),  $\theta = 0$  (dashed curves),  $\theta = 0.15\pi$  (solid curves). Basically, the ring diameter increases gradually and the ring dark soliton becomes grayer with the propagation distance. But the ring dark soliton may first shrink for positive soliton angle ( $\theta > 0$ ) until the soliton contrast reaches the maximum (i.e., blackness = 1.0) before it diverges again. The initial contraction of the ring dark soliton can be explained by the transverse velocity of

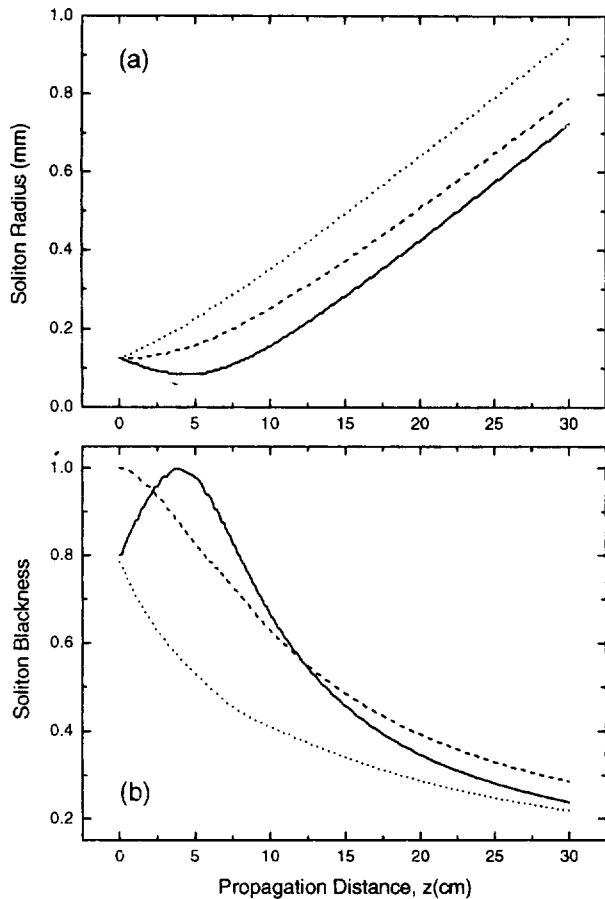


FIG. 3. The variation of the soliton radius (a) and the soliton blackness (b) of three different ring dark solitons with the same ring radius of  $\rho_0 = 125\mu\text{m}$  as a function of the propagation distance:  $\theta = -0.15\pi$  (dotted curves),  $\theta = 0$  (dashed curves),  $\theta = 0.15\pi$  (solid curves).

gray solitons [5]. When the dark soliton shrinks, at the turning point the dark ring changes its radial velocity ( $\equiv d\rho_0(z)/dz$ : the slope of the ring radius) and then it diverges along a shifted trajectory [8]. The radial velocities of the ring dark solitons are nearly constant after the distance of  $z = 10\text{cm}$ .

In practical experiments, an infinite-width background field is not physical. So, we try the numerical simulations for the propagation of the ring dark solitons superimposed on a finite-width background beam which is taken to be a Gaussian beam,  $\exp(-2\rho^2/R_0^2)$  with  $R_0 = 500\mu\text{m}$ . Fig. 4 shows the propagation of the ring dark soliton having the same parameters as the case of Fig. 2 except for the Gaussian background. The beam width increases and its top flattens with the propagation distance due to the self-defocusing non-linearity. The trajectory of the dark ring is similar to that of Fig. 2. but the soliton contrast is much different. We plot in Fig. 5 the variation of the soliton radius (a) and the soliton blackness (b) for three different soliton angles as in the case of Fig. 3:  $\theta = -0.15\pi$  (dotted curves),  $\theta = 0$  (dashed curves),  $\theta = 0.15\pi$

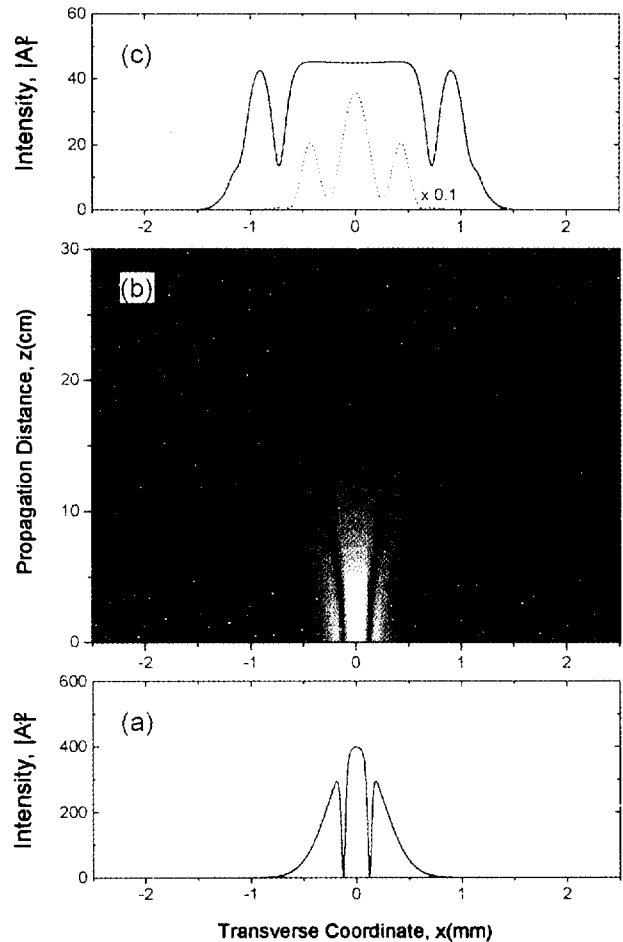


FIG. 4. The propagation of a ring dark soliton with the same parameters as Fig. 1 except for the finite-width background of a Gaussian beam,  $\exp(-2\rho^2/R_0^2)$  with  $R_0 = 500\mu\text{m}$ : (a) the intensity profile of the input ring dark soliton, (b) the propagation of the ring dark soliton presented as a gray-scale image, and (c) the intensity profile of the output ring dark soliton.

(solid curves). Comparing the variation of the soliton radius in Fig. 3 and Fig. 5, the radial velocity becomes smaller for  $\theta \geq 0$  and larger for  $\theta < 0$  when the ring dark soliton is superimposed on a finite-width background. The difference is not so much. However, the variation of the soliton blackness depends strongly on whether the ring dark soliton is imposed on a finite-width background or not. The soliton blackness decreases slowly when the soliton is superimposed on a finite-width background. At the distance of  $z = 30\text{cm}$ , the soliton blackness is about 0.3 in the case of infinite-width background and it is about 0.7 in the case of finite-width Gaussian background. We think that this difference is caused by the broadening of the soliton width which weakens the soliton property. Expansion of the ring dark soliton radius beyond of the width of the background beam gives the peculiar variation of the soliton blackness near the distance of  $z = 25\text{cm}$ .

In addition, we carried out the numerical simulations

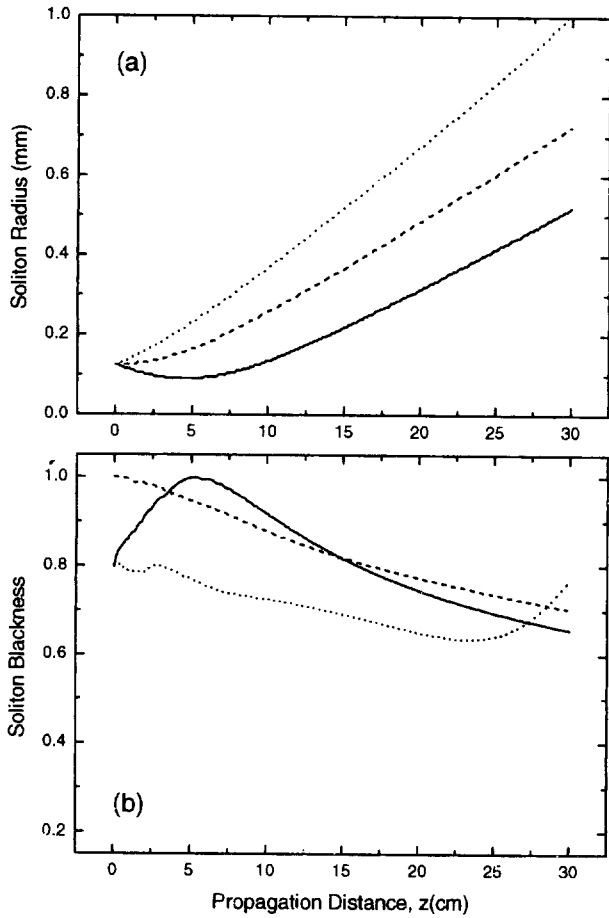


FIG. 5. The variation of the soliton radius (a) and the soliton blackness (b) of three different ring dark solitons with the same ring radius of  $\rho_0 = 125\mu\text{m}$  as a function of the propagation distance when they are superimposed on a Gaussian background beam,  $\exp(-2\rho^2/R_0^2)$  with  $R_0 = 500\mu\text{m}$ :  $\theta = -0.15\pi$  (dotted curves),  $\theta = 0$  (dashed curves),  $\theta = 0.15\pi$  (solid curves).

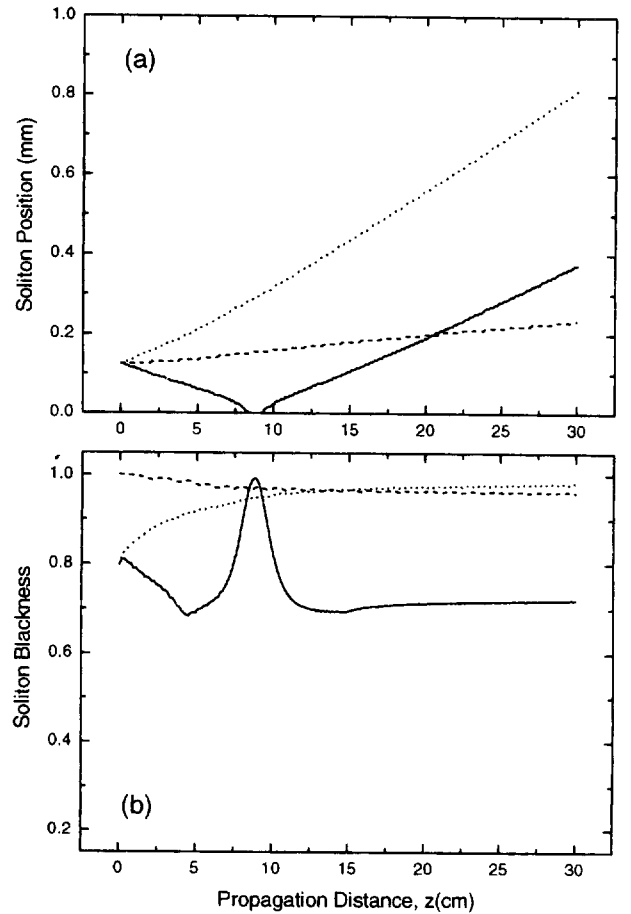


FIG. 6. The variation of (a) the soliton position and (b) the soliton blackness as a function of the propagation distance for the three dark soliton pairs of Eq. (3) with different soliton angles when they are superimposed on a Gaussian background beam,  $\exp(-2\rho^2/R_0^2)$  with  $R_0 = 500\mu\text{m}$ :  $\theta = -0.15\pi$  (dotted curves),  $\theta = 0$  (dashed curves),  $\theta = 0.15\pi$  (solid curves).

of the propagation of an odd dark soliton pair in the form of stripes, which has similar field profiles with

$$A(Z = 0, x) = \begin{cases} A_0(\cos\theta \tanh\zeta + i \sin\theta), \\ A_0(\cos\theta \tanh\zeta - i \sin\theta), \end{cases}$$

where  $2\rho_0$  is equal to the separation distance between two dark stripes. Fig. 6 shows the variation of the soliton position (a) and the soliton blackness (b) when the soliton stripes are superimposed on the Gaussian background. The initial parameters are the same as for the case of Fig. 5. When  $\theta < 0$  the trajectory of the dark soliton stripes is similar to that of the ring dark solitons but the soliton contrast becomes better. However, the trajectory is much different when  $\theta \geq 0$

The transverse velocity of the black soliton stripes is very small compared with that of the ring black soliton and the blackness remains nearly unchanged. An outstanding feature is that the dark soliton stripes col-

ring dark soliton in one transverse direction:

$$\begin{cases} \zeta = A_0(x - \rho_0) \cos\theta, & x \geq 0 \\ \zeta = A_0(x + \rho_0) \cos\theta, & x < 0 \end{cases} \quad (3)$$

lide and cross each other with a shift of the trajectory when  $\theta > 0$  [12]. Such a crossing is not observed in the case of ring dark solitons.

Up to now, ring dark solitons of even type, which are generated by the amplitude modulation of an input beam, have been demonstrated with circular amplitude mask of metal film dots [9] since the concept of the ring dark solitons was first introduced by Kivshar and Yang [8]. We are going to excite ring dark solitons of odd type which are generated by the phase modulation of an input beam. It is necessary to determine the disk size of the phase mask relative to the input laser beam. We proceed with numerical calculations on the

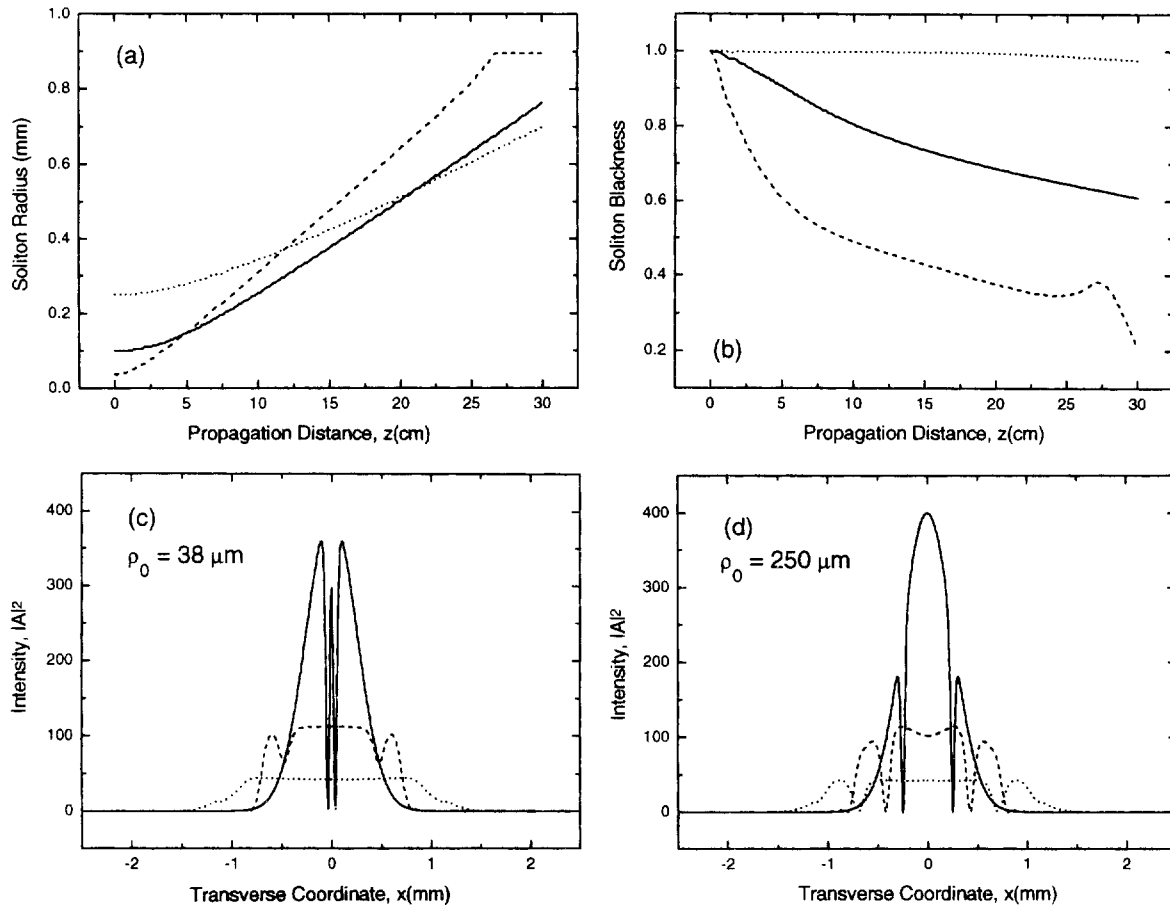


FIG. 7. The variation of (a) the soliton position and (b) the soliton blackness of three ring black solitons ( $\theta=0$ ) with different soliton radii as a function of the propagation distance when they are superimposed on a Gaussian background beam,  $\exp(-2\rho^2/R_0^2)$  with  $R_0 = 500\mu\text{m}$ :  $\rho_0 = 38\mu\text{m}$  (dashed curves),  $\rho_0 = 100\mu\text{m}$  (solid curves),  $\rho_0 = 250\mu\text{m}$  (dotted curves). And we plot the intensity profiles for (c)  $\rho_0 = 38\mu\text{m}$  and (d)  $\rho_0 = 250\mu\text{m}$  at the propagation distances:  $z = 0\text{cm}$  (solid curves),  $z = 15\text{cm}$  (dashed curves), and  $z = 30\text{cm}$  (dotted curves).

propagation of odd ring dark solitons superimposed on the Gaussian background with various value of the ring radius of Eq.(2). Fig. 7 shows the numerical results for three different soliton radii:  $\rho_0 = 38\mu\text{m}$  (dashed curves),  $\rho_0 = 100\mu\text{m}$  (solid curves), and  $\rho_0 = 250\mu\text{m}$  (dotted curves). Notice that the black solitons ( $\theta = 0$ ) are launched and the radius of the Gaussian beam is  $500\mu\text{m}$  ( $e^{-2}$  of the intensity). The variation of the soliton radius and the soliton blackness is shown in Figs. 7(a) and 7(b), respectively. As the soliton radius decreases, the ring dark soliton diverges rapidly and the change of the soliton blackness is large. It is noticeable that the soliton blackness for  $\rho_0 = 250\mu\text{m}$  remains nearly unchanged. The peculiar behavior for  $\rho_0 = 38\mu\text{m}$  observed near the distance of  $z = 25\text{cm}$  is because the dark ring runs out of the background beam. We show the evolution of intensity profiles in Fig. 7(c) and Fig. 7(d) for  $\rho_0 = 38\mu\text{m}$  and  $\rho_0 = 250\mu\text{m}$ , respectively. The propagation distance is  $z = 0\text{cm}$ ,  $15\text{cm}$ , and  $30\text{cm}$  for solid curves, dashed curves, and dotted curves. The intensity profiles for  $\rho_0 = 100\mu\text{m}$

are similar to the case of Fig. 4. When the ring radius is much smaller than the background beam size, the ring dark soliton diverges rapidly and disappears when it exceeds the width of the background beam (Fig. 7(c) if the propagation distance is large. On the other hand, when the ring radius is comparable to the background beam size, the ring dark soliton preserves the soliton properties with a broadened soliton width (Fig. 7(d)). Therefore we can say that phase masks with large disk radius are good for the experimental demonstration of odd ring dark solitons.

In conclusion, we investigated numerically the propagation properties of ring dark solitons. Self-dynamics of the soliton formation makes the ring dark solitons diverge and become grayer with the propagation distance. We observed that the ring dark solitons may disappear when they are excited on a finite-width background beam. Numerical results showed that phase masks with large disk radius comparable to the background beam width are good for the experimental excitation of odd ring dark solitons.

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