Advanced Idealized Structural Units Considering Excessive Tension-Deformation Effects

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Abstract

In this paper, three kinds of the existing idealized structural units, namely the idealized beam-column unit, the idealized unstiffened plate unit and the idealized stiffened plate unit are expanded to deal with the excessive tension-deformation effects. A simplified mechanical model for the stress-strain relationship of steel members under tensile load is suggested. The 1/3-scale hull model for a leander class frigate under sagging moment tested by Dow is analyzed, and it is shown that the excessive tension-deformation is a significant factor affecting the progressive collapse behavior, particularly in the post-collapse range.

1 Introduction

When the safety of a ship structure is considered, the analysis of the ultimate collapse behavior for the whole structure is essential. To predict the maximum load-carrying capacity, the analysis may be performed up to the ultimate collapse state. However, the post-collapse behavior should also be analyzed to estimate the absorbed energy capacity which is calculated by intergrating the area below the load-displacement curve of the structure[1].

So far, to analyze the progressive collapse behavior of a ship structure using the idealized structural unit method(ISUM), several idealized structural units[1-3] have been developed taking into account ductile-collapse behavior. In these units, however, the importance of an excessive tension-deformation behavior after yielding has been disregarded.

As sagging moment increases, for instance, deck structure will collapse by compression, while excessive tension-deformation will be formed at the bottom part. It is considered that the overall collapse behavior of a ship's hull is dependent on tension-deformation behavior as well as ductile-collapse behavior due to axial compression or shearing force.

In this paper, three kinds of the existing idealized structural units, namely the idealized beam-column unit, the idealized unstiffened plate unit and the idealized stiffened plate unit are expanded to deal with the excessive tension-deformation effects. For this purpose, a simplified mechanical model for the stress-strain relationship of steel members under tensile load is suggested.

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The 1/3-scale hull model of a leander class frigate under sagging moment tested by Dow[4] is analyzed using the present theory, and it is observed that the tension-deformations of individual members play an important role on the progressive collapse behavior, particularly in the post-collapse range of the whole structure.

2 Theoretical Formulation

Since the detailed formulation for the idealized structural units including the ductile-collapse behavior due to axial compression and shearing force is found in the previous papers[1-3], only a brief description and additional formulation are given here.

2.1 Basic Idealizations

(1) ISUM Modelling for Ship Structure

In ISUM modelling, the basic structural member composing the object structure is chosen as the idealized structural unit.

Heavy longitudinal and transverse members supporting plate panels are modelled as the idealized beam-column unit which has only two end nodal points, shown in Fig. 1. In a stiffened panel, a number of one-sided stiffeners are attached to the plate in the longitudinal and/or transverse directions. The stiffener spacing is usually equal and geometric/material properties of each stiffener are supposed to be the same in each direction. The stiffened panel is modelled as the idealized stiffened plate unit, indicated in Fig. 2. Also when the panel has no stiffener, it is modelled as the idealized unstiffened plate unit, shown in Fig. 3. The idealized unstiffened and stiffened plate units have only four corner nodal points.

(2) Idealization of Structural Behavior

If a structural member is subjected to axial compression or shearing force, it will buckle with an increase in the applied load and shows a large deflection behavior. As a result, its in-plane stiffness decreases and it reaches the ultimate collapse state. In the post-collapse range, the internal stress goes down if the compression-deformation is continued.

On the other hand, when a member is subjected to tensile loads, its behavior is different from that of compressed members. Fig. 4 represents a typical stress-strain curve for a steel member under tensile load. It is observed that as the tensile load increases, the member yields without lateral deflection. In the existing idealized structural units, behavior of the yielded unit has been treated as the dotted line (segment CH) in Fig. 5. Because of the strain-hardening effect, however, the member can carry further loading even after yielding, and reaches the ultimate tensile strength. When there is an excessive tension-deformation, the actual cross-sectional area decreases and the nominal stress goes down with an increase in the tension-deformation. After that, the ductile fracture will occur if the tensile strain of the member exceeds the critical fracture strain which depends on material properties, initial crack damage, etc.

The actual stress-strain curve for structural members is idealized as the solid line in Fig. 5. Then, if the characteristics of each segment in Fig. 5 are known, the stress-strain curve can be traced as:

(i) Compressed Member :

OA(Elastic Large Deflection Behavior) → A(Ductile-Collapse Strength) → AB(Post-Collapse Behavior)

(ii) Tensiled Member :

OC(Linear Elastic Behavior) → C(Yield Strength) → CD(Post-Yield Behavior) → D(Ultimate Tensile Strength) → DE(Post-Tensile Strength Behavior) → E(Ductile-Fracture Strength) → EF-FG(Post-Fracture Behavior)

In the next section, the detailed formulations for each segment or failure criteria are described. Table 1 indicates the definition of failure modes of the idealized structural units.

2.2 Idealized Beam-Column Unit

(1) General

As mentioned, heavy longitudinal and transverse supporting members are modelled by the idealized beam-column unit, indicated in Fig. 1. End conditions of the unit are assumed to be pin-jointed. When a ship's hull is subjected to longitudinal bending moments, this unit will carry the axial loads.

In the present study, a deflected column is replaced by an equivalent straight unit which has the reduced stiffness due to lateral deformations. As a result, rotational degree of freedoms are not necessary to be included in the numerical formulations. The nodal force increment $\{\Delta R\}_{bc}$ and the nodal displacement increment $\{\Delta U\}_{bc}$ of the unit in the local coordinates are then defined by (see Fig. 1)

$$\begin{aligned} \{\Delta R\}_{bc} &= \{\Delta R_{x1} \Delta R_{x2}\}^T \\ \{\Delta U\}_{bc} &= \{\Delta u_1 \Delta u_2\}^T \end{aligned} \tag{1}$$

Applying the principle of virtual work, the stiffness equation of the unit will be given by

$$\{\Delta R\}_{bc} = [K]_{bc}\{\Delta U\}_{bc} \tag{2}$$

where $[K]_{bc}$ is the stiffness matrix of the idealized beam-column unit in the local coordinates, and that will change according to failure or loading condition. In the following, $[K]_{bc}$ for each segment in Fig. 5 is formulated.

(2) Segment OA : Elastic Large Deflection Behavior

An initially deflected pin-ended column is considered. The geometric configuration may take the following form:

$$\begin{aligned} w_o &= \delta_o \sin\left(\frac{\pi}{L}\right) x \\ w &= \delta \sin\left(\frac{\pi}{L}\right) x \end{aligned} \tag{3}$$

where δ_o = initial deflection amplitude
δ = unknown added-deflection amplitude
L = member length

Applying the principle of minimum potential energy, the unknown amplitude will be found.

$$\delta = \frac{\delta_o}{1 - R/R_E} \tag{4}$$

where R_E = Euler's buckling load

Using the above solution, total end-shortening considering the large deflection effect will be given by

$$u = \frac{RL}{EA_b} + \frac{\pi^2 \delta_o^2}{4L(1 - R/R_E)^2} - \frac{\pi^2 \delta_o^2}{4L} \tag{5}$$

where A_b = cross-sectional area of the unit
E = Young's modulus

The incremental relationship between the axial force and the end-shortening is then given by

$$\Delta R_x = \eta_E \Delta u \tag{6}$$

where $\eta_E = \frac{1}{\frac{L}{EA_b} + \frac{\pi^2 \delta_o}{2LR_E(1-R/R_E)^3}}$

Accordingly, considering the equilibrium condition, the stiffness matrix $[K]_{bc}$ in this range will be obtained.

(3) Criterion of Point A : Ductile-Collapse Strength

Several criteria for checking buckling or collapse of the beam-column member have been suggested. The well-known Perry-Robertson formula[5] taking into account the initial imperfection effects is used.

(4) Segment AB : Post-Collapse Behavior

For the collapsed unit, the following relationship between the axial force and the end-shortening, using the concept of plastic hinge collapse mechanism can be used[6].

$$\frac{R}{R_p} = \left[\left\{\frac{R_p}{2M_p}\right\}^2 Lu + 1\right]^{1/2} - \frac{R_p}{2M_p}(Lu)^{1/2} \tag{7}$$

where R_p = fully plastic axial load $(= \sigma_o A_b)$
M_p = fully plastic bending moment $(= Z_p \sigma_o)$
Z_p = plastic section modulus
σ_o = yield stress

The incremental form of the above equation will become

$$\Delta R = \eta_u \Delta u \tag{8}$$

where

$$\eta_u = \frac{R_p^3 L}{8}\left[M_p^2\left\{\left(\frac{R_p}{2M_p}\right)^2 Lu + 1\right\}^{1/2}\right] - \frac{R_p^2 L}{4M_p(Lu)^{1/2}}$$

Accordingly, considering the equilibrium condition, the stiffness matrix $[K]_{bc}$ in the post-ultimate range will be obtained.

(5) Segment OC : Linear Elastic Behavior

In this case, the behavior of the unit is linear and the stiffness matrix will be the same as that of the truss element.

(6) Criterion of Point C : Yield Strength

When the axial stress σ_x reaches the yield stress σ_o, the unit yields.

$$\sigma_x \geq \sigma_o \tag{9}$$

(7) Segment CD : Post-Yield Behavior

Even after yielding, the steel member can carry further tensile loading because of the strain-hardening effect. As shown in Fig. 5, the gradient of the stress-strain curve for the yielded unit can be simplified by

$$E_p = \alpha_p E \tag{10}$$

where E_p = tangent modulus of the yielded unit
α_p = coefficient

In principle, coefficient α_p can be obtained from the tensile test result. Thus, the stiffness matrix in this range will be given by

$$[K]_{bc} = \frac{E_p A_b}{L}\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} \tag{11}$$

(8) Criterion of Point D : Ultimate Tensile Strength

When the axial stress exceeds the ultimate tensile stress σ_T, the ultimate tensile strength is reached.

$$\sigma_x \geq \sigma_T \tag{12}$$

(9) Segment DE : Post-Tensile Strength Behavior

After the axial stress reaches the ultimate tensile strength, the internal stress decreases with increases in the tension-deformation as shown in Fig. 5. The gradient of the stress-strain curve in this range can be simplified by

$$E_f = \alpha_f E \tag{13}$$

where E_f = tangent modulus in the post-tensile strength range
α_f = coefficient

In principle, coefficient α_f can also be obtained from the tensile test result of the member and the stiffness matrix in this case is given by

$$[K]_{bc} = \frac{E_f A_b}{L}\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} \tag{14}$$

(10) Criterion of Point E : Ductile-Fracture Strength

If the axial tensile strain exceeds the critical fracture strain of the unit depending on material property, initial crack, etc. the ductile-fracture will occur.

$$\varepsilon_x \geq \varepsilon_{fcr} \tag{15}$$

where ε_{fcr} is the critical fracture strain of the unit, which is obtained from the tensile test result. Here, even for an initially cracked unit, ε_{fcr} can be obtained by the tensile test.

(11) Segment EF and FG : Post-Fracture Behavior

If the unit is fractured, it is not able to carry the tensile load further. In the subsequent loading step, therefore, the stiffness matrix of the fractured unit is equal to zero such that the internal stress increment is zero for the rest of the analysis. Also, the accumulated stress in the fractured unit should be released[20].

2.3 Idealized Unstiffened and Stiffened Plate Units

(1) General

In the present method, the deflected plate panel is replaced by an equivalent "flat" plate panel which has the reduced in-plane stiffness due to lateral deformation. As a result, rotational degrees of freedom at each nodal point can be removed as in the idealized beam-column unit. Therefore, the total degrees of freedom at each nodal point are just three axial displacements which are u, v and w in the x, y and z direction, respectively. Thus, the nodal force increment $\{\Delta R\}_{pl}$ and the nodal displacement increment $\{\Delta U\}_{pl}$ in the local coordinates are defined(see Fig. 2 and 3).

$$\begin{aligned} \{\Delta R\}_{pl} &= \{\Delta R_{x1}\Delta R_{y1}\Delta R_{z1}\cdots\Delta R_{x4}\Delta R_{y4}\Delta R_{z4}\}^T \\ \{\Delta U\}_{pl} &= \{\Delta u_{u1}\Delta u_{v1}\Delta u_{w1}\cdots\Delta u_{u4}\Delta u_{v4}\Delta u_{w4}\}^T \end{aligned} \tag{16}$$

Also the average stress increment of the unit is calculated by

$$\{\Delta\sigma\} = [D]\{\Delta\varepsilon\} \tag{17}$$

where $\{\Delta\sigma\} = \{\Delta\sigma_{xav}\Delta\sigma_{yav}\Delta\tau_{xyav}\}^T$

$[D]$ = average stress-average strain matrix

Applying the principle of virtual work, the stiffness equation of the unit will be given after unbalance forces are eliminated:

$$\{\Delta R\}_{pl} = [K]_{pl}\{\Delta U\}_{pl} \tag{18}$$

where $[K]_{pl}$ = stiffness matrix of the idealized plate unit

$= \int([B_p]^T[D][B_p] + [G]^T[\sigma_b][G])d\text{vol}$

$$[\sigma_b] = \begin{bmatrix} \sigma_{xav} & \tau_{xyav} \\ \tau_{xyav} & \sigma_{yav} \end{bmatrix}$$

It is observed from the above equation that once the [D] matrix is known, the stiffness matrix of the unit is calculated. In this regard, the following description is focused on the derivation of the [D] matrix.

(2) Segment OA : Elastic Large Deflection Behavior

An actual plate member always has initial deflection. When compressive loads are predominant, in-plane stiffness of an initially deflected plate panel decreases from the beginning with the increase in the applied load. Also, the present method attempts to replace the deflected plate by an equivalent flat plate accounting for the reduction of in-plane stiffness due to lateral deformaion.

(i) Idealized Unstiffened Plate Unit

Using the effective width formulation[8], the [D] matrix of deflected plate units is given by :

$$[D] = [D]^B_{pl} \tag{19}$$

where $[D]^B_{pl}$ is the stress-strain matrix for the deflected plate panel.

(ii) Idealized Stiffened Plate Unit

It is assumed that stiffeners keep straight until they buckle and thus the stress-strain matrix is calculated by adding the deflected plate and intact stiffeners as [3]:

$$[D] = [D]^B_{sp} = [D]^B_{pl} + [D]^{EXY}_{st} \tag{20}$$

where
$$[D]^{EXY}_{st} = \begin{bmatrix} En_{sx}A_{sx}/bt & 0 & 0 \\ 0 & En_{sy}A_{sy}/at & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

n_{sx}= number of longitudinal stiffeners
n_{sy}= number of transverse stiffeners
A_{sx}= cross-sectional area of one longitudinal stiffener
A_{sy}= cross-sectional area of one transverse stiffener

a = plate length
b = plate width
t = plate thickness

(3) Criterion of Point A : Ductile-Collapse Strength

Considering the membrane stress distribution of deflected plate elements under applied loads, the ductile-collapse criteria are as follows:

(i) Idealized Unstiffened Plate Unit

If the equivalent stress in the corner or the transverse mid-edge or the longitudinal mid-edge of the unit reaches the yield stress, the unstiffened plate unit will be assumed to collapse[13].

(ii) Idealized Stiffened Plate Unit

Collapse patterns for stiffened plate panels are divided into three categories such as local collapse, panel collapse and overall collapse[5]. Here, the local collapse of plate part between stiffeners is checked by using the same criterion for the unstiffened plate unit. Also the panel and the overall collapses are treated as a buckling problem of columns with effective platings[1].

If longitudinal stiffeners collapse while transverse stiffeners are still intact, the panel collapse between transverse stiffeners will occur. On the other hand, if transverse stiffeners collapse while longitudinal stiffeners are still intact, the panel collapse between lonitudinal stiffeners will occur. Also, if both longitudinal and transverse stiffeners collapse at the same time, then the overall collapse occurs.

For checking the collapse of stiffeners with effective platings, several formulas have been suggested[15]. In the present study, the well-known Perry-Robertson formula is employed, as in the idealized beam-column unit. It should be noted that the present idealized stiffened plate unit does not take into account the flexural-torsional collapse pattern of stiffeners.

(4) Segment AB : Post-Collapse Behavior

After the unit collapses under axial compression, the internal stress decreases if the compression-deformation is continued. General formulation of the stress-strain relationship in this range have been described by the author[3] or Ueda et al[14]. However, a more simplified derivation is made as follows:

(i) Idealized Unstiffened Plate Unit

The equivalent stress of a collapsed unit should be kept constant and the change of membrane stress components in the unit may be negligible as far as the loads are applied continuously. The average membrane strain components of the collapsed unit are calculated with the assumption that the Poisson's ratio effect of the collapsed unit is neglected.

$$\begin{aligned} \varepsilon_{xav} &= \sigma^u_{xmax}/E \\ \varepsilon_{yav} &= \sigma^u_{ymax}/E \end{aligned} \tag{21}$$

where σ^u_{xmax} and σ^u_{ymax} are maximum membrane stresses just before or after ultimate collapse of the unit in the x and y direction, respectively.

Using the effective width formulations in the x and y directions[8], the average membrane stress components of the collapsed unit are obtained by

$$\begin{aligned} \sigma_{xav} &= b_e/b \; \sigma^u_{xmax} \\ \sigma_{yav} &= a_e/a \; \sigma^u_{ymax} \end{aligned} \tag{22}$$

where b_e = effective plate width
a_e = effective plate length

With the increase in the compression-deformation, the effective plate width and length of the collapsed unit also decrease continuously, and it is assumed that the reduction tendency of the effective plate width/length with an increase in the applied loads is the same with that in the pre-collapse range[15], that is,

$$\begin{aligned} b_e/b &= \sigma^*_{xav}/\sigma^*_{xmax} \\ a_e/a &= \sigma^*_{yav}/\sigma^*_{ymax} \end{aligned} \tag{23}$$

where the asterisk indicates a virtual amount of the stress.

Under the assumption that the effects due to initial imperfection, Poisson's ratio and aspect ratio are neglected for the collapsed unit, the virtual maximum stresses are calculated by [16]

$$\begin{aligned} \sigma^*_{xmax} &= 2\sigma^*_{xav} - \sigma_{xcr} = E\varepsilon^*_{xav} \\ \sigma^*_{ymax} &= 2\sigma^*_{yav} - \sigma_{ycr} = E\varepsilon^*_{yav} \end{aligned} \tag{24}$$

where σ_{xcr} = buckling stress in longitudinal compression
σ_{ycr} = buckling stress in transverse compression

Substitution of Eq. (24) into Eq. (23) yields the effective plate width and length as a function of membrane strain components.

$$\begin{aligned} \frac{b_e}{b} &= \frac{1}{2}\left\{1 + \frac{\sigma_{xcr}}{E\varepsilon_{xav}}\right\} \\ \frac{a_e}{a} &= \frac{1}{2}\left\{1 + \frac{\sigma_{ycr}}{E\varepsilon_{yav}}\right\} \end{aligned} \tag{25}$$

Also, substituting Eq. (25) into Eq. (22), the relationship between average axial stress and average axial strain of the collapsed unit is given

$$\begin{aligned} \sigma_{xav} &= \frac{1}{2}\left\{1 + \frac{\sigma_{xcr}}{E\varepsilon_{xav}}\right\} \sigma^u_{xmax} \\ \sigma_{yav} &= \frac{1}{2}\left\{1 + \frac{\sigma_{ycr}}{E\varepsilon_{yav}}\right\} \sigma^u_{ymax} \end{aligned} \tag{26}$$

The above equation is rewritten by the incremental form as:

$$\begin{aligned}\Delta\sigma_{xav} &= -\frac{\sigma^u_{xmax}}{2}\frac{\sigma_{xcr}}{E\varepsilon^2_{xav}}\Delta\varepsilon_{xav}\\ \Delta\sigma_{yav} &= -\frac{\sigma^2_{ymax}}{2}\frac{\sigma_{ycr}}{E\varepsilon^2_{yav}}\Delta\varepsilon_{yav}\end{aligned} \tag{27}$$

On the other hand, it is assumed that the shearing modulus of the collapsed unit is nearly zero, that is,

$$\Delta\tau_{xyav} = 0\cdot\Delta\gamma_{xyav} = 0 \tag{28}$$

Accordingly, the relationship between the average stress increment and the average strain increment will be obtained.

$$[D] = [D]^U_{pl} \tag{29}$$

(ii) Idealized Stiffened Plate Unit

For the idealized stiffened plate unit, the post-collapse behavior is different according to the collapse mode.

1) after local collapse

In this case, the stiffness of the unit is calculated by adding the collapsed plate part and the intact longitudinal/transverse stiffeners as:

$$[D] = [D]^U_{sp} = [D]^U_{pl} + [D]^{EXY}_{st} \tag{30}$$

2) after longitudinal panel collapse (longitudinal stiffeners collapsed)

In this case, the stiffness of the unit is calculated by adding the collapsed plate part, the intact transverse stiffeners and the collapsed longitudinal stiffeners as:

$$[D] = [D]^U_{sp} = [D]^U_{pl} + [D]^{EY}_{st} + [D]^{UX}_{st} \tag{31}$$

where $$[D]^{EY}_{st} = \begin{bmatrix} 0 & 0 & 0\\ 0 & En_{sy}A_{sy}/at & 0\\ 0 & 0 & 0\end{bmatrix}$$

$[D]^{UX}_{st}$ = stress-strain matrix of the collapsed longitudinal stiffener which is the same with the idealized beam-column unit

3) after transverse panel collapse (transverse stiffeners collapsed)

In this case, the stiffness of the unit is calculated by adding the collapsed plate part, the intact longitudinal stiffeners and the collapsed transverse stiffeners as:

$$[D] = [D]^U_{sp} = [D]^U_{pl} + [D]^{EX}_{st} + [D]^{UY}_{st} \tag{32}$$

where $$[D]^{EXY}_{st} = \begin{bmatrix} En_{sx}A_{sx}/bt & 0 & 0\\ 0 & 0 & 0\\ 0 & 0 & 0\end{bmatrix}$$

$[D]_{st}^{UY}$ = stress-strain matrix of the collapsed transverse stiffener which is the same with the idealized beam-column unit

4) after overall collapse

In this case, the stiffness of the unit is calculated by adding the collapsed plate part and the collapsed longitudinal/transverse stiffeners as:

$$[D] = [D]_{sp}^{U} = [D]_{pl}^{U} + [D]_{st}^{U} \tag{33}$$

where $[D]_{st}^{U}$ = stress-strain matrix of the collapsed longitudinal/transverse stiffener

(5) Segment OC : Linear Elastic Behavior

In this case, behavior of the unit is linear, and the [D] matrix is given as follows:

(i) Idealized Unstiffened Plate Unit

$$[D] = [D]_{pl}^{E} = \frac{E}{1-\nu^2}\begin{bmatrix} 1 & \nu & 0 \\ \nu & 1 & 0 \\ 0 & 0 & (1-\nu)/2 \end{bmatrix} \tag{34}$$

(ii) Idealized Stiffened Plate Unit

The stiffness of the unit is calculated by adding the flat plate part and the intact longitudinal/transverse stiffeners.

$$[D] = [D]_{sp}^{E} = [D]_{pl}^{E} + [D]_{st}^{EXY} \tag{35}$$

(6) Criterion of Point C : Yield Strength

If the equivalent stress reaches the yield stress, the unit will be yielded. Here, for checking the yield strength, the following von Mises' condition is employed.

$$f_y = \sigma_{xav}^2 - \sigma_{xav} \cdot \sigma_{yav} + \sigma_{yav}^2 + 3\tau_{xyav}^2 - \sigma_o^2 = 0 \tag{36}$$

(7) Segment CD : Post-Yield Behavior

Because of the strain-hardening effect, the yielded unit can still carry further tensile loading. In the present method, therefore, the elasto-plastic stress-strain relationship taking into account the strain-hardening effect is derived as in the same manner with the conventional FEM. If unloading in the yielded unit occurs, the elastic stress-strain relationship is employed.

(i) Idealized Unstiffened Plate Unit

$$\begin{aligned} [D] &= [D]_{pl}^{p} \\ &= [D]_{pl}^{E} - \frac{[D]_{pl}^{E}\{\partial f_y/\partial\sigma\}\{\partial f_y/\partial\sigma\}^T[D]_{pl}^{E}}{E_p + \{\partial f_y/\partial\sigma\}^T[D]_{pl}^{E}\{\partial f_y/\partial\sigma\}} \end{aligned} \tag{37}$$

where E_p indicates the tangent modulus accounting for the strain-hardening effect and f_y is the yield function in Eq. (36).

(ii) Idealized Stiffened Plate Unit

$$
\begin{aligned}
[D] &= [D]_{sp}^{p} \\
&= [D]_{sp}^{E} - \frac{[D]_{sp}^{E}\{\partial f_y/\partial\sigma\}\{\partial f_y/\partial\sigma\}^T[D]_{sp}^{E}}{E_p + \{\partial f_y/\partial\sigma\}^T[D]_{sp}^{E}\{\partial f_y/\partial\sigma\}}
\end{aligned}
\tag{38}
$$

(8) Criterion of Point D : Ultimate Tensile Strength

If the equivalent stress exceeds the ultimate tensile stress of the material, the unit will reach the ultimate tensile strength.

$$
f_T = \sigma_{xav}^2 - \sigma_{xav}\cdot\sigma_{yav} + \sigma_{yav}^2 + 3\tau_{xyav}^2 - \sigma_T^2 = 0 \tag{39}
$$

(9) Segment DE : Post-Tensile Strength Behavior

As far as the tension-deformation is continued, the internal stress decreases in this range. As in the idealized beam-column unit, the tangent modulus can be quantified based on the tensile test result.

(i) Idealized Unstiffened Plate Unit

The basic relationship between stress and strain is

$$
\frac{\sigma_{xav} - \nu\sigma_{yav}}{\varepsilon_i} = \frac{\sigma_{yav} - \nu\sigma_{xav}}{\varepsilon_i} = \frac{\tau_{xyav}}{2(1+\nu)\gamma_{av}} = \frac{\sigma_i}{\varepsilon_i} \tag{40}
$$

where σ_i, ε_i = equivalent stress, strain

It is assumed that the relationship between the equivalent stress and the equivalent strain is linear in this range.

$$
\frac{\sigma_i}{\varepsilon_i} = E_f \tag{41}
$$

where E_f = tangent modulus in the post-tensile strength range

Accordingly, the stress-strain matrix reads

$$
\begin{aligned}
[D] &= [D]_{pl}^{F} \\
&= \frac{E_f}{1-\nu^2}\begin{bmatrix} 1 & \nu_f & 0 \\ \nu_f & 1 & 0 \\ 0 & 0 & (1-\nu_f)/2 \end{bmatrix}
\end{aligned}
\tag{42}
$$

where ν_f denotes Poisson's ratio in the post-tensile strength range and for steels it may be taken as $\nu_f = 0.5$ because the material is in fully plastic regime.

(ii) Idealized Stiffened Plate Unit

$$
[D] = [D]_{sp}^{F} = [D]_{pl}^{F} + [D]_{st}^{FXY} \tag{43}
$$

where
$$
[D]_{st}^{FXY} = \begin{bmatrix} E_f n_{sx} A_{sx}/bt & 0 & 0 \\ 0 & E_f n_{sy} A_{sy}/at & 0 \\ 0 & 0 & 0 \end{bmatrix}
$$

(10) Creterion of Point E : Ductile-Fracture Strength

As in the idealized beam-column unit, if the tensile strain of the unit exceeds the critical fracture strain, the ductile-fracture will occur.

$$\varepsilon_{eq} \geq \varepsilon_{fcr} \tag{44}$$

where ε_{eq} is the equivalent tensile strain in the unit and ε_{fcr} is the critical fracture strain which is obtained from the tensile test result. As memtioned, the effect of initial crack damage can also be included in the determination of the critical fracture strain.

(11) Segment EF and FG : Post-Fracture Behavior

The fractured unit is not able to carry the tensile load further. In the subsequent loading step, therefore, the modulus of material is equal to zero, as in the idealized beam-column unit. In this case, the stress-strain matrix for both unstiffened and stiffened plate units becomes zero and also the accumulated membrane stress in the fractures unit should be released[20].

3 Analysis of the 1/3-scale Frigate Test Model

Dow[4] carried out the progressive collapse test for the 1/3-scale hull model of a leander class frigate under sagging moment. The ISSC'94 committe III.1 [19] has analyzed the progressive collapse behavior of this model. Many researchers have participated in this work and the same definitions for the geometry and the initial imperfections of the model were suggested.

3.1 Test Model

Dimensions of the model are $18m$ length, $4.1m$ beam and $2.8m$ depth. Fig. 6 shows midship section of the test model. The material properties of the structural members are defiend by

E = 207 GPa (Young's modulus)
σ_o = 245 MPa (yield stress)
ν = 0.3 (Poisson's ratio)

Basically, the material is assumed to be linear elastic, perfectly plastic by the ISSC committee[19]. Also the present study aims to investigate the nonlinear behavior due to the excessive tension-deformation and for this purpose the following magnitudes which are selected from the typical tensile test results of materials are assumed by

E_p = 0.01 E (tangent modulus after yielding)
E_f = $-E$ (tangent modulus after ultimate tensile strength)
ε_{fcr} = 0.15 (critical rupture strain)

The initial deflection of the plates between stiffeners is suggested to be in the shape of one-half sine wave in both longitudinal and transverse directions with the maxium deflection at midpoint given by

$$\delta/b = 0.030 \quad \text{for deck plating}$$
$$= 0.010 \quad \text{for side shell plating}$$

where δ = magnitude of initial deflection
b = width of plate between stiffeners

The residual stress in the plate between stiffeners is suggested to have the well-known square shape which forms a self-equilibrium pattern. The magnitude of the residual stress in the two outer zones is equal to the tension yield stress while the residual stress magnitude of the middle zone in the longitudinal direction is compressive, given by Table 3 of reference [4], namely

$$\sigma_{rx} = -125MPa \quad \text{for } 2mm \text{ plate}$$
$$= -147MPa \quad \text{for } 3mm \text{ plate}$$

where σ_{rx} = magnitude of the residual stress in the longitudinal direction

By the way, since the welding is performed along the transvere as well as the longitudinal edge of the plate panels, the transverse residual stress which also affects the stiffness and strength of the structure will exist. According to the measured data for center panels, the residual stress in the transvere direction is about 19 % of the longitudinal one. In this regard, the compressive residual stress magnitude in the transverse direction can be assumed by

$$\sigma_{ry} = 0.19\sigma_{rx} = -23.75MPa \quad \text{for } 2mm \text{ plate}$$
$$= -27.93MPa \quad \text{for } 3mm \text{ plate}$$

where σ_{ry} = magnitude of the residual stress in the transverse direction

3.2 Discussion of Results

The computer program ALPS/ISUM [21] has been developed based on the present theory. Procedure for the progressive collapse analysis of the plated structures such as ships is found in the previous papers[17, 18]. When a ship's hull is under bending moment, the position of neutral axis changes and the computer program also takes into account this effect automatically.

Fig. 7 shows the ALPS/ISUM model for the test model. Midship part in one frame spacing is taken as the extent of the modelling and a full-hull is modelled. All of the three kinds of the idealized structural elements, described in this paper are used for this modelling. Here, the "hard element" is introduced, in which buckling does not occur, but yielding will take place. In this analysis, plate parts at the deck-side corner, 2nd deck girder, 3rd deck, 4th deck, center girder and side girder which are rigidly connected with transverse frames are considered as the "hard element".

As sagging moment increases, the idealized structual units will show various failure modes, defined in Table 1. Symmetric boundary condition at the center line of the bottom part and restriction of rigid-body motion are prescribed. The bending curvature for the hull is applied incrementally.

Fig. 8 compares the sagging moment-curvature curve for the test model between the Dow's experimental result and the ALPS/ISUM solution. The existing numerical results obtained by using other computer programs such as the Dow's FABSTRAN/NS94[4], the Dow's ASAS-NL[4], the Yao's HULLST[10] are also compared. Here, both FABSTRAN/NS94 and HULLST are based on the Smith's method[9], but the ASAS-NL code is applying the nonlinear finite-element method. The solution by ALPS/ISUM is coincidental with the exprimental result.

Figs. 9 and 10 and Table 2 indicate solutions by ALPS/ISUM assuming $E_p = 0.01E$ and $E_f = -E$. Fig. 9 shows the average stress-displacement curve for typical members under compression or tension. The computed moment of inertia for the unreduced ideal midship section was $6.237 \times 10^{10} mm^4$.

Fig. 10 represents change of the horizontal neutral axis height. The computed position of the neutral axis above the base line for the unreduced ideal midship section was $1419.8mm$. Since the actual members have the initial imperfections the real position of the neutral axis is different from the ideal one. The computed position of the neutral axis for the real midship section was $1223mm$.

Furthermore, if the failure of structural members occurs with an increase in the applied load, the position of the neutral axis changes. At first it moves downward due to collapse of the deck structure, and then it moves upward due to yielding of the bottom structure. The failured region will be expanded to the mid-height part of the section. In the present modelling, the bottom girders are considered to be hard and the neutral axis position slightly moves downward again. Table 2 indicates the failure history of the test model with an increase in the applied sagging moment. In this analysis, the critical fracture strain of the material is assumed to be 15 % , but for this model the ductile-fracture (failure mode number = 7) did not occur.

Fig.11 investigates the effect of the post-tensile strength behavior on the progressive collapse behavior of the whole structure. Parametric analysis is performed varying the magnitude of E_f. It is observed that the progressive collapse behavior is the same for each other up to the ultimate collapse strength, but in the post-collapse range the behavior is different by the value of E_f. As the absolute value of E_f increases, the gradient of the "unloading path" becomes larger. The solution obtained assuming $E_p = 0.01E$ and $E_f = -0.5E$ is very similar with the Dow's FABSTRAN/NS94 result.

4 Concluding Remarks

The importance of excessive tension-deformation effects after yielding has been disregarded in the existing idealized structural units although the ductile-collapse due to axial compression and shearing force is treated carefully. The existing idealized structural units are improved so as to include the excessive tension-deformation effects. For this purpose, a simplified mechanical model for the stress-strain relationship of steel members under tensile load has been suggested. Through an illustration example for a large-sized test hull model, the role of the excessive tension-deformation of local members on the overall collapse behavior has been demonstrated. It is concluded that the excessive tension-deformation of individual members after the ultimate tensile strength is also a significant factor affecting the progressive collapse behavior of the whole structure, particularly in the post-ultimate collapse range.

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Table 1: Definition of failure modes for the idealized structural units

Model No.	Beam-column Unit	Unstiffened Plate Unit	Stiffened Plate Unit
0	Failure-Free	Failure-Free	Failure-Free
1	Collapse	Collapse	Local Collapse
2	-	-	Panel Collapse (between trans.)
3	-	-	Panel Collapse (between longi.)
4	-	-	Overall Collapse
5	Yield Strength	Yield Strength	Yield Strength
6	Ultimate Tensile Strength	Ultimate Tensile Strength	Ultimate Tensile Strength
7	Ductile-Fracture	Ductile-Fracture	Ductile-Fracture

Table 2: Failure history for the 1/3-scale frigate test model

Applied Sagging	Plate		Beam-Column	
Moment (ton-m)	Unit No.	Failure mode	Unit No.	Failure mode
486.67			11, 12	1
6002.29	13	2		
640.46	12, 14	2		
800.12	9, 17	2		
898.95	1, 2 24, 25	5		
946.47	8, 18	2		
953.39	3, 23	5		
976.20	4, 22	5		
987.38	7, 19	2		
*987.75	1, 2 24, 25	6		
962.49	3, 23	6		
915.63	4, 22	6		
896.25	5, 21	5		

* Ultimate collapse strength has been reached.

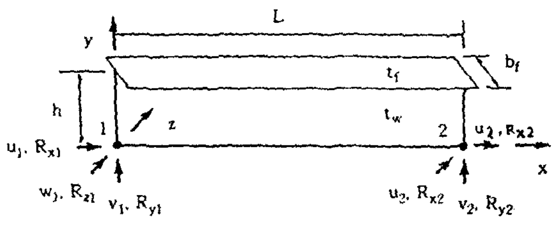

Figure 1: Idealized beam-column unit

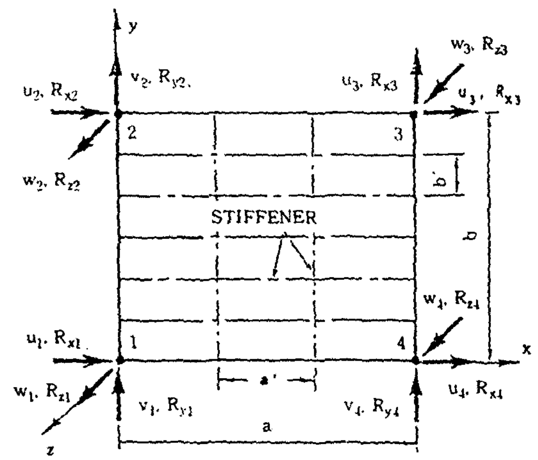

Figure 2: Idealized stiffened plate unit

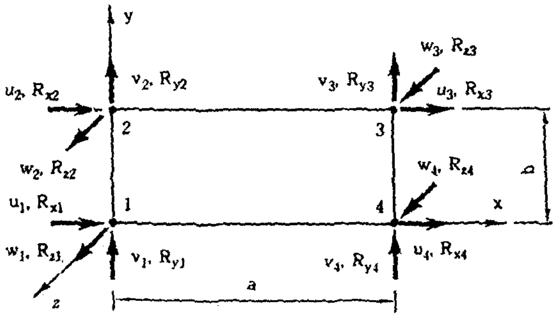

Figure 3: Idealized unstiffened plate unit

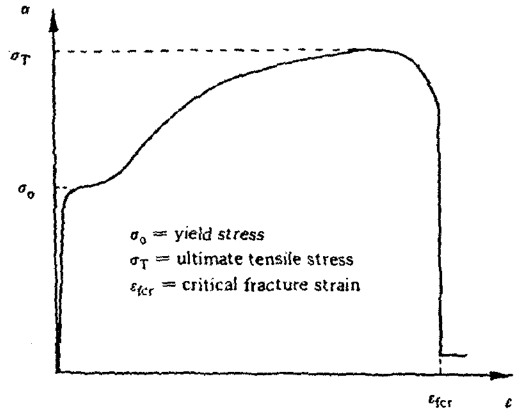

Figure 4: A typical tensile test result for steel material

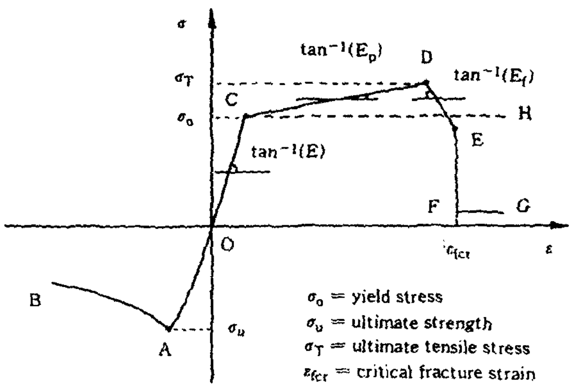

Figure 5: Idealization of nonlinear behavior for steel members

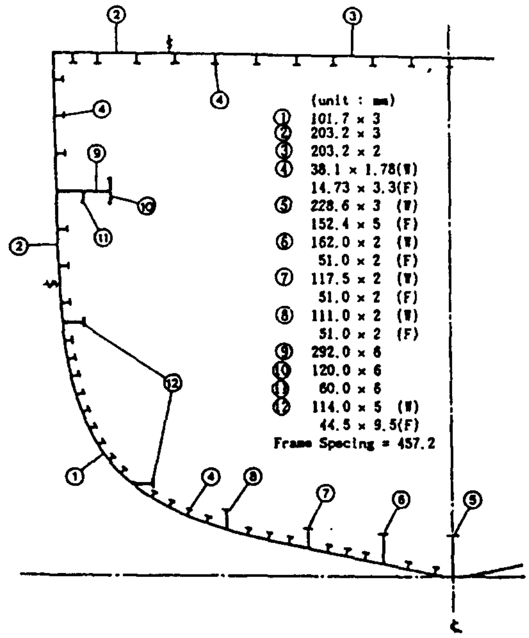

Figure 6: Midship section of the 1/3-scale frigate test model

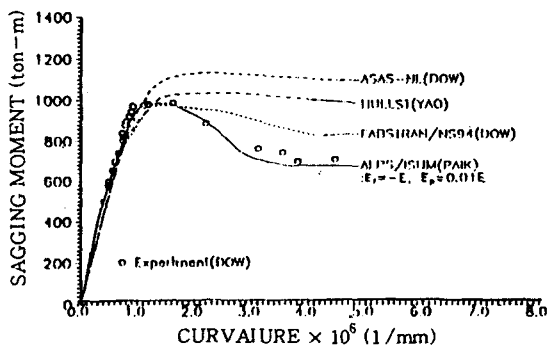

Figure 8: Sagging moment-curvature response for the 1/3-scale frigate test model

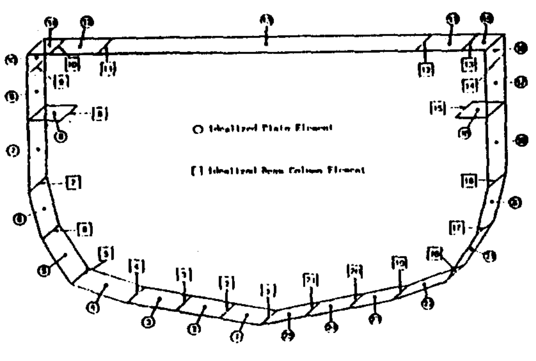

Figure 7: ALPS/ISUM modelling for the 1/3-scale frigate test model

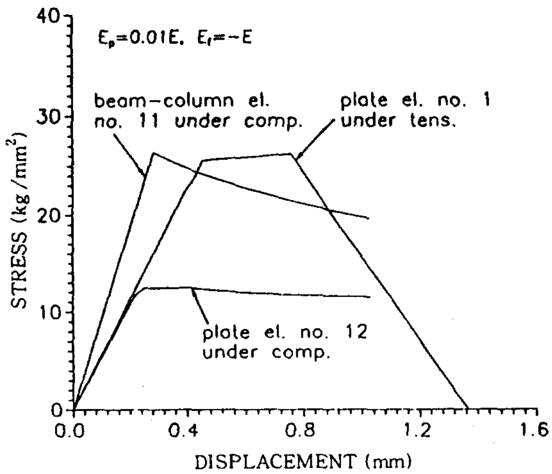

Figure 9: Axial stress-displacement curve of typical compressed and tensiled members for the 1/3-scale frigate test model

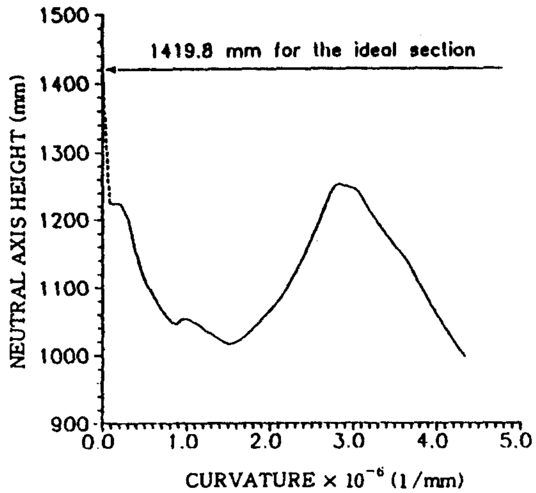

Figure 10: Change of the neutral axis position with increase in the sagging moment for the 1/3-scale frigate test model

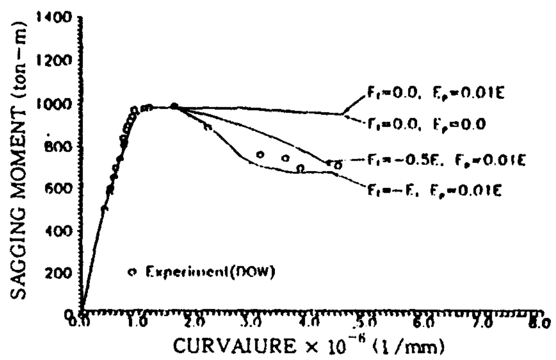

Figure 11: Influence of the excessive tension-deformation on the progressive collapse behavior for the 1/3-scale frigate test model