Estimation of Hydrodynamic Derivatives by Parallel Processing of Second Order Filter

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Abstract

Unknown parameters can be determined by system identification techniques. Extended Kalman filter method was introduced as a real time estimator of hydrodynamic derivatives but it has the problem named the coefficient drift.

In this study, 2nd order filter estimates hydrodynamic derivatives in Abkowitz model In order to reduce the coefficient drift, parallel processing is used. The measured state and ship trajectory are compared with the estimated values.

Parallel processing of 2nd order filter gives very similar results to parallel processing of extended Kalman filter. Parallel processing cannot not remove the coefficient drift perfectly, but it reduces the estimation error.

1 Introduction

System identification determines, from a given input/output record of vehicle test response, an estimate of the physical model which relates to the observed data. The actual processing of the data requires three major steps: model structure determination, parameter estimation, and model validation. The model structure determination is a process of selecting a mathematical form for the equations of motion. In ship hydrodynamics, the mathematical model is determined by Taylor series expansion of hydrodynamic forces and moments in equations of motion of a ship. Abkowitz model, MMG model, and Nomoto's K-T model are widely used. There are many methods of the parameter estimation. Nomoto's model is identified by Fujino [4] with the phase portrait in 1972. The output error method is applied by Trankle in 1985, which minimizes the fit error between the measured ship trajectory and the trajectory estimated by a simulation. Abkowitz applied extended Kalman filter to the estimation of hydrodynamic derivatives successfully. In this method, unknown parameters are treated as additional differential equations to be integrated along with equations of the ship motion. The extended Kalman filter method has the advantage of being able to provide parameter estimates in real time, but its convergence to the best parameter values seem to be more difficult to control. Identifiability is enhanced by careful attention. The solutions

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of the coefficient drift and the error cancellation are keys to successful identification. To solve the coefficient drift problem, technique of parallel processing and over and under initial estimates were developed. Kang, et al. [7] concluded that the coefficient drift was caused by ratio of coefficients. Ahn [2] applied 2nd order filter, but did not remove the coefficient drift perfectly. In this study, 2nd order filter estimates hydrodynamic derivatives in Abkowitz model. In order to reduce the coefficient drift, parallel processing is used. The measured state and ship trajectory are compared with the estimated values. The estimation error between by the 2nd order filter and by the extended Kalman filter is little. Parallel processing cannot prevent perfectly the simultaneous drift of the hydrodynamic derivatives. But it reduces the estimation error.

2 Mathematical Model

6-DOF motions of a ship are reduced to 3-DOF planar motions under some assumptions as follows: - Froude number is low, and trim and sinkage are small. - The amplitudes of roll, heave and pitch motions are small. - The acceleration is linear in the hydrodynamic forces. - Current is not considered. - Manoeuvring coefficients are time invariant. Coordinates systems, the inertial reference frame $o_o x_o y_o$ and the body fixed coordinates oxy, are shown in Figure 1. Where G is the longitudinal center of gravity, and δ is the rudder angle. u and v are x and y component of the total velocity U. X, Y and N means surge force, sway force and yaw moment respectively.

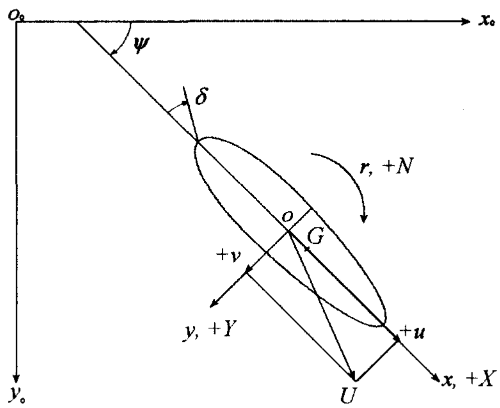

Figure 1: Coordinate systems

The mathematical model to describe the motion of a ship is Abkowitz model which is modified by the identification with the extended Kalman filter method. The modified Abkowitz model is as follows:

$$\dot{u} = \frac{f_1}{m - X_{\dot{u}}}$$

$$\dot{v} = \frac{(I_z - N_{\dot{r}})f_2 - (mx_G - Y_{\dot{r}})f_3}{f_4}$$

$$\dot{r} = \frac{(m - Y_{\dot{v}})f_3 - (mx_G - N_{\dot{v}})f_2}{f_4}$$

where f_i are shown as below:

$$\begin{aligned}
f_1 = \; & \eta_1' \left[\frac{\rho}{2}L^2\right] u^2 + \eta_2' \left[\frac{\rho}{2}L^3\right] nu + \eta_3' \left[\frac{\rho}{2}L^4\right] n^2 - C_R' \left[\frac{\rho}{2}Su^2\right] \\
& + X_{vv}' \left[\frac{\rho}{2}L^2\right] v^2 + X_{ee}' \left[\frac{\rho}{2}L^2c^2\right] e^2 \left(X_{rr}' + m'x_G'\right) \left[\frac{\rho}{2}L^4\right] r^2 \\
& + \left(X_{vr}' + m'\right) \left[\frac{\rho}{2}L^3\right] vr + X_{vvrr} \left[\frac{\rho}{2}L^4U^{-2}\right] v^2r^2
\end{aligned}$$

$$\begin{aligned}
f_2 = \; & Y_0' \left[\frac{\rho}{2}L^2 \left(\frac{u_{A\infty}}{2}\right)^2\right] + Y_v' \left[\frac{\rho}{2}L^2U\right] v - Y_\delta'(c - c_0) \left[\frac{\rho}{2}L^2\right] v \\
& + (Y_r' - m'u') \left[\frac{\rho}{2}L^3U\right] r + \frac{Y_\delta'}{2}(c - c_0) \left[\frac{\rho}{2}L^3\right] r \\
& + Y_\delta' \left[\frac{\rho}{2}L^2c^2\right] \delta + Y_{vvv}' \left[\frac{\rho}{2}L^2U^{-1}\right] v^3 + Y_{vvr}' \left[\frac{\rho}{2}L^3U^{-1}\right] v^2r \\
& + Y_{rrv}' \left[\frac{\rho}{2}L^4U^{-1}\right] r^2v + Y_{rrr}' \left[\frac{\rho}{2}L^5U^{-1}\right] r^3 + Y_{eee}' \left[\frac{\rho}{2}L^2c^2\right] e^3
\end{aligned}$$

$$\begin{aligned}
f_3 = \; & N_0' \left[\frac{\rho}{2}L^3 \left(\frac{u_{A\infty}}{2}\right)^2\right] + N_v' \left[\frac{\rho}{2}L^3U\right] v - N_\delta'(c - c_0) \left[\frac{\rho}{2}L^3\right] v \\
& + (N_r' - m'x_G'u') \left[\frac{\rho}{2}L^4U\right] r + N_\delta'(c - c_0) \left[\frac{\rho}{2}L^4\right] r \\
& + N_\delta' \left[\frac{\rho}{2}L^3c^2\right] \delta + N_{vvv}' \left[\frac{\rho}{2}L^3U^{-1}\right] v^3 + N_{vvr}' \left[\frac{\rho}{2}L^4U^{-1}\right] v^2r \\
& + N_{rrv}' \left[\frac{\rho}{2}L^5U^{-1}\right] r^2v + N_{rrr}' \left[\frac{\rho}{2}L^6U^{-1}\right] r^3 + N_{eee}' \left[\frac{\rho}{2}L^3c^2\right] e^3
\end{aligned}$$

$$\begin{aligned}
f_4 = \; & (m' - Y_{\dot{v}}') \left[\frac{\rho}{2}L^3\right] (I_z' - N_{\dot{r}}') \left[\frac{\rho}{2}L^5\right] \\
& - (m'x_G' - N_{\dot{v}}') \left[\frac{\rho}{2}L^4\right] (m'x_G' - Y_{\dot{r}}') \left[\frac{\rho}{2}L^4\right]
\end{aligned}$$

This simulation model has 36 hydrodynamic derivatives(or manoeuvring coefficients). Details about Abkowitz model are shown in Abkowitz [1] and Hwang [6].

3 Second Order Filter

Extended Kalman filter is the recursive formed, unbiased and minimum variance filter which is applied to nonlinear systems. The concept of 2nd order filter is that the filter equations contain 2nd order term of Taylor expansion with respect to the last estimated state vector while extended Kalman filter linearizes the system and measurement equations. 2nd order terms are obtained by analytic differentiation, while Ahn [2] computed numerically. The manoeuvring sea trials are the continuous time nonlinear system with the discrete time measurement as follows:

$$\dot{\underline{x}}(t) = \underline{f}[t, \underline{x}(t)] + \underline{w}(t)$$
$$\underline{z}_k = \underline{h}_k[\underline{x}(t_k)] + \underline{v}_k$$

The system dynamics function vector $\underline{f}$ and the measurement function vector $\underline{h}_k$ are differentiable to the state variable $\underline{x}$. The process noise $\underline{w}(t)$ is the Gaussian white noise of the spectral density $Q(t)$ and the measurement noise $\underline{v}_k$ is the Gaussian white noise of the covariance R_k. The continuous-discrete 2nd order filter can be summarized as follows:

$$\dot{\hat{\underline{x}}}(t) = \hat{\underline{f}}[t, \hat{\underline{x}}(t)] + \frac{1}{2}\underline{\partial}^2[\underline{f}, P(t)]$$
$$\dot{P}(t) = F[t, \hat{\underline{x}}(t)]P(t) + P(t)F[t, \hat{\underline{x}}(t)]^T + Q(t)$$
$$\hat{\underline{x}}(t_k^+) = \hat{\underline{x}}(t_k^-) + K_k\Big\{\underline{z}_k - \underline{h}_k[\hat{\underline{x}}(t_k^-)] - \frac{1}{2}\underline{\partial}^2[\underline{h}_k, P(t_k^-)]\Big\}$$
$$P(t_k^+) = \Big\{I - K_k H_k[\hat{\underline{x}}(t_k^-)]\Big\}P(t_k^-)$$

Where

$$K_k = P(t_k^-)H_k[\underline{x}(t_k^-)]^T\Big\{H_k[\hat{\underline{x}}(t_k^-)]P(t_k^-)H_k[\hat{\underline{x}}(t_k^-)]^T + A_k + R_k\Big\}^{-1}$$
$$A_k = \frac{1}{4}E\Big\{\underline{\partial}^2[\underline{h}_k, P(t_k^-)]\underline{\partial}^2[\underline{h}_k, P(t_k^-)]^T\Big\} - \frac{1}{4}\underline{\partial}^2[\underline{h}_k, P(t_k^-)]\underline{\partial}^2[\underline{h}_k, P(t_k^-)]^T$$
$$F[t, \hat{\underline{x}}(t)] = \frac{\partial \underline{f}}{\partial \underline{x}}[t, \hat{\underline{x}}(t)]$$
$$H_k[\hat{\underline{x}}(t_k^-)] = \frac{\partial \underline{h}_k}{\partial \underline{x}}[\hat{\underline{x}}(t_k^-)]$$
$$\partial^2[\underline{f}, P(t)] = trace\Big\{\left[\frac{\partial^2 f_i}{\partial x_p \partial x_q}\right] P(t)\Big\}$$

This algorithm is valid in any time interval from t_{k-1} to t_k. Where, minus and plus signs mean values before and after measurement. In order to estimate the unknown parameters, the state variable is augmented by the unknown parameters.

$$\underline{x}(t) = \begin{bmatrix} \underline{x}(t) \\ \cdots\cdots \\ \underline{\theta} \end{bmatrix}$$

The state augmented 2nd order filter can estimate, from given input/output data, not only the original state variables but also the manoeuvring coefficients.

4 Parallel Processing

When extended Kalman filter was utilized for the parameter estimation, the simultaneous drift of hydrodynamic coefficients was found. It was caused by the cancellation of the coefficients contributions to the hydrodynamic forces and moments and highly relates to ratios of coefficients. Parallel processing helps 2nd order filter to obtain a better estimation of the coefficients. Parallel processing involves passing the data of two different manoeuvring tests simultaneously through the parameter estimation process. The state variable is augmented by combinations of two or more state valiable from any different manoeuvring tests. The dynamics of the state augmented system for parallel processing is represented as below:

$$\dot{\underline{x}}(t) = \begin{bmatrix} \dot{\underline{x}}_1(t) \\ \cdots\cdots \\ \dot{\underline{x}}_2(t) \\ \cdots\cdots \\ \dot{\underline{\theta}} \end{bmatrix} = \begin{bmatrix} \underline{f}[\underline{x}_1(t), \underline{\theta}] + \underline{w}_1(t) \\ \cdots\cdots\cdots\cdots\cdots\cdots \\ \underline{f}[\underline{x}_2(t), \underline{\theta}] + \underline{w}_2(t) \\ \cdots\cdots\cdots\cdots\cdots\cdots \\ \underline{0} \end{bmatrix}$$

Even though the system equation is nonlinear, the measurement equation is linear because the original state variables can be measured directly.

$$\underline{z}_k = \begin{bmatrix} \underline{z}_{1k} \\ \cdots \\ \underline{z}_{2k} \end{bmatrix} = \begin{bmatrix} \underline{x}_1(t_k) + \underline{v}_1(t_k) \\ \cdots\cdots\cdots\cdots\cdots \\ \underline{x}_2(t_k) + \underline{v}_2(t_k) \end{bmatrix}$$

5 Estimation Result

The Esso-Osaka is adopted in this paper, but any sea trial data are not provided except principal dimensions of the ship. The measurement data are obtained from computer simulations of $10^0/10^0$ zigzag manoeuver, $20^0/20^0$ zigzag manoeuver, and 35^0 turning circle manoeuver. The original state variables are surge velocity, sway velocity and yaw rate. The state vector is augmented by unknown manoeuvring coefficients and by state variables for another manoeuver. All the hydrodynamic derivatives are estimated simultaneously. Result of 2nd order filter method is compared with the extended Kalman filter method. Parallel processing result is compared with single processing result of each manoeuver. Figure 2 shows parallel processing results of the extended Kalman filter method and the 2nd order filter method. Little difference cannot say that 2nd order filter is superior to extended Kalman filter. Figure 3 and Figure 4 shows estimated hydrodynamic derivatives by single and parallel processing of 2nd order filter. Parallel processing of $10^0/10^0$ zigzag manoeuver and 35^0 turning circle manoeuver gives less error results than any single processing, but the simultaneous drift is not removed. Some manoeuvring sea trial is suitable to the estimation of some special coefficients, and this relation is not consistent for initial guesses.

From Figure 5 to Figure 7, simulation results by given state variables and manoeuvring parameters are compared with those by estimated values. Figure 5 shows that, in a point of view of state variables, the best simulation of $10^0/10^0$ zigzag manoeuver is obtained by the estimation by $10^0/10^0$ zigzag manoeuver data. The next is done by parallel processing. But, Figure 5 says that the estimation by 35^0 turning manoeuver gives the best

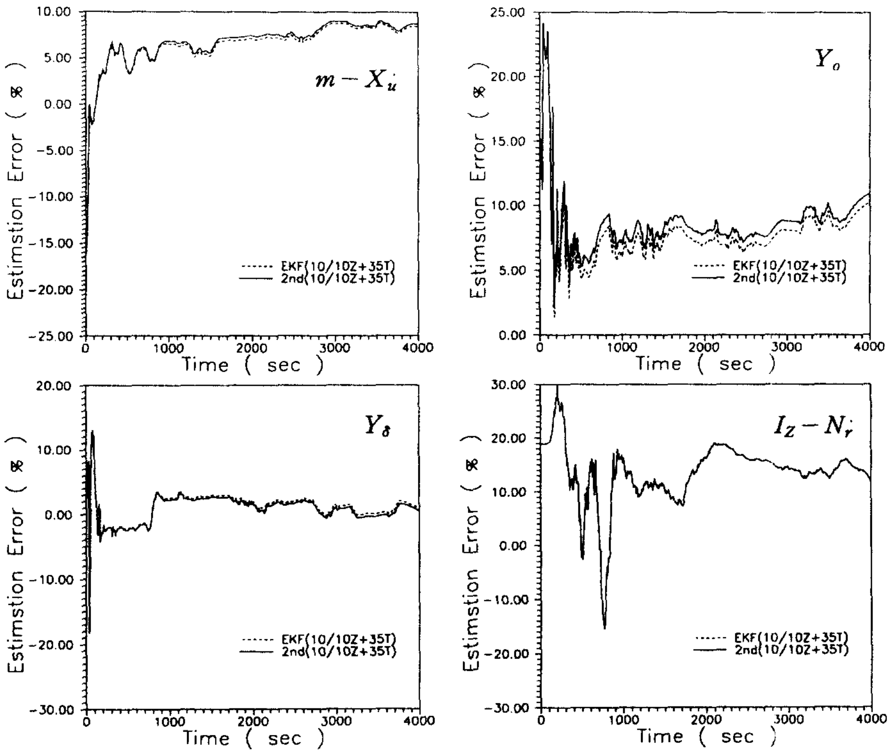

Figure 2: Parallel processing results of extended kalman filter and 2nd order filter

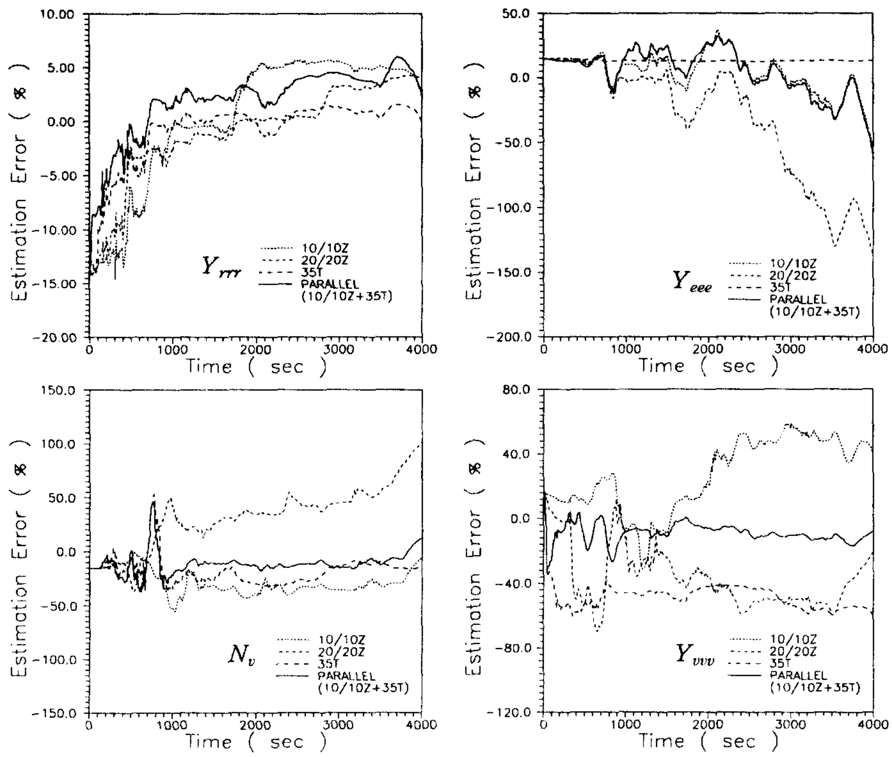

Figure 3: Results of single and parallel processing of 2nd order filter

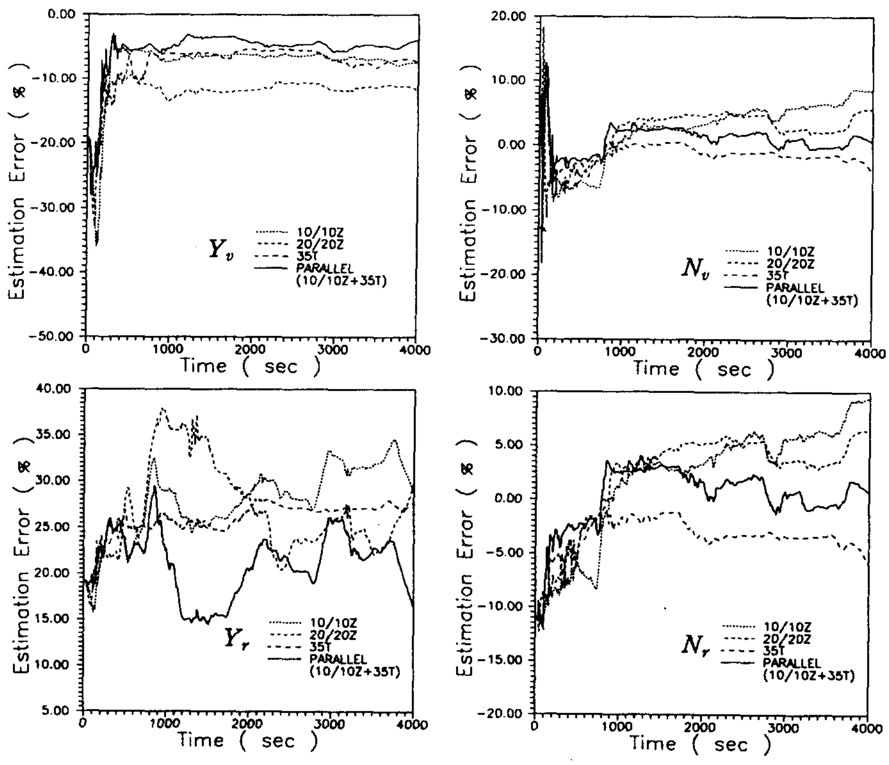

Figure 4: Results of single and parallel processing of 2nd order filter

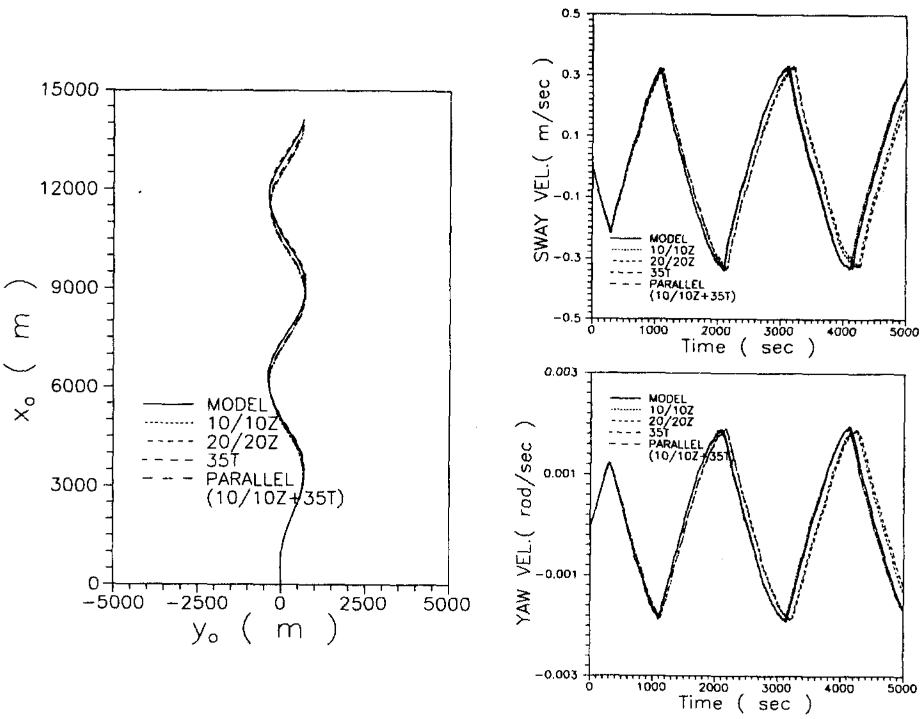

Figure 5: $10^0/10^0$ zigzag manoeuver

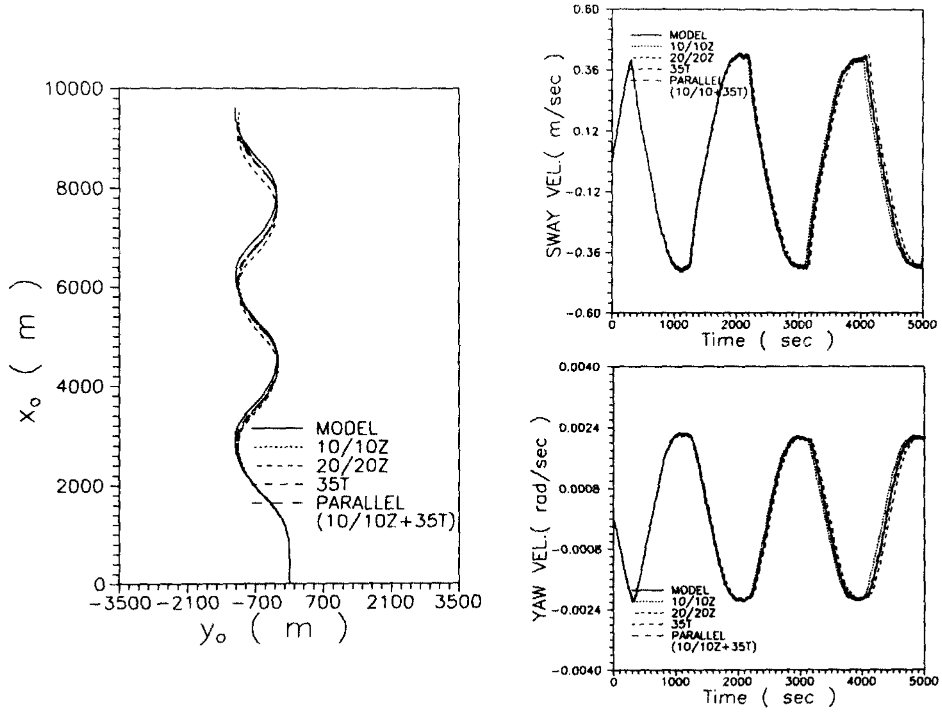

Figure 6: $20^0/20^0$ zigzag manoeuver

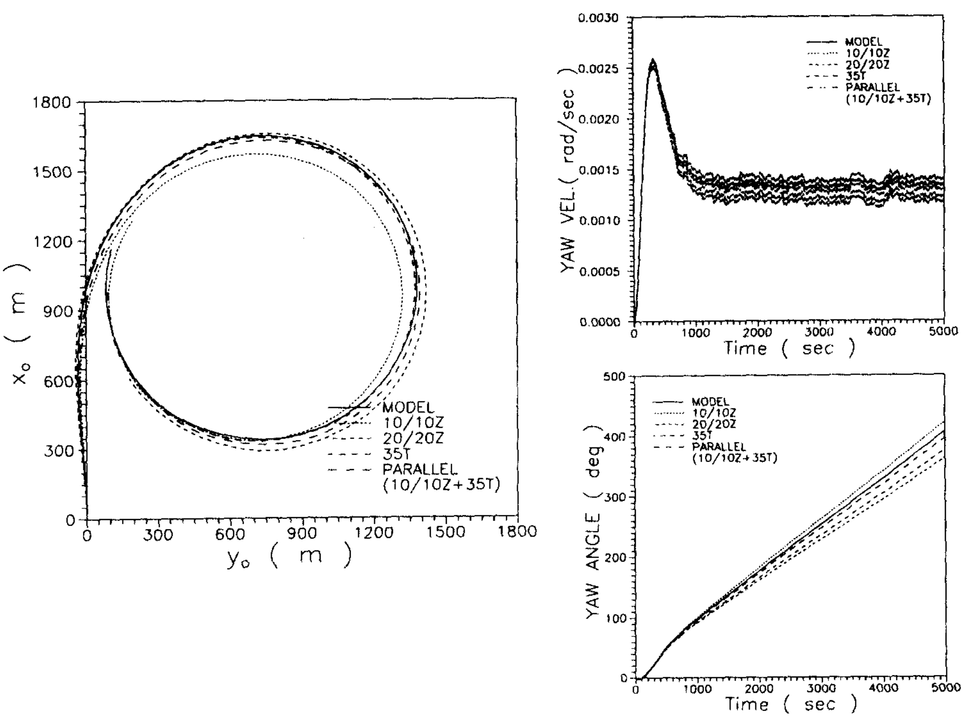

Figure 7: 35^0 turning circle manoeuver

trajectory because errors in each state variables are cancelled in the trajectory. The result by parallel processing is much closer than by $10^0/10^0$ zigzag manoeuver. In Figure 6, parallel processing is the best in any comparison. All the state variables are matched by parallel processing in Figure 7, and the estimation by parallel processing is as good as by 35^0 turning manoeuver in Figure 7.

6 Conclusions

The difference of the estimation between by the 2nd order filter and by the extended Kalman filter is little. So higher order filter than the extended Kalman filter is not necessary to estimate the hydrodynamic derivatives in manoeuvring equations. Parallel processing cannot prevent perfectly the simultaneous drift of the hydrodynamic derivatives. But it reduces the estimation error, and gives satisfactory trajectories. To remove the simultaneous drift, nonlinear or higher order filter is less effective than parallel processing. The parameter estimation becomes more accurate when many manoeuvring tests are processed parallel.

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