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# On Mathematical Representation and Integration Theory for GIS Application of Remote Sensing and Geological Data

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## Abstract

In spatial information processing, particularly in non-renewable resource exploration, the spatial data sets, including remote sensing, geophysical, geological and geochemical data, have to be geocoded onto a reference map and integrated for the final analysis and interpretation. Application of a computer based GIS (Geographical Information System or Geological Information System) at some point of the spatial data integration/fusion processing is now a logical and essential step. It should, however, be pointed out that the basic concepts of the GIS based spatial data fusion were developed with insufficient mathematical understanding of spatial characteristics or quantitative modeling framework of the data. Furthermore many remote sensing and geological data sets, available for many exploration projects, are spatially incomplete in coverage and introduce spatially uneven information distribution. In addition, spectral information of many spatial data sets is often imprecise due to digital rescaling. Direct applications of GIS systems to spatial data fusion can therefore result in seriously erroneous final results. To resolve this problem, some of the important mathematical information representation techniques are briefly reviewed and discussed in this paper with consideration of spatial and spectral characteristics of the common remote sensing and exploration data. They include the basic probabilistic approach, the evidential belief function approach (Dempster-Shafer method) and the fuzzy logic approach. Even though the basic concepts of these three approaches are different, proper application of the techniques and careful interpretation of the final results are expected to yield acceptable conclusions in each case. Actual tests with real data (Moon, 1990a; An et al., 1991, 1992, 1993) have shown that implementation and application of the methods discussed in this paper consistently provide more accurate final results than most direct applications of GIS techniques.

## 1. Introduction

In non-renewable resource exploration, certain measurements and observations, such as earth's gravitational field anomaly, geochemical lake-sediment data or mapping of rock formations form a 'spatial data set'. To quantize these types of spatial information, there have recently been several statistical and AI/EXPERT system approaches proposed and applied (McCammon, 1986; Agterberg, 1989; Bonham-Carter and Agterberg, 1989; An, 1989; Moon, 1990b). Among the statistical and AI/EXPERT system approaches, Bayesian probabilistic approaches (Agterberg, 1989; Bonham-Cater and Agterberg, 1989; Bonham-Carter et al.,1988; Singer and Kuoda, 1988; etc.) and the Dempster-Shafer type orthogonal sum rule approach of evidential belief have been directly applied for mineral exploration problems (Moon, 1989; Chung and Moon, 1990; Moon, 1990a; Moon and An, 1991). Based on a somewhat similar reasoning process, the weight of evidence modelling method is presented and successfully applied to mineral potential mapping (Bonham-Carter et al., 1990). A fuzzy logic approach has been applied for mineral exploration research in recent papers by An et al. (1991, 1992), Moon and An (1990) and Moon (1992). Application of the Dempster-Shafer method has also been reported for a mixed multi-spectral data situation including remote sensing data (Lee et al., 1987; Kim and Swain, 1989). Most of the early developments in mathematical applcaions in exploration geology is reviewed by Bonham-Carter (1990). Wang (1989) applied the fuzzy set theory in conjunction with a remote sensing study and Blonda et al. (1989) applied the fuzzy logic approach in classifying multi-temporal remotely sensed image data. Goodenough et al. (1989) and An et al. (1992) applied a rule based AI/EXPERT approach to a mixed multi-sensor remote sensing problem.

In the following discussion, digital map inventory preparation, creation of base maps, resampling techniques, interpolation, geo-coding and other basic GIS functions are assumed to be understood and carried out. In addition, the geophysical exploration data we wish to integrate are assumed to be two dimensional spatial information, even when most of them originate from three dimensional bodies, describing three dimensional exploration targets. Traditional remote sensing techniques have treated only the map/surface features or the surface projections of subsurface anomalies whereas the geophysical exploration techniques can in fact detect and image geological bodies at subsurface depths. Simplification of the complex three dimensional information into two dimensional information layers does simplify mathematical discussion and the following discussion focuses on only two dimensional aspects. However, generalization of the problem into the three dimensional framework is rather trivial and will be left to a future discussion.

## 2. Representation of Exploration Information

Geological and geophysical data required for resource exploration can be qualitative, quantitative, pictorial, textural or any combination of these. Remote sensing images and various maps carry information about the exploration target in widely ranging temporal and spatial windows. If one considers the actual spatial extent of the exploration target, information resolution can be defined in terms of a mathematical function or a set of basis functions that describes the actual target. Pixel and information resolution of common geological and geophysical data used in resource exploration varies considerably from sensor to sensor. The widely varying spatial resolution can pose serious problems for certain types of exploration information integration tasks. In the GIS type overlaying of information layers, grossly different resolutions introduce serious difficulty in interpolation and geocoding of the averaged pixel values. Resolution differences sometimes require different representation schemes which are not always compatible to others during the information representation stage. In this respect, the manner with which the exploration information can be described and organized must be consistent and justifiable, so that exploration personnel's understanding and formulation of the problem, with respect to the exploration data on hand, can be precise and technically accurate (Argialas and Harlow, 1990). Relationships between models, whether they represent a geological, geophysical or AI/EXPERT system, should first be identified and categorized based on the spatial and terrestrial spectral characteristics of the exploration data. The role of models is then to categorize the visual information into usable forms for meaningful reasoning and, therefore, to enable an interpreter to effectively analyze the objective properties of the exploration targets being studied. Among many levels of abstraction of the exploration data, only the spatial exploration data will be focused on in the following discussion.

Given several layers of remote sensing, geophysical, geological and other auxiliary data, the information levels or the exploration evidences usually have varying degrees of certainty about the target existence or possibilities of the exploration targets. Some of these will be proved to be either correct(*true*) or incorrect(*false*), based on the follow up ground truth study. If one can assign a probability value or a degree of belief to each piece or type of evidence, the numerical value with varying degrees of certainty about the chosen exploration target is in fact represented by a choice of the interpreter's information or partial belief functions(Shafer, 1976). Philosophical argument on the interpreter's or the explorationist's partial belief towards the chosen propositions often poses considerable ambiguity and mathematical quantification of ones belief raises some fundamental questions among certain users. However, these philosophical and conceptual discussion is out of scope for this paper and will be left for future discussion elsewhere.

### 2.1 Basic Framework for Information Representation

Suppose that n geoscience maps are assembled for exploration of a specific deposit type in a prospecting area A. Each layer of geophysical and geological map data is regarded as an exploration evidence, denoted by $E_k$ (k=1,2, ..., n). The whole data set of n map layers in A is represented by :

$$E = \{E_1, E_2, \ \ldots \ , E_n\} \qquad (1)$$

For each layer (exploration evidence) $E_k$ , we define a mapping $d_k$ :
for each observation e in $E_k$ ,

$$d_k : E_k \rightarrow [\ 0,\ 1] \qquad (2)$$

where $d_k$ (e) represents the geologist's partial belief that an observed value, e, in $E_k$ is related to a mineral deposit or the exploration target at the observation point in A. Of course, the method of defining such a mapping from exploration evidence is not only difficult, but also is a very subjective task, depending on the geologist.

In addition, all the sub-areas covered by the same survey or observation, e, are represented by the same value $d_k$(e). Hence, from this type of representation, we are not able to discuss the probabilities or possibilities that a sub-area or a pixel associated with a specific size, contains a mineral deposit as discussed in Chung and Moon (1991). Also $d_k$ must be defined for every observation e of $E_k$.

Let us now define a proposition ET (exploration target) that a point p in A belongs to a deposit of the type and $\neg$ET states the opposite of ET which implies that a point p is not located in any deposit. We will discuss, among many others, three different interpretations of the mapping $d_k$ in terms of ET and $\neg$ET. Note that the point p is not associated with any size scale.

## 2.2 Probabilistic Interpretation

Suppose that $d_k(e)$ is defined as the probability that e of $E_k$ is related to having a deposit at a point p where the observation e is made. Then dk (e) should be interpreted as the conditional probability, denoted by Probk {e | ET} that the observation e of Ek is made at a point p conditioned by p containing in a target deposit.

In this interpretation, what we really wish to have is the conditional probability, denoted by Probk {ET | e}, that p is contained in a deposit, given that the observation e was made at p. Using Bayes' theorem, we obtain :

$$Prob_k\{ET|e\} = \frac{Prob_k\{ET\}Prob_k\{e|ET\}}{Prob_k\{e\}} \tag{3}$$

where $Prob_k\{ET\}$ is the prior probability that any point in A contains a deposit and $Prob_k\{e\}$ is the probability that the observation e is made at a point p in A regardless whether p contains a deposit or not. It should be noted here that both $Prob_k\{ET\}$ and $Prob_k\{e\}$ can easily be estimated in A and do not play any crucial roles in the discussions that follow. Particularly, $Prob_k\{ET\}$ should be a constant for all k, because it is not related to any specific type of evidence.

Using the probability rule, we also have

$$Prob_k\{\neg ET|e\} = 1 - Prob_k\{ET|e\} \tag{4}$$

## 2.3 Dempster - Shafer Belief Function Interpretation

In this approach(Shafer, 1976), the mapping $d_k$ (e) for each piece of evidence, k, is defined as the degree to which the observation e of $E_k$ supports the proposition ET and the approach denotes by $Bel_k(ET|e)$ a Belief function. We also define another mapping called a Plausibility function, denoted by Plsk(ET|e), which represents the degree to which the observation e of $E_k$ is plausible for ET. The difference between these two mappings, $Pls_k(ET|e) - Bel_k(ET|e)$ represents the ignorance of one's belief of ET, given evidence e in $E_k$(Shafer, 1976); the two mappings have the following relationship :

$$Pls_k(ET|e) = 1 - Bel_k(\neg ET|e) \quad (5)$$

Compared to the probabilistic interpretation in the previous section, one would expect the following relationship(Wally, 1987) :

$$Pls_k(ET|e) \geq Prob_k(ET|e) \geq Bel_k(ET|e) \quad (6)$$

If the observation e has perfect information on ET, one should expect that $Pls_k(ET|e) - Bel_k(ET|e) = 0$ and $Pls_k(ET|e) = Bel_k(ET|e)$. In this case, $Bel_k$ is called the Bayesian Belief function and one would expect that all three mappings, $Pls_k(ET|e)$, Probk(ET|e) and $Bel_k(ET|e)$ should have identical properties.

### 2.4 Fuzzy Logic Interpretation

In this interpretation, the mapping dk for each piece of evidence, Ek, represents the degree of "compatibility" of the observation e at p for ET (Zadeh, 1965). The following membership function :

$$U_k(ET|e) = d_k(e) \quad (7)$$

represents the degree of certainty that p is a member of the set of points which belong to a deposit, given e in A. A membership mapping $U_k(ET|e)$ close to 1 implies that p is likely to contain a deposit.

## 3. Integration of Data

Consider a set of n observed values $\{e_1, e_2, ..., e_n\}$ of n pieces of evidence $\{E_1, E_2, ... E_n\}$ at a point p in A. Suppose $\{d_1, d_2, ..., d_n\}$ are defined on $\{E_1, E_2, ... , E_n\}$. Then we have n representations $\{d_1(e_1), d_2(e_2), ... , d_n(e_n)\}$ at the point p where the observations are made for ET. We wish to integrate these n representations into one single function. The integration rules depend on the interpretation of the mappings.

### 3.1 Probabilistic Approach

As discussed in Section 2.2, we have $d_k(e_k) = Prob_k\ \{e_k|ET\}$ for all $k = 1, 2, ... , n$. Using these values, we wish to have the conditional probability, denoted by Prob $\{ET|e_1, e_2, ... e_n\}$ , that p contains a deposit given that the observations $\{e_1, e_2, ... , e_n\}$ are made at p. In general, it is impossible to obtain the conditional probability from $\{d_k(e) = Prob_k\{e_k$

$|ET\}\}$ ($k = 1,2, \ldots, n$) without having the joint probability distribution function Prob $\{ET, e_1, e_2, \ldots, e_n\}$. Here we discuss two possible approximations of the Prob $\{ET|e_1, e_2, \ldots, e_n\}$ from $\{ d_1(e_1), d_2(e_2), \ldots, d_n(e_n) \}$.

The simplest and easiest approximation of Prob $\{ ET \mid e_1, e_2, \ldots, e_n \}$ is :

$$\sum_{k=1}^{n} \frac{d_k(e_k)}{n} = \sum_{k=1}^{n} \frac{\text{Prob}_k\{e_k|ET\}}{n} . \tag{8}$$

However, using the Bayes' theorem, we have

$$\text{Prob}\{ET|e_1, e_2, \ldots, e_n\} = \frac{\text{Prob } \{ET\}\{\text{Prob } \{e_1, e_2, \ldots, e_n|ET\}}{\text{Prob } \{e_1, e_2, \ldots, e_n\}} \tag{9}$$

where Prob $\{ET\}$ is the prior probability that a point in A contains a deposit, Prob $\{e_1, e_2, \ldots, e_n\}$ is the probability that the observations $\{e_1, e_2, \ldots, e_n\}$ occur in A and both probabilities can be estimated from $\{E_1, E_2, \ldots, E_n\}$. From Eq.(9) above, it is obvious that Eq.(8) has no justification as an approximation.

Another approximation would be the use of Bayes' theorem with the conditional independence assumption (Agterberg, 1989; McCammon, 1986; Bonham-Carter and Agterberg, 1989). Under the conditional independence assumption, we have

$$\text{Prob } \{e_1, e_2, \ldots, e_n \mid ET\} = \prod_{k=1}^{n} \text{Prob}_k\{e_k \mid ET\} . \tag{10}$$

By substituting Eq.(10) into Eq.(9), we obtain

$$\begin{aligned} \text{Prob}\{ET|e_1, e_2, \ldots, e_n\} &= \frac{\text{Prob } \{ET\} \prod_{k=1}^{n} \text{Prob\_k } \{e_k \mid ET\}}{\text{Prob } \{e_1, e_2, \ldots, e_n\}} \\ &= \frac{\text{Prob } \{ET\} \prod_{i=1}^{n} d_k(e_{ki})}{\text{Prob } \{e_1, e_2, \ldots, e_n\}} \end{aligned} \tag{11}$$

The linear approximation in Eq.(8) and the Bayesian estimator with conditional independence in Eq.(11) are completely different and they are really not compatible.

### 3.2 Dempster - Shafer Approach

Suppose that we have $\{Bel_1(ET|e_1), Bel_2(ET|e_2), \ldots, Bel_n(ET|e_n)\}$ and $\{Pls_1(ET|e_1), Pls_2(ET|e_2), \ldots, Pls_n(ET|e_n)\}$ in a Dempster - Shafer belief function representation. Then we wish to define $Bel(ET|e_1, e_2, \ldots, e_2)$ and $Pls(ET|e_1, e_2, \ldots, e_n)$ which represent the combination of n beliefs of the existence of ET at p where the n observations $\{e_1, e_2, \ldots, e_n\}$ are made. Using Dempster's combination rule (Shafer, 1976), we can obtain $Bel(ET|e_1, e_2, \ldots, e_n)$ from $\{Bel1(ET|e_1), Bel_2(ET|e_2), \ldots, Bel_n(ET|e_n)\}$.

Dempster's rule for combining $Bel_1(ET|e_1)$ and $Bel_2(ET|e_2)$ into $Bel(ET|e_1, e_2)$ is :

$$\begin{aligned} Bel(ET|e_1, e_2) &= \frac{ab + a(1-b-b') + b(1-a-a')}{1-ab-a'b'} \\ Pls(ET|e_1, e_2) &= 1 - \frac{a'b' + a'(1-b-b') + b'(1-a-a')}{1-ab-a'b'} \end{aligned} \tag{12}$$

where $a = Bel_1(ET|e_1)$, $a' = 1 - Pls_1(ET|e_1)$, $b = Bel_2(ET|e_2)$ and $b' = 1 - Pls_2(ET|e_2)$. By repeating Eq.(12) n-1 times, we obtain $Bel(ET|e_1, e_2, \ldots, e_n)$ and $Pls(ET|e_1, e_2, \ldots, e_n)$ from $\{Bel_1(ET|e_1), Bel_1(ET|e_2), \ldots, Bel_n(ET|e_n)\}$ and $\{Pls_1(ET|e_1), Pls_2(ET|e_2), \ldots, Pls_n(ET|e_n)\}$.

### 3.3 Fuzzy Set Approach

Suppose that we have the membership functions, $U_k(ET|e_k)$ for all $k = 1, 2, \ldots, n$ for all pieces of evidence $\{e_1, e_2, \ldots e_n\}$ at a point p. We wish to define a membership function $U(ET|e_1, e_2, \ldots e_n\}$ from n membership functions $U_k(ET|ek)$. This can be done using many different types of operators available in fuzzy set theory (Zadeh, 1965). Some of the basic operations that are most frequently applied to spatial information are as follows :

(1) Min - operator

$$U(ET|e_1, e_2, \ldots e_n) = \text{minimum}\ \{U_1(ET|e_1), U_2(ET|e_2), \ldots, U_n(ET|e_n)\} \tag{13}$$

(2) Max - operator

$$U(ET|e_1, e_2, \ldots e_n) = \text{maximum}\ \{U_1(ET|e_1), U_2(ET|e_2), \ldots, U_n(ET|e_n)\} \tag{14}$$

(3) Algebraic sum operator

$$\begin{aligned} U(ET|e_1, e_2, \ldots e_n) &= \sum_{k-1}^{n} U_k(ET|e_k) - \sum_{k=1}^{n}\sum_{j=k+1}^{n} U_k(ET|e_k)U_j(ET|e_j) \\ &+ \ldots + \ldots \\ &+ (-1)^n U_1(ET|e_1) \ldots U_n(ET|e_n). \end{aligned} \tag{15}$$

(4) $\gamma$ - operator (Zimmermann and Zysno, 1980)

$$U(ET|e_1, e_2, \ldots, e_n) = [\prod_{k=1}^{n} U_k(ET|e_k)]^{(1-\gamma)} - [1 - \prod_{k=1}^{n}(1-U_k(ET|e_k))] \tag{16}$$

where $0 \le \gamma \le 1$

## 4. Discussion and Conclusion

Most of the newly available computer based GISs are very effective for overlaying digital (raster and vector) information and provide efficient tools for quantitative modeling of spatial data. However, the intrinsic nature of impreciseness and incomplete coverage of geological, geophysical and remote sensing data sets often used in Earth observation science and resource exploration requires a proper quantitative information representation of the data sets involved. Exploration data for nonrenwable reources usually have varying range of spatial and depth resolutions and we need a carefully studied mathematical information representation scheme, which is independent of spatial resolution.

The probabilistic approach (Bayesian), evidential belief function approach (Dempster-Shafer) and fuzzy logic approach have proved to be very effective whether they are applied through a rule based AI/EXPERT system or through a straight data driven inference system. The fuzzy logic approach is conceptually different from the above two approaches. However, it can be very effective when the proposition (e.g. environmental and/or exploration target) itself is vague and when it is applied to single node problems. Uncertainty of information can not be handled easily in the traditional probabilistic approaches in most cases. Vagueness of evidence can be processed and interpreted in a pseudo-probabilistic approach. However, incomplete spatial coverage, which is very common in many resource exploration projects, poses a serious problem with the probabilistic approach. The evidential belief function method (Dempster-Shafer method) can be formulated to handle spatially incomplete coverage more appropriately, where the same problem can sometimes result in unexpected and erroneous results with the probabilistic approaches. If missing data points and incomplete survey coverage become important, plausibility and ignorance of each data set have to be defined as precisely as possible and the evidential belief function approach has advantage as it is currently formulated. The fuzzy logic method can handle incompleteness of spatial coverage adequately but the fuzzy reasoning process at the data fusion stage has over- and/or under-estimation problems.

The integrated final probability, computed either using the traditional Bayes rule approach or from the evidential belief function approach, represents a real probability of certainty towards the chosen target proposition. Similarly the integrated final membership function represents a possibility of truthfulness towards the target proposition. However, quantitative relationships between the fuzzy logic representation and the traditional probability and the belief function representation cannot be established at this time. They require further study. The problem of conditional dependency between the data sets poses a serious uncertainties for these quantitative approaches and requires detailed investigation in future (Moon, 1989; An et al., 1992). Another problem with applying fuzzy approach to spatial data processing is the large number of available fuzzy operators upon which the final membership function critically depends. There also exists unresolved problem in error and uncertainty propagation in the target regions with incomplete data coverage (An et al., 1993).

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