Failure Detection Filter for the Sensor and Actuator Failure in the Auto-Pilot System

SangHyun Suh*

Abstract

Auto-Pilot System uses heading angle information via the position sensor and the rudder device to control the ship's direction. Most of the control logics are composed of the state estimation and control algorithms assuming that the measurement device and the actuator have no fault except the measurement noise. But such asumptions could bring the danger in real situation. For example, if the heading angle measuring device is out of order the control action based on those false position information could bring serious safety problem. In this study, the control system including improved method for processing the position information is applied to the Auto-Pilot System. To show the difference between general state estimator and F.D.F., BJDFs for the sensor and the actuator failure detection are designed and the performance are tested. And it is shown that bias error in sensor could be detected by state-augmented estimator. So the residual confined in the 2-dimension in the presence of the sensor failure could be unidirectional in output space and bias sensor error is much easier to be detected.

1 Introduction

The automatic navigation technology of the ship has been developed due to the requirement of the safety and the speediness, the accuracy. System failure detection and reconstruction of the fault system became highlighted to increase the reliability of the automated ship. Also the research field related to the failure detection of the ocean engineering system became paid attention when the oil drilling or deep ocean exploit requires the high tech and high risk drilling ship appears.

In 1980, when Balchen. et al[1] designed the control system, he took into account of the possible failure of the measurement system. That is, he introduced a failure detection technique by putting an error limit to the measurement value to replace the off the limit measurement value with the estimation. This was very simple failure detection method to be found very usfull when it was adapted to the diving and support vessel "Sea way Eagle" in North Sea Ocean oil exploit site. But the frequent measurement blocking phenomenan made the reliability of the failure detection system very poor. This was big problem at the real field work so the failure detection technique was seriously paid attention from that

*Member, Maneuvering and Control Lab., KRISO

time. Afterwords, the fuilure detection filter theory adapted to the airplane engine failure, was applied to the failure detection of the actuator of SWATH ship in 1990 to be found applicable[2].

In this paper, the detection filter theory, which detects the failure in the system by generating the analytic redundancy using system dynamics, is introduced among many of the failure detection techniques. For we have goed reference model which is result of the study of the system dynamics in the ship maneuverability. And then the new failure detection filter using state augmented estimator in the case of the sensor failure was derived. Also it was applied to ship auto-pilot system to be shown useful for the failure detection.

2 Failure Detection Technique

2.1 Problem Modelling

A major analytic contribution to failure detection in linear systems is due to Beard (Beard(1971))[and Jones (Jones(1973))[4]. The approach used by Beard is similar to that used in state estimation theory and linear observer theory.

It is assumed that the system to be monitored is represented by a linear, time-invariant model with observable dynamics. The state space representation of the system is,

$$\begin{aligned} \dot{\mathbf{x}}(t) &= A\mathbf{x}(t) + B\mathbf{u}(t), \\ \mathbf{y}(t) &= C\mathbf{x}(t), \end{aligned} \tag{1}$$

where $\mathbf{x}(t)$ is the state n-vector; $\mathbf{u}(t)$ is the input l-vector; and $\mathbf{y}(t)$ is the output m-vector. The matrix A is the $n \times n$ system matrix, B is the $n \times l$ control matrix and C is the $m \times n$ output matrix. The basic state model equations of the detection filter are similar to eq. (1) except for the presence of an output error feedback term; i.e.,

$$\begin{aligned} \dot{\hat{\mathbf{x}}}(t) &= A\hat{\mathbf{x}}(t) + B\mathbf{u}(t) + D[\mathbf{y}(t) - \hat{\mathbf{y}}(t)], \\ \hat{\mathbf{y}}(t) &= C\hat{\mathbf{x}}(t), \end{aligned} \tag{2}$$

with $\hat{\mathbf{x}}(t)$ as the filter state, and $\hat{\mathbf{y}}(t)$ the expected measurement output.

The output residual is defined as the differences between the actual sensor outputs and the detection filter outputs; i.e.,

$$\mathbf{q}(t) = \mathbf{y}(t) - \hat{\mathbf{y}}(t). \tag{3}$$

Also the state error , $e(t)$, is defined to represent the difference between the actual system state and the detection filter state; i.e.,

$$e(t) = \mathbf{x}(t) - \hat{\mathbf{x}}(t). \tag{4}$$

Using eqs. (1) through (4), we obtain the state error equation and the residual $\mathbf{q}(t)$ as following,

$$\dot{e}(t) = [A - DC]e(t), \tag{5}$$
$$\mathbf{q}(t) = Ce(t). \tag{6}$$

Proper choice of the feedback gain matrix D is the goal of detection filter design. This choice must satisfy the following requirements. One requirement is that $[A - DC]$ be stable so that any deviation in $e(t)$ introduced by incorrect initial conditions will die out, allowing $\hat{\mathbf{x}}(t)$ to approach $\mathbf{x}(t)$. Another is that when a selected component of the system fails, the residual generated by the detection filter have a specific, time-invariant direction in the residual space. Thus, it will be readily recognizable as a result of the particular failure in the real system.

2.2 Failure Models

In the event of a failure, (A, B, C) and eqs. (1) upon which the reference model is based no longer describe the system dynamics, and $\hat{\mathbf{y}}(t)$ will deviate from $\mathbf{y}(t)$. By assuming a suitable mathematical model for failures of the various system components, we can examine the corresponding behavior of the residual $\mathbf{q}(t)$. Three general cases of failure will be considered: actuator failures, system dynamics changes, and sensor failures.

2.2.1 Actuator Failure

If failure occurs in the i-th component of the actuator, the failure must result in a change in the $u_i(t)$ with unknown magnitude, $n(t)$. Then the real system with an actuator failure in the i-th actuator component becomes,

$$\dot{\mathbf{x}}(t) = A\mathbf{x}(t) + B\mathbf{u}(t) + \mathbf{b}_i n(t), \tag{7}$$

where $\mathbf{b}_i$ is the i-th column of control matrix B and u_i is the i-th component of $\mathbf{u}(t)$. The failure does not alter the reference model, so the difference between the system state and the filter state becomes,

$$\dot{e}(t) = [A - DC]e(t) + \mathbf{b}_i n(t). \tag{8}$$

The vector $\mathbf{b}_i$ will be called the event vector associated with the i-th actuator; $n(t)$ is the scalar time function that multiplies the event vector. We also have,

$$\mathbf{q}(t) = Ce(t). \tag{9}$$

2.2.2 Sensor Failure Model

Since sensors are not modeled in the state equation, sensor failure must be represented by changes in only in the measurement equation. The changes are simple, though, because all possible failures of any of these sensors can be modeled by one term composed of the appropriate unit vector times a ime-varying scalar. For instance, failure of the j-th sensor is modeled by,

$$\begin{aligned} \dot{\mathbf{x}}(t) &= A\mathbf{x}(t) + B\mathbf{u}(t), \\ \mathbf{y}(t) &= C\mathbf{x}(t) + \mathbf{E}_j n(t). \end{aligned} \tag{10}$$

Here, $\mathbf{E}_j$ is the j-th unit vector in the output space, and $n(t)$ is again an arbitrary scalar time function. Since the failure term appears in the measurement equation, we call eq. (10) the sensor failure model.

In contrast to the other failures that we have described, those modeled by the sensor failure model produce a residual vector that cannot be held fixed in direction in output space. It can, however, always be constrained to a plane that is uniquely associated with the particular failed sensor. The output vector $\mathbf{y}(t)$ is fed into the reference model through the term $D[\mathbf{y}(t) - \hat{\mathbf{y}}(t)]$, so a failure of the j-th sensor alters the model by,

$$\mathbf{d}_j n(t) = D\mathbf{E}_j n(t), \tag{11}$$

where $\mathbf{d}_j$ is the j-th column of D. Consequently, the equations for the residuals become,

$$\begin{aligned} \dot{e}(t) &= [A - DC]e(t) - \mathbf{d}_j n(t), \\ \mathbf{q}(t) &= Ce(t) + \mathbf{E}_j n(t). \end{aligned} \tag{12}$$

3 Detection Filter Theory

Assuming the system measurements are perfect, the reference model output differs from the system output when a failure has occurred. Beard showed that this output error can be used to modify the state of the reference model in such a manner that the output error generated by certain system failures has a fixed direction in output space so that the output residual is easily recognizable and can be associate with the particular failure.

Proper choice of the feedback gain matrix D is the goal of detection filter design. This choice must satisfy the following requirements. One requirement is that $[A - DC]$ be stable so that any deviation in $e(t)$ introduced by incorrect initial conditions will die out, allowing $\hat{\mathbf{x}}(t)$ to approach $\mathbf{x}(t)$. Another is that when a selected component of the system fails, the residual generated by the detection filter have a specific, time-invariant direction in the residual space. Thus, it will be readily recognizable as a result of the particular failure in the real system.

The important feature of the detection filter is that the settled-out error $e(t)$ for a single event maintains a fixed direction in the state space. Of course, this also means that $Ce(t)$ maintains a fixed direction in the output space.

The design of detection filters is primarily concerned with being able to specify certain properties of the matrix $(A - DC)$ by choice of D. It is known that if (A, C) is an observable pair, then all n eigenvalues of $(A - DC)$ can be arbitrarily specified by choice of D (Kailath (1980)).

3.1 Beard-Jones Detection Filter for Actuator Failures

The failure associated with the event vector **f** is detectable if and only if,

(1) the residual $\mathbf{q}(t)$ generated by $\mathbf{f}n(t)$ maintains a fixed direction in the output space, and

(2) the eigenvalues of $A - DC$ can be specified almost arbitrarily by the proper choice of the detection filter gain matrix D.

However, the results of detection theory say that if the plant is observable; i.e., if (A, C) is an observable pair, event information in the sense of Definition 1 can still be produced by some detection filter. The limited capacity might mean that it may take a number of different filters to provide all the event information.

From linear system theory, it is known that the solution of eq. (9) $e(t)$ must be contained in the controllable subspace of **f**; i.e,

$$W = [\mathbf{f} \quad (A - DC)\mathbf{f} \quad \cdots \quad (A - DC)^{n-1}\mathbf{f}]. \tag{13}$$

From eqs. (9) and (13), it is concluded that the output residual $\mathbf{q}(t)$ will be constrained to the subspace spanned by the columns of CW. Condition (1) of Definition 1 can, therefore, be revised to required that D should satisfy,

$$rank \;\; CW = 1, \tag{14}$$

while maintaining the freedom to assign the eigenvalues of $(A - DC)$. This is the distinguishable aspect by which the detection filter differs from the general state observer; the detection filter gain D must satisfy Condition(1) for some **f**. So it is refered to as a detection filter for that **f**.

By Condition(2), the matrix $(A - DC)$ should at least be stable so that the initial condition term in the solution of eq. (14) will die out. Also, it would be desirable to have enough freedom to specify the eigenvalues of $(A - DC)$ to be able to influence the time required for $Ce(t)$ to settle out.

Once the detection filter which satisfys the eq. (14) has been designed, the output residual $Ce(t)$ is unidirectional. The output residual is a linear transform of $e(t)$, which is spanned by column vectors of W, through C. It is defined that a vector space called "the event space" is composed of the vectors representing all possible failures, real or ficticious, that would induce a unidirectional output residual along $C\mathbf{f}$, given that the detection filter is designed to monitor the failure driven by **f**. The basis vectors for the event space are generated by g, the detection generator, as a cyclic subspace with respect to $(A - DC)$. If the dimension of the event space is ν, then the basis vectors for the event space are,

$$\{\mathbf{g}, \;\; (A - DC)\mathbf{g}, \;\; \cdots, (A - DC)^{\nu-1}\mathbf{g}\}. \tag{15}$$

If g is chosen as follows;

$$\begin{bmatrix} C \\ CA \\ \cdot \\ \cdot \\ CA^{\nu-2} \end{bmatrix} \mathbf{g} \;=\; \mathbf{0}, \tag{16}$$

$$CA^{\nu-1}\mathbf{g} \neq \mathbf{0},$$

then the following holds:

$$\{\mathbf{g}, \ (A-DC)\mathbf{g}, \ \cdots, \ (A-DC)^{\nu-1}\mathbf{g}\} = \{\mathbf{g}, \ A\mathbf{g}, \ \cdots, \ A^{\nu-1}\mathbf{g}\}. \quad (17)$$

Thus, if the g and ν are found, the event vector f can be expressed as a combination of basis vectors for the event space,

$$\mathbf{f} = \alpha_1\mathbf{g} + \alpha_2 A\mathbf{g} + \cdots + A^{\nu-1}\mathbf{g}. \quad (18)$$

Multiplying C on both sides of eq. (18), and with the aid of eq. (16),

$$C\mathbf{f} = CA^{\nu-1}\mathbf{g}. \quad (19)$$

The Caylie-Hamilton Theorem[5] says that if ν eigenvalues of $(A - DC)$ are given by the roots of

$$s^{\nu} + p_{\nu}s^{\nu-1} + \cdots + p_2 s + p_1 = 0, \quad (20)$$

then $(A-DC)^{\nu}\mathbf{g} + p_{\nu}(A-DC)^{\nu-1}\mathbf{g} + \cdots + p_2(A-DC)\mathbf{g} + p_1\mathbf{g} = \mathbf{0}$. Using eq. (16) and eq. (18) yields,

$$DCA^{\nu-1}\mathbf{g} = DC\mathbf{f} = p_1\mathbf{g} + p_2 A\mathbf{g} + \cdots + p_{\nu}A^{\nu-1}\mathbf{g} + A^{\nu}\mathbf{g}. \quad (21)$$

Here the p_i are at the designer's will and if ν is given, the detection gain D can be calculated by using the general solution form;

$$\begin{aligned} D &= [p_1\mathbf{g} + \cdots + p_k A^{k-1}\mathbf{g} + A^k\mathbf{g}][(C\mathbf{f})^T C\mathbf{f}]^{-1}(C\mathbf{f})^T \\ &+ D_0[I - C\mathbf{f}[(C\mathbf{f})^T C\mathbf{f}]^{-1}(C\mathbf{f})^T], \end{aligned} \quad (22)$$

as the general solution. Note that $[(C\mathbf{f})^T C\mathbf{f}]$ is a nonzero scalar since $C\mathbf{f} \neq \mathbf{0}$. For D given by eq. (22),

$$\begin{aligned} A - DC &= A - [p_1\mathbf{g} + \cdots + p_{\nu}A^{\nu-1}\mathbf{g} + A^{\nu}\mathbf{g}][(C\mathbf{f})^T C\mathbf{f}]^{-1}(C\mathbf{f})^T \\ &+ D_0[I - C\mathbf{f}[(C\mathbf{f})^T C\mathbf{f}]^{-1}(C\mathbf{f})^T]C \\ &= A_0 - D_0 C_0, \end{aligned} \quad (23)$$

where,

$$A_0 = A - [p_1\mathbf{g} + \cdots + p_{\nu}A^{\nu-1}\mathbf{g} + A^{\nu}\mathbf{g}][(C\mathbf{f})^T C\mathbf{f}]^{-1}(C\mathbf{f})^T, \quad (24)$$

$$C_0 = [I - C\mathbf{f}[(C\mathbf{f})^T C\mathbf{f}]^{-1}(C\mathbf{f})^T]C. \quad (25)$$

When Condition(1) $rank\ CW = 1$ is satisfied, ν-eigenvalues of $(A - DC)$ are specified. The number of eigenvalues of $(A_0 - D_0C_0)$ to be specified by free choice of D_0, which is same as the remaining number of the eigenvalues of the $(A - DC)$ once D is chosen to satisfy the Condition(1), is given by,

$$q' = rk \begin{bmatrix} C_0 \\ C_0 A_0 \\ \vdots \\ C_0 A_0^{n-1} \end{bmatrix} = rk \begin{bmatrix} C_0 \\ C_0 K_0 \\ \vdots \\ C_0 K_0^{n-1} \end{bmatrix}, \tag{26}$$

where C_0 is defined by eq. (25) and,

$$K_0 = A - A\mathbf{f}[(C\mathbf{f})^T C\mathbf{f}]^{-1}(C\mathbf{f})^T C. \tag{27}$$

Therefore, the number of eigenvalues that can be assigned when D is constrained to a solution of eq. (14), is independent of the choice of g or ν. Now the detectability of f depends on the existence of the detection generator, g, whose order is $\nu = n - q'$, the maximal detection order. Beard[3] proved the existance and uniqness of this maximal detection generator.

3.2 State Augmented FDF for Sensor Failure

For some types of failures like the bias error in teh sensor, we could also utilize the state augmentation method. This occurrence can be represented by measurement process described by,

$$\mathbf{y}(t) = C\mathbf{x}(t) + \mathbf{b}(t), \tag{28}$$

where b(t) is an m-dimensional vector whose value is unknown. By state augmentation, a new augmented state vector $\mathbf{x}_a(t) = [\mathbf{x}(t),\ \mathbf{b}(t)]^T$ is defined with dimension of augmented state $(n + m)$. In general, the augmented state dynamics can be modelled by,

$$\dot{\mathbf{b}}(t) = A_{21}\mathbf{x}(t) + A_{22}\mathbf{b}(t). \tag{29}$$

Then, the augmented system becomes,

$$\begin{bmatrix} \dot{\mathbf{x}}(t) \\ \dot{\mathbf{b}}(t) \end{bmatrix} = \begin{bmatrix} A & \mathbf{0} \\ A_{21} & A_{22} \end{bmatrix} \begin{bmatrix} \mathbf{x}(t) \\ \mathbf{b}(t) \end{bmatrix} + \begin{bmatrix} G \\ \mathbf{0} \end{bmatrix} \mathbf{u}(t), \tag{30}$$

$$\mathbf{y}(t) = [C \quad I] \begin{bmatrix} \mathbf{x}(t) \\ \mathbf{b}(t) \end{bmatrix}. \tag{31}$$

where I is the identity matrix.

Furthermore, since the system of eq. (30) has no bias error itself, the state vector estimate is automatically compensated for the presence of the measurement bias. First, let's think about the development of an observer to estimate the bias error in sensor. If b(t) is an unknown constant parameter, then it can be modeled by the vector differential equation,

$$\dot{\mathbf{b}}(t) = \mathbf{0}.$$

Augmenting the system to the order of $n+m$ yields,

$$\begin{aligned} \begin{bmatrix} \dot{\mathbf{x}}(t) \\ \dot{\mathbf{b}}(t) \end{bmatrix} &= \begin{bmatrix} A & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} \mathbf{x}(t) \\ \mathbf{b}(t) \end{bmatrix} + \begin{bmatrix} G \\ 0 \end{bmatrix} \mathbf{u}(t), \\ \mathbf{y}(t) &= \begin{bmatrix} C & I \end{bmatrix} \begin{bmatrix} \mathbf{x}(t) \\ \mathbf{b}(t) \end{bmatrix}, \end{aligned} \tag{32}$$

and an observer can be designed for this augmented system; i.,e.,

$$\begin{bmatrix} \dot{\hat{\mathbf{x}}}(t) \\ \dot{\hat{\mathbf{b}}}(t) \end{bmatrix} = \begin{bmatrix} A & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} \hat{\mathbf{x}}(t) \\ \hat{\mathbf{b}}(t) \end{bmatrix} + \begin{bmatrix} G \\ 0 \end{bmatrix} \mathbf{u}(t) + K[\mathbf{y}(t) - C\hat{\mathbf{x}}(t) - \hat{\mathbf{b}}(t)].$$

By examining the $\hat{\mathbf{b}}(t)$, the bias errors in the sensors can be detected.

4 Application to Auto-Pilot System

Let's test the performance of the detection filter which uses the model reference of the ship auto-pilot system, centerpiece of the ship automatic navigation. First, it is necessary to describe the ship motion equiped with auto-pilot system in the mathematical model. We assume that the ship auto-pilot system be composed of the magnetic compass or gyro compass to measure the heading angle and the controller which decides the command rudder angle from the difference between the actural heading and the desired heading, and the actuator which rotate the rudder. So the whole system has directional stability only. The block diagram of the system is shown in Figure 1.

4.1 State-Space Motion Equation

It is known that the ship maneuvering motion has strong nonlinearity, especially in case of turning with large rudder angle which make very complicated flow phenomenon around the ship hull. But when the auto-pilot system is operated in the ocean-going situation, the rudder angle is confined within 10 degree. And it is reasonable to assume that the ship motion be linear. In this linearization of the mathematical model, the validity test is necessary. For such a closed-loop system as the ship with auto-pilot system, the validity range of the linearized model must be estimated. In this paper, linearized sway-yaw motion equations are chosen for which every hydrodynamic coefficients are well known. The followings are mathematical description of linearized sway-yaw motions ;

$$\begin{aligned} -Y_v v + (m - Y_{\dot{v}})\dot{v} - (Y_r - m)r - (Y_{\dot{r}} - mx_G)\dot{r} &= Y_\delta \delta \\ -N_v v + (N_{\dot{v}} - mx_G)\dot{v} - (N_r - mx_G)r - (I_Z - N_{\dot{r}})\dot{r} &= N_\delta \delta \end{aligned} \tag{33}$$

here $v, \dot{v}, r, \dot{r}, \delta$ are sway velocity, acceleration, yaw velocity, acceleration, and rudder angle respectively. And every definitions of the hydrodynamic coefficients are those of ITTC's. Above maneuvering motion equations are rewrittn as a state-space variable equation;

$$\frac{d}{dt}\begin{bmatrix} v(t) \\ r(t) \\ \phi(t) \end{bmatrix} = \begin{bmatrix} a_{11} & a_{12} & 0 \\ a_{21} & a_{22} & 0 \\ 0 & 1 & 0 \end{bmatrix}\begin{bmatrix} v(t) \\ r(t) \\ \phi(t) \end{bmatrix} + \begin{bmatrix} b_{11} \\ b_{21} \\ 0 \end{bmatrix}\delta(t) \tag{34}$$

$$y(t) = \phi(t) = [0 \;\; 0 \;\; 1]\begin{bmatrix} v(t) \\ r(t) \\ \phi(t) \end{bmatrix}$$

here ϕ is heading angle. Actually there is time lag through the steering system between the real rudder angle and therudder command. Generally this time lag can be expressed as 1st order time lag system, say, $\dot{\delta}(t) = -\tau\delta(t) + \tau\delta_c(t)$. Also there is a rudder angle feedback unit with which the actual rudder angle can be measured. If this actuator model is involved in the state space equation, the whole state-space equation became,

$$\begin{aligned} \dot{\mathbf{x}}(t) &= A\mathbf{x}(t) + B\mathbf{u}(t), \\ \mathbf{y}(t) &= C\mathbf{x}(t) \end{aligned} \tag{35}$$

where $x = [v, r, \phi, \delta]^T$ and

$$A = \begin{bmatrix} a_{11} & a_{12} & 0 & b_{11} \\ a_{21} & a_{22} & 0 & b_{22} \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & -\tau \end{bmatrix}, B = \begin{bmatrix} 0 \\ 0 \\ 0 \\ \tau \end{bmatrix}.$$

And the measurement equation is composed of the heading angle by the compass and the rudder angle by the rudder angle feedback unit as following,

$$C = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}.$$

4.2 Controller Design

The algorithm for auto-pilot system,$\delta_c = f(v, r, \phi, \delta)$ the function f is modelled as a P.I.D controller, and the feedback gains are determined for the stable closed loop system. It is necessary to test the controllability of the system before the controller is designed. If the controllability matrix.

$$\theta = [B \; AB \; ... \; A^{n-1}B]$$

has full rank, say, same rank as the system matrix A, then the system is controllable. So eq(32) has rank 4 in this system and is controllable. And the simplified closed loop system without rudder angle feedback unit and rudder dynamics could be modelled as a SISO (single input - single output) system and the feedback gain could be obtained by pole placement technique. eq(30) could be expressed in Laplace domain as $G(s) = C(sI - A)^{-1}B$ following,

$$G(s) = \frac{a_{21}b_{11} + b_{22}(s - a_{11})}{s(s^2 - (a_{11} + a_{22})s + a_{11}a_{22} - a_{12}a_{21})} \tag{36}$$

And the variation of the closed loop pole due to the variation of the feedback gain K is shown on Figure 2.

4.3 State Estimator Design

It is not possible that all the state variable could be measured in the measurement equation C. So the possibility of estimation of the rest state variable must be checked by using the measurable state variable and system dynamics. In case of the system composition for which the state estimation is impossible, it is necessary to consist the minimum measurement device for the state feedback controller possible. It is known that the observability of the system could be found by the checking of the rank of the observability matrix Ω is following ;

$$\Omega = \begin{bmatrix} C \\ CA \\ \cdots \\ CA^{n-1} \end{bmatrix} \tag{37}$$

And when Ω has the full rank, this system is said observable. When the system is observable, the state variable could be estimated by state estimator. It is known by Seperation Theorem [5] that if the state estimations are used in place of the real state variable in the observable system, the stability of the whole system including state estimator could be judged by the stabilities of each controller and state estimator. Then the dynamics of the state estimator could be ;

$$\dot{\hat{x}}(t) = A\hat{x}(t) + Bu(t) + K_E(y(t) - \hat{y}(t)) \tag{38}$$

When state estimation error $e(t)$ is defined as $e(t) = x(t) - \hat{x}(t)$, K_E is obtained to satisfy $e(t) \rightarrow 0$ if $t \rightarrow \infty$. The state estimation could replace the real state variable and if the estimator is designed to have the much faster dynamics than that of the controller, the effect of the dynamics of the estimator could be negligible for the whole system. We could see the estimator design problem is dual of the controller design. So K_E is decided as for the system (A^T, C^T) to be controllable like the KC is in (A,C). In Figure 3, the block diagram of the controller and state estimator is shown.

4.4 FDF(Failure Detection Filter) Design

The simple state estimator is designed i the above section and the F.D.F is designed in this section to be tested as a state estimator in case of no system failure and as a failure detection filter in cas of the system failure. The detection filter which is designed as a BJDF in section 3 is as following ;

$$D = \begin{bmatrix} -91.677 & 154.168 \\ 87.032 & -214.623 \\ 22.135 & -51.430 \\ 1.459 & 0 \end{bmatrix} \tag{39}$$

The simulation results which compare the function of FDF and simple state estimator given actuator failure f (Bias Error Ramdom Disturbance) are shown in Figure 4, Figure 5. As shown in Figure 4, the residual generated by the FDF has unidirection [1 0] in the output space to show that the system has the actuator failure. But in Figure 5, the residual generated by simple state estimator gives the state estimation only, without any information related to the failure in the system. Moreover the state estimation is carried out using wrong measurement value that came from the system failure. And it is dangerous to use those state estimation for the state feedback controller. Also we can see the bias error in the sensor could be detected by the augmented state estimator in Figure 6.

5 Conclusions

There are many approaches for the failure detection in the dynamic system. When the system dynamics is expressed mathematically and its validity is proved like the ship maneuvering equation, it is found that the failure detection and identification technique by analytical redundancy using reference model is useful. In detail, the failure detection filter theory was introduced and the failures in the system were mathematically modelled. Especially the state augmented estimator in which bias error is included as a new augmented state, detects the sensor bias error. Though it is restricted, the difficulty of the sensor failure detection has been removed by the state angmented estimator which change the sensor failure into actuator failure to be found easy to detect the failure. Finally the failure detection filter and state estimator were applied to the auto-pilot system of the ship and numerical simulation was made to show the differences.

References

[1] Balchen, J., G., Jenssen, N. A., and Saelid,S., "Dynamic positioning Using Kalman Filtering and Optimal Control Theory", *Proceedings of the IFAC/IFIP Symposium on Automation in Offshore Oil Field Operations*, North Holland Publishing, Amsterdam, 1976

[2] Ryan R. Kim "Failure Detection and Control of an Automatic Control System in SWATH" *Proceedings 9th Ship Control System Symposium*,

[3] Beard,R.V., "Failure Accommodation in Linear Systems Through Self-Reorganization", *Report MIT-71-1*, Man Vehicle Laboratory, MIT, Feb., 1971

[4] Jones, H.L., Failure Detection in Linear Systems, PhD. Dissertation, MIT, Sept., 1973

[5] Kailath, F., *Linear Systems*, Prentice-Hall, 1980

[6] Deckert, J. C., M.N. Desai, J.J. Deyst and A.S. Willsky, "F-8 DFBW Sensor Failure Identification using Analytic Redundancy", *IEEE Trans. Automatic Control*, Vol. 22, 1977

[7] Patton, R.J., P.M. Frank and R.N. Clark *Fault Diagnosis in Dynamic Systems : Theory and Applications*, Prentice-Hall, 1989

[8] *Matrix User's Guide ; Engineering Analysis and Control Design*, Integrated System Inc., 1987

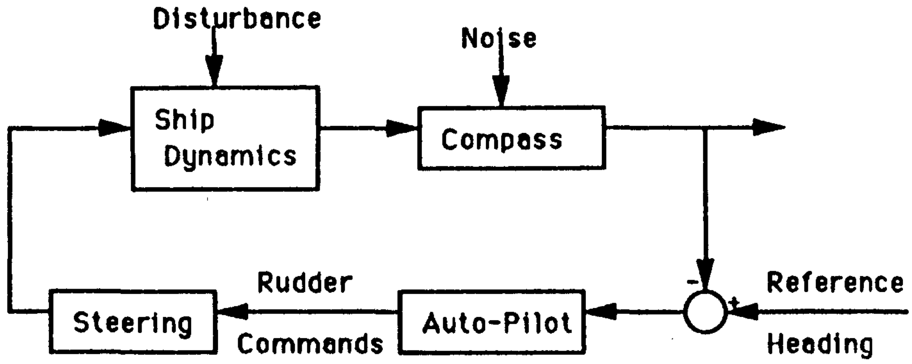

Figure 1: Auto-Pilot system composition

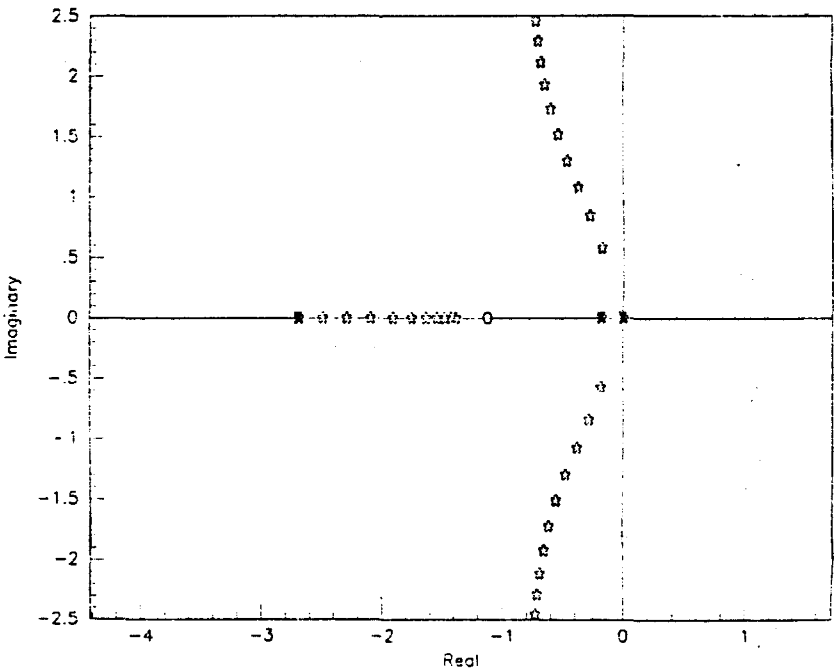

Figure 2: Closed-loop system poles due to feedback gain K_c changes

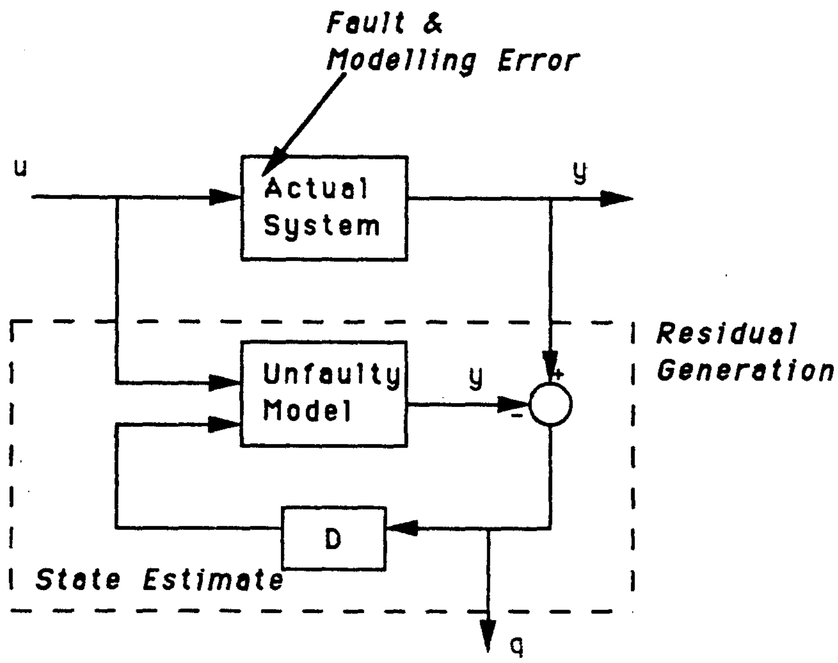

Figure 3: Basic configuration of residual generation through state estimation

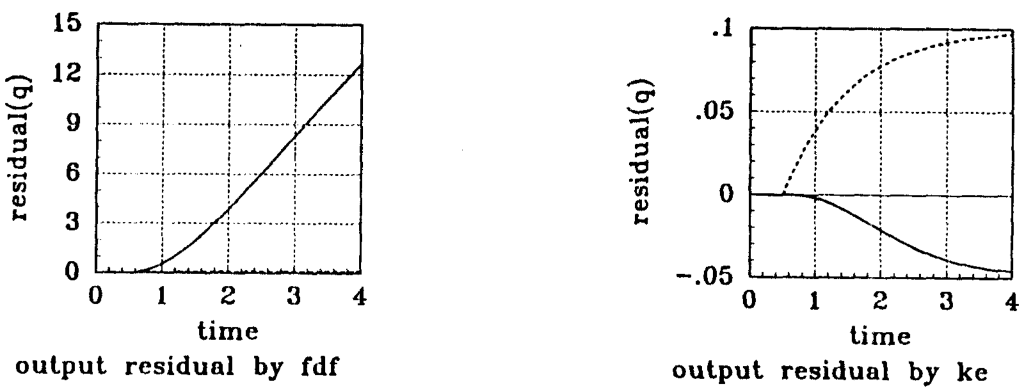

Figure 4: Time history of residual by detection filter and simple state estimator due to bias error

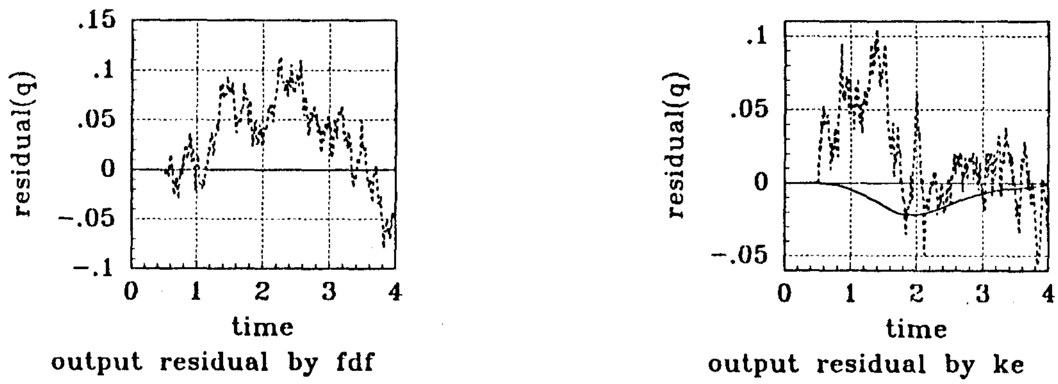

Figure 5: Time history of residual by detection filter and simple state estimator due to random disturbance

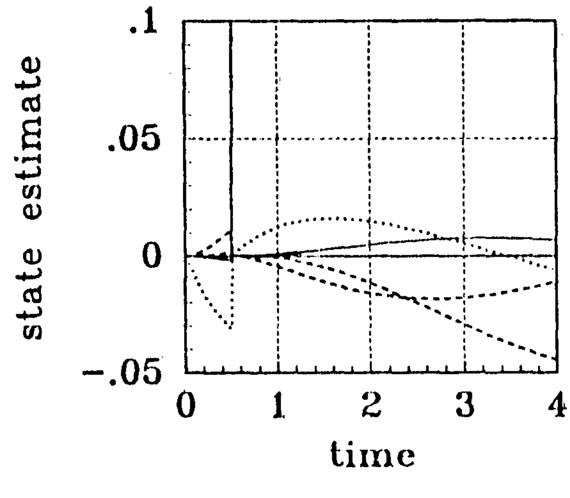

Figure 6: State estimate of augmented system