Dynamics and Motion Control of an Underactuated Manipulator

비구동 관절을 가지는 매니퓰레이터의 동력학과 운동제어

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요 약 : 본 논문에서는 비구동 관절을 가지는 2링크 매니퓰레이터의 동력학 해석과 운동제어를 제1적분을 기초로 하여 전개하고 있다. 매니퓰레이터의 운동이 제1적분의 적분상수에 의해서 기술되는 것을 보이고, 제1적분을 이용하여 매니퓰레이터의 동력학을 해석하고 있다. 그리고 해석된 동력학을 적극적으로 이용하는 운동제어 알고리즘을 구성하고 시뮬레이션을 통하여 유효성을 확인하고 있다. 끝으로 비구동 관절에 마찰이 작용하는 경우, 브레이크등의 보조수단을 이용하지 않고도 매니퓰레이터의 제어가 가능함을 보이고 있다.

Keywords : dynamics, motion control, underactuated manipulator, non-linear motion, friction

I. Introduction

The underactuated manipulator whose some joints do not have actuator has some interesting features. First, a light weight manipulator can be made by implementing simple hinge or holding brake instead of some joint actuators. The reduction of manipulator weight is strongly required for space robots. Second, the underactuated manipulator easily overcome actuator failure due to unexpected accident. The fault-tolerant control is highly desirable for robots in remote or hazardous environments.

Recently some researches for analysis and control of the underactuated manipulator have been presented[1-6]. In the control of the underactuated manipulator, we should consider the coupling characteristics of manipulator dynamics and that the manipulator is uncontrollable. For more effective motion control of the underactuated manipulator, we have to know the dynamic characteristics of the manipulator. But, in general it is difficult to analyze the dynamics of the manipulator because of its strong non-linear characteristics.

From the past it is known that first integrals of non-linear differential equations give qualitative information about the behavior of the underlying dynamical systems [7-10]. In this paper we present the analysis of the dynamic characteristics of the underactuated two-link manipulator based on the first integral approach. Also simple and effective motion control algorithm of the manipulator is given with numerical example. The first integral approach can make us analyze the dynamics of the underactuated two-link manipulator. As a result we can make simple control algorithm using the dynamic charac- teristics.

In section 2 we derive the equations of motion. In section 3 we find the first integral for the manipulator and illustrate that the motion of the manipulator is described by the integral constant of the first integral. The dynamics in forced motion actuated only at the second joint and the two configurations in free motion are presented. In section 4 a simple motion control algorithm with numerical example is made using the dynamic characteristics analyzed in section 3. And the controllability of the manipulator without any additional equipment such as a brake is discussed under friction effect.

접수일자 : 1997. 3. 19., 수정완료 : 1997. 10. 30.
유기호: 한국과학기술원 전기 및 전자공학과

II. Equations of motion

The two-link manipulator actuated only at the second joint which has planar motion without friction and damping effect is modeled in Fig.1,

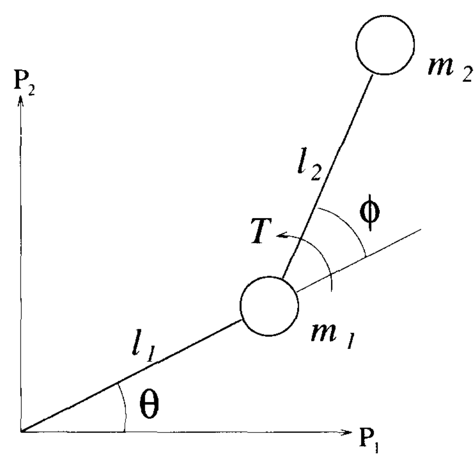

Fig. 1. Model of an underactuated two-link manipulator.

where

ϕ is the angle of the first joint,

θ is the angle of the second joint related to the first link,

$m_1(m_2)$ is the mass of the link 1(link 2),

$l_1(l_2)$ is the length of the link 1(link 2), and

T is the actuated torque at the second joint.

The kinetic energy of the manipulator, $\tilde{E}_k$, can be expressed

$$\tilde{E}_k = \frac{1}{2}(m_1+m_2)l_1^2\left(\frac{d\theta}{dt}\right)^2 + \frac{1}{2}m_2\Big\{l_2^2\Big(\frac{d\theta}{dt} + \frac{d\phi}{dt}\Big)^2 + 2l_1l_2\frac{d\theta}{dt}\Big(\frac{d\theta}{dt}+\frac{d\phi}{dt}\Big)\cos\phi\Big\}. \quad (1)$$

The equations of motion of the manipulator can be written as follows using Lagrangians:

$$\begin{aligned}(1+\varkappa\lambda^2+2\lambda\cos\phi)\ddot{\theta} \\ +(1+\lambda\cos\phi)\ddot{\phi} \\ -\lambda\dot{\phi}(2\dot{\theta}+\dot{\phi})\sin\phi = 0,\end{aligned} \quad (2)$$

$$(1+\lambda\cos\phi)\ddot{\theta}+\ddot{\phi}+\lambda\dot{\theta}^2\sin\phi = \beta, \quad (3)$$

where

$$\tau=\omega t,\quad \omega^2=\frac{g}{l_2},\quad \varkappa=\frac{m_1+m_2}{m_2},\quad \lambda=\frac{l_1}{l_2},\quad \beta=\frac{T}{m_2gl_2},$$

$$\dot{\theta}=\frac{d\theta}{d\tau},\quad \ddot{\theta}=\frac{d^2\theta}{d\tau^2},\quad \dot{\phi}=\frac{d\phi}{d\tau},\quad \ddot{\phi}=\frac{d^2\phi}{d\tau^2},$$

where g is acceleration of gravity. In the same manner, the energy equation (1) has dimensionless form as

$$E_k=\frac{1}{2}\varkappa\lambda^2\dot{\theta}^2+\frac{1}{2}(\dot{\theta}+\dot{\phi})^2+\lambda\dot{\theta}(\dot{\theta}+\dot{\phi})\cos\phi, \quad (4)$$

where

$$E_k \equiv \frac{\tilde{E}_k}{m_2gl_2}.$$

Solving (2) and (3) for $\ddot{\theta}$ and $\ddot{\phi}$ leads to

$$\begin{aligned}\ddot{\theta}= &\frac{1}{\lambda^2(\varkappa-\cos^2\phi)}\{-(1+\lambda\cos\phi)\beta \\ &+\{(\dot{\theta}+\dot{\phi})^2+\lambda\dot{\theta}^2\cos\phi\}\lambda\sin\phi\},\end{aligned} \quad (5)$$

$$\begin{aligned}\ddot{\phi}= &\frac{1}{\lambda^2(\varkappa-\cos^2\phi)}\{(1+\varkappa\lambda^2\cos\phi)\beta \\ &-\{\varkappa\lambda^2\dot{\theta}^2+(\dot{\theta}+\dot{\phi})^2+2\lambda\dot{\theta}(\dot{\theta}+\dot{\phi})\cos\phi \\ &\lambda\dot{\phi}^2\cos\phi\}\lambda\sin\phi\}.\end{aligned} \quad (6)$$

III. Descriptions of motion and dynamics

In this section, we show that the motion of the manipulator can be described schematically by the integral constant of the first integral (it is called the trajectory parameter in this paper). And we show the dynamic characteristics of the manipulator based on the analytic approach by the first integral[10].

1. Descriptions of motion by the first Integral approach

We consider the two-link manipulator as a discrete time system. The control input is taken as a discrete values and the non-dimensional sampling time is chosen appropriately. The control action is started from the time τ_0 applying a constant torque, and switched by other value at the time $\tau_1, \tau_2, \ldots, \tau_n$. Here ϕ_i and E_{k_i} $(i=0, 1, \ldots, n)$ are the angle of the second joint and the kinetic energy of the manipulator respectively at the time τ_i. Also let the interval $\tau_i \leq \tau < \tau_{i+1}$ be control interval i, and β_i is a constant torque at the interval. Then, the kinetic energy of the control interval n is defined as follows:

$$\begin{aligned}E_k= &\beta_n(\phi-\phi_n)+\sum_{i=n-1}^{0}\beta_i(\phi_{i+1}-\phi_i)+E_{k_0} \\ = &\beta_n\phi+c_{k_n},\end{aligned} \quad (7)$$

where

$$c_{k_n}=-\beta_n\phi_n+E_{k_n}. \quad (8)$$

In here, we find the first integral for the second joint equation. Since the first three terms in the parentheses {} of (6) equal two times of the kinetic energy (4), we have

$$\begin{aligned}\ddot{\phi}= &\frac{1}{\lambda^2(\varkappa-\cos^2\phi)}\{(1+\varkappa\lambda^2+2\lambda\cos\phi)\beta \\ &-(2E_k+\lambda\dot{\phi}^2\cos\phi)\lambda\sin\phi\}.\end{aligned} \quad (9)$$

To find the first integral, we consider the following auxiliary equation from (9):

$$\ddot{\phi}=-\frac{\dot{\phi}^2\sin\phi\cos\phi}{\varkappa-\cos^2\phi}. \quad (10)$$

Integrating (10), we have

$$\dot{\phi}=\sqrt{\frac{\varkappa-1}{\varkappa-\cos^2\phi}}\,\dot{\phi}_*, \quad (11)$$

where $\dot{\phi}_*$ is the the angular velocity of the second joint of the manipulator at $\phi=0$. Next, we consider (9) including the neglected terms. Assuming that $\dot{\phi}_*$ is the function of time τ, (11) can be expressed by

$$\dot{\phi}_*(\tau)=\sqrt{\frac{\varkappa-\cos^2\phi}{\varkappa-1}}\,\dot{\phi}. \quad (12)$$

Differentiating (12) and substituting (7) into (12), we have

$$\begin{aligned}\dot{\phi}_*(\phi)\,d\dot{\phi}_*= &\frac{1}{\lambda(\varkappa-1)}\Big\{\frac{(1+\varkappa\lambda^2+2\lambda\cos\phi)\beta_n}{\lambda} \\ &-2(\beta_n\phi+c_{k_n})\sin\phi\Big\}d\phi.\end{aligned} \quad (13)$$

Integrating (13), we obtain

$$\begin{aligned}\frac{1}{2}\dot{\phi}_*(\phi)^2= &\frac{(1+\varkappa\lambda^2+2\lambda\cos\phi)\beta_n\phi+2\lambda c_{k_n}\cos\phi}{\lambda^2(\varkappa-1)} \\ &+c_{t_n},\end{aligned} \quad (14)$$

where c_{t_n} is a constant and determined by

$$\begin{aligned}c_{t_n}= &\frac{1}{\lambda^2(\varkappa-1)}\{(1+\varkappa\lambda^2+2\lambda\cos\phi_n) \\ &(\beta_{n-1}-\beta_n)\phi_n+2\lambda(c_{k_{n-1}}-c_{k_n}) \\ &\cos\phi_n\}+c_{t_{n-1}}.\end{aligned} \quad (15)$$

In (14) since $\dot{\phi}_*$ expresses the forced motion actuated by the torque β_n, we call it the trajectory parameter. Here c_{k_n} and c_{t_n} are determined by (8) and (15) respectively, and then the motion of the second link can be obtained by (14). Fig. 2 shows the plots of (14) for $\beta_i=1, 0, -1$ in case of $\varkappa=2$ and $\lambda=1$. The plots

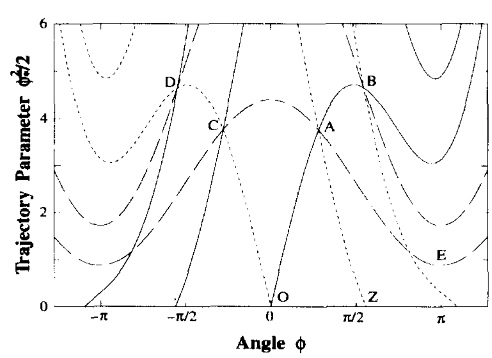

Fig. 2. The variations of the trajectory parameter $\dot{\phi}_*$ in case of $\varkappa=2$, and $\lambda=1$, where the solid line is $\beta_i=1$, the dotted line is $\beta_i=-1$, and the dotted chain is $\beta_i=0$.

represent the forced motion, started from the point O with $\phi_0=0$ and $E_{k_0}=0$, actuated by a constant torque, and then switched by other constant torque at the point A, B, C and D respectively. As long as the actuated torque and switched point are given, we can take the motion trajectory schematically using the figure. As a extreme example, if we assume the mass of the manipulator as $m_2 \gg m_1$, we have to take the very long motion trajectory to reach the any target position such as Z.

On the other hand, according to the conditions for the integrability of the free joint dynamic equation[2], (5) can be integrated twice to

$$\theta = \theta_0 - \frac{\phi-\phi_0}{2} - \frac{1-x\lambda^2}{\sqrt{(1+x\lambda^2)^2-4\lambda^2}} \left\{ \tan^{-1}\left\{ \sqrt{\frac{1+x\lambda^2-2\lambda}{1+x\lambda^2+2\lambda}} \tan\frac{\phi}{2} \right\} + \tan^{-1}\left\{ \sqrt{\frac{1+x\lambda^2-2\lambda}{1+x\lambda^2+2\lambda}} \tan\frac{\phi_0}{2} \right\} + n\pi \right\}, \quad (2n-1)\pi \le \phi \le (2n+1)\pi,\ n=1,2,\cdots \tag{16}$$

Equation (16) represents the motion of the passive joint of the manipulator and shows that there is one-to-one mapping between θ and ϕ.

As shown in the above illustrations, we can describe completely the motion of the manipulator actuated at the second joint by (14) and (16).

2. Dynamic characteristics

In this section, we present some important dynamic characteristics of the manipulator based on the proposed analytic approach in the previous section.

2.1. Forced motion

The forced motion actuated at the second joint has a specific motion characteristics. Now we introduce the specific motion characteristics as a form of theorem.

Theorem 1 : Assume that the initial kinetic energy is zero, i.e., $E_{k_0}=0$. Then if the angular velocity of the second joint $\dot{\phi}$ is zero, the two-link manipulator will stop.

Proof : Equation (14) showing the variation of the trajectory parameter in the control interval n can be expanded by (8) and (15), and arranged as follows:

$$\frac{1}{2}\dot{\phi}_*(\phi)^2 = \frac{1+x\lambda^2}{\lambda^2(x-1)}(E_k - E_{k_0}) + \frac{2}{\lambda(x-1)}(E_k\cos\phi - E_{k_0}\cos\phi_0) + \frac{1}{2}\dot{\phi}_*(\phi_0)^2. \tag{17}$$

If (17) is 0, the angular velocity of the second joint $\dot{\phi}$ will be 0 also. Rewriting the right side of (17), we have

$$\frac{E_k}{\lambda(x-1)}\left\{2+\frac{x}{\lambda}(\lambda-\frac{1}{x})^2+\frac{1}{\lambda}(1-\frac{1}{x}) + 2\cos\phi\right\} - \frac{E_{k_0}}{\lambda(x-1)}\left\{2+\frac{x}{\lambda}(\lambda-\frac{1}{x})^2 + \frac{1}{\lambda}(1-\frac{1}{x})+2\cos\phi_0\right\} + \frac{1}{2}\dot{\phi}_*(\phi_0)^2 \tag{18}$$

Since the initial condition $E_{k_0}=0$ and the possible values of the physical parameters are $x>1$ and $\lambda>0$, (18) cannot be 0 without $E_k=0$. Therefore it is proven that the two-link manipulator stops when the angular velocity of the second joint is 0. ■

On the other hand, in the case the initial kinetic energy E_{k_0} is not zero, the two-link manipulator has kinetic energy; nevertheless the angular velocity of the second joint $\dot{\phi}$ is zero.

2.2. Free motion

Next, we show that the free motion can be classified into two configurations. The one is oscillation and the other is rotation. We consider the free motion in control interval n. Then, since the actuated torque β_n is zero, the equation of the variation of the trajectory parameter (17) is

$$\frac{1}{2}\dot{\phi}_*(\phi)^2 = \frac{2E_{k_n}\cos\phi}{\lambda(x-1)} + p, \tag{19}$$

where p is a constant and can be expressed as

$$p = \frac{(1+x\lambda^2)(E_{k_n}-E_{k_0}) - 2\lambda E_{k_0}\cos\phi_0}{\lambda^2(x-1)} + \frac{1}{2}\dot{\phi}_*(\phi_0)^2. \tag{20}$$

Equation (19) expresses the free motion of the second link of the manipulator. Since the left side of (19) is positive, we have

$$\frac{2E_{k_n}\cos\phi}{\lambda(x-1)} + p \ge 0. \tag{21}$$

In (21), we can consider the following two conditions:

$$(a)\quad \frac{2E_{k_n}}{\lambda(x-1)} \ge p, \qquad (b)\quad \frac{2E_{k_n}}{\lambda(x-1)} < p. \tag{22}$$

Under the condition (a), (21) is satisfied on a constrained range of ϕ. In this case, the second link of the manipulator oscillates between the two points defined by the solutions of $\frac{2E_{k_n}\cos\phi}{\lambda(x-1)}+p=0$. On the other hand, under the condition (b), (21) is satisfied on all range of ϕ. In this case, the second link of the manipulator rotates continuously. When the condition (a) is satisfied, we call it the oscillation mode, and when the condition (b) is satisfied, we call it the rotation mode. The two configurations of the motion are illustrated in Fig. 3.

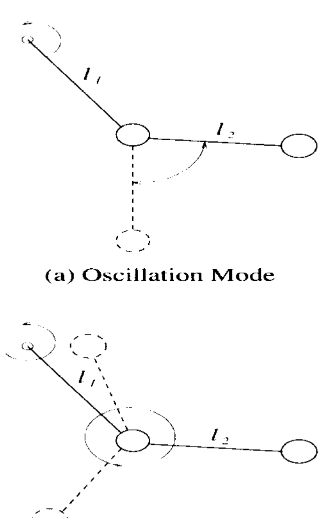

Fig. 3. Two configurations in free motion.

Threorem 2 : Assume that the initial kinetic energy is zero, i.e., $E_{k_0}=0$. Then a free motion occurring after a forced motion is always rotation.

Proof : Consider the free motion in control interval *n*. By the same manner of (21) and (22), we investigate the sign of the following equation:

$$\frac{2E_{k_0}}{\lambda(x-1)}-p= \frac{E_{k_0}}{\lambda(x-1)}\left\{-\frac{x}{\lambda}(\lambda-\frac{1}{x})^2-\frac{1}{\lambda}(1-\frac{1}{x})\right\} + \frac{(1+x\lambda^2+2\lambda\cos\phi_0)E_{k_0}}{\lambda^2(x-1)}-\frac{1}{2}\dot{\phi}_*(\phi_0)^2. \tag{23}$$

Since the condition $E_{k_0}=0$ and the possible values of the physical parameters are $x>1$ and $\lambda>0$, (23) is always less than 0. Thus we can say that the type of the free motion is rotation. ■

Also in the case that the initial kinetic energy E_{k_0} is not 0, the types of the free motion is determined by (23). If the dynamical condition of the manipulator makes (23) to be less than 0, the rotating motion will appear. On the other hand, if the condition makes (23) greater than 0, the oscillating motion will appear

IV. Motion control

In this section we show the simple and effective control algorithm with numerical example using the the dynamic characteristics showed in the previous section. Also the controllability under friction effect will be discussed.

1. Simple control algorithm

It is reported that the manipulator considered in this paper is uncontrollable[1]. The manipulator has one-to-one mapping between θ and ϕ. If the one joint of the manipulator is positioned, the position of the other is determined depending on the pre-positioned joint. However, it is confirmed that the output of the passive joint is controllable. As mentioned before, actuating the active joint, we can control the two-joints one by one. First, we control the passive joint using dynamic coupling by the motion of the active joint. Second, we control the active joint while fixing the passive joint. At this time, it is necessary to implement the brake at the passive joint to hold the rotation induced by the motion of the active joint. In motion control, the actuated torque pattern is made using dynamic characteristics of the manipulator analyzed in the previous section. The simple motion control algorithm is presented as follows.

Positioning the passive joint : At first, we determine the motion of the active joint to positioning the passive one using (16). If the target position of the passive joint is given, we can determine the temporary target position of the active one to generate appropriate dynamic coupling for the positioning the passive one. Second, we make the actuated torque pattern for the passive joint to reach the target position with the state that the both joint velocities are zero. Focusing on the theorem 1, we can take the appropriate torque pattern for the manipulator to be stopped at the target position of the passive joint by making the active joint velocity to be zero. Considering the above condition, the actuated torque pattern can be determined by the arranged form of (7):

$$\phi_t=\frac{1}{\beta_n}\left\{\beta_n\phi_n-\sum_{i=n-1}^{0}\beta_i(\phi_{i+1}-\phi_i)-E_{k_0}\right\}, \tag{24}$$

where ϕ_t is a target position of the active joint. Also focusing the theorem 2. we can take the torque pattern that has free motion interval. Although we take the free motion interval in the actuated torque pattern, we can make the active joint moves toward the temporary target position because there is only rotation.

Positioning the active joint : We control the active joint while acting the brake at the passive one to hold the rotation induced by the motion of the active joint. Then, we can consider the manipulator as a one-link manipulator. Thus we can make the torque pattern easily considering the kinetic energy.

Optimization of the actuated torque pattern : The controlled motion of the manipulator is composed of forced motion and free motion. We can make various torque patterns for a control objective. We optimize the actuated torque pattern for energy consumption E and control time T. The estimative function J is defined by

$$J=k_e\frac{E}{E_{\max}}+k_t\frac{T}{T_{\min}}, \tag{25}$$

where k_e and k_t are constants, and $E_{\max}$ and $T_{\min}$ are energy consumption and control time respectively in the controlled motion without free motion. We determine the actuated torque pattern minimizing the estimative function J by the simple computation.

In this study we consider the manipulator as a conservative system. If a manipulator has a friction which cannot be neglected, we should implement a feedback scheme to the proposed control algorithm.

2. Numerical example

The parameters of the two-link manipulator used in the simulation are $x=2$ and $\lambda=1$. And the actuated constant torques are used 1, 0, -1 to simplify the problem. The manipulator is in the initial configuration $(\theta,\phi,\dot{\theta},\dot{\phi})=(0,0,0,0)$. The goal is to bring the manipulator to $(\theta,\phi,\dot{\theta},\dot{\phi})=(\frac{\pi}{2},\frac{\pi}{4},0,0)$. The constant k_e and k_t are determined as same value in this example. The two constants are in trade-off relations for low energy consumption and high speed motion control. The result of the simulation is shown in Fig. 4, where the symbol * shows that the holding brake of the passive joint is active from the point. We can see that the objective of the motion control is carried out successfully.

3. Controllability under friction effect

If the friction act on the passive joint, it is not necessary to implement any additional equipment such as a brake for controlling all the joints to a specific position. For control of the passive joint, we use the dynamic coupling of the active one[1]. We give the desired motion of the passive joint as $\theta_d(\tau)$. The $\theta_d(\tau)$ is a sufficiently smooth function which satisfies the initial conditions and the final conditions. Substituting the

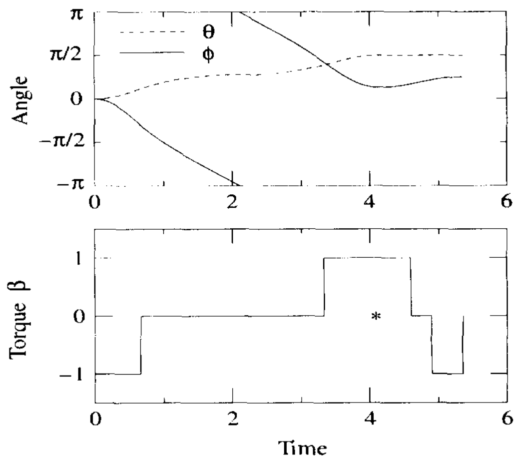

Fig. 4. A numerical example : motion of the manipulator and the actuated torque.

desired motion $\theta_d(\tau)$ into the equations of motion which consider the friction effect, we have the following equations:

$$\begin{aligned}(1+\kappa\lambda^2+2\lambda\cos\phi)\ddot{\theta}_d \\ +(1+\lambda\cos\phi)\ddot{\phi} \\ -\lambda\dot{\phi}(2\dot{\theta}_d+\dot{\phi})\sin\phi+\zeta_1\dot{\theta}_d = \alpha_{fr},\end{aligned} \tag{26}$$

$$\begin{aligned}(1+\lambda\cos\phi)\ddot{\theta}_d+\ddot{\phi}+\lambda\dot{\theta}_d^2\sin\phi+\zeta_2\dot{\phi} \\ =\beta+\beta_{fr},\end{aligned} \tag{27}$$

where $\alpha_{fr}(\beta_{fr})$ is the friction torque of the passive (active) joint. From these equations, the torque β is obtained as follows:

$$\begin{aligned}\beta=\frac{1}{1+\lambda\cos\phi}\{-\lambda^2(\kappa-\cos^2\phi)\ddot{\theta}_d \\ +\lambda\dot{\phi}(2\dot{\theta}_d+\dot{\phi})\sin\phi-\zeta_1\dot{\theta}_d+\alpha_{fr}\} \\ +\lambda\dot{\theta}_d^2\sin\phi+\zeta_2\dot{\phi}-\beta_{fr}.\end{aligned} \tag{28}$$

Substituting the current position and velocity of the active joint to (28), we can calculate the torque β, so that we can control the passive joint from the initial position to the final position. Next step is the control of the active joint. The active joint is moved to the final position with sufficiently small torque which does not affect the passive one at rest. To keep the rest state of the passive joint, the angular velocity and acceleration of the passive joint should be zero, and the torque β has to satisfy the following inequality:

$$\alpha_{fr}\geq|(\beta+\beta_{fr}+\zeta_2\dot{\phi})(1+\lambda\cos\phi)-\lambda\dot{\phi}^2\sin\phi|. \tag{29}$$

At this time the equation of motion of active joint becomes

$$\ddot{\phi}=\beta+\beta_{fr}-\zeta_2\dot{\phi}. \tag{30}$$

Thus we can calculate the actuated torque to control the manipulator to a desired position based on the above equations.

V. Conclusion

A dynamic analysis and motion control of an underactuated two-link manipulator have been presented. The dynamic analysis of an underactuated two-link manipulator based on first integral approach was introduced. Also a simple control algorithm based on the dynamic characteristics of the manipulator was made. The proposed control algorithm is simple and clear, and can make effective motion control using the dynamic characteristics positively. Finally the controllability of the underactuated manipulator under friction effect was discussed.

※ The author wishes to thank Prof. H. Inooka of Tohoku University for his proper and earnest advices, and continuous encouragement about this study.

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