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■ 논 문 ■

Application of the Fuzzy Set Theory to Uncertain Parameters in a Countermeasure Model

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ABSTRACT

A method for estimating the effectiveness of each protective action against a nuclear accident has been proposed using the fuzzy set theory. In most of the existing countermeasure models in actions under radiological emergencies, the large variety of possible features is simplified by a number of rough assumptions. During this simplification procedure, a lot of information is lost which results in much uncertainty concerning the output of the countermeasure model. Furthermore, different assumptions should be used for different sites to consider the site specific conditions. In this study, the diversity of each variable related to protective action has been modelled by the linguistic variable. The effectiveness of sheltering and evacuation has been estimated using the proposed method. The potential advantage of the proposed method is in reducing the loss of information by incorporating the opinions of experts and by introducing the linguistic variables which represent the site specific conditions.

Key words: countermeasure, sheltering, evacuation, fuzzy set, linguistic variable, fuzzy reasoning

Introduction

The necessity of emergency response planning has increased since the accident of TMI-2 in the USA. One of the essential parts of emergency planning is how to decide countermeasures including sheltering, evacuation, interdiction, and decontamination. Decision makers should have access to rational decision model that can be used to support the selection of appropriate off-site protective action in the event of nuclear reactor accident[1,2]. The effects of these protective actions may be dependent on the site-specific conditions such as shielding factors, population density, and road conditions. The variables representing these conditions have a wide distribution of values due to the complexity of real situation. Representative values of these variables are used in most existing countermeasure models for simplicity and tractability. The simplification procedure inevitably results in the loss of information and the occurrence of uncertainty[3].

There are several facts which may cause the uncertainty in a countermeasure model:

1) Real situation is very often not crisp and deterministic due to uncertainty or vagueness. The future state of a system might not be known completely due to the lack of information.
2) The human behavior is dependent on the situation. There is a lot of uncertainty due to the assumed values of parameters such as the fraction of population with different shielding factor, driving speed, etc. Therefore, different values in the existing countermeasure model should be assumed for different site.
3) The shielding factors may be different with respect to site as well as direction from the source even in the same site. But, the site specific factors which are dependent on the effect of wind direction used to be not reflected in the most existing models.

The concept of fuzzy set provides the mathematical formulations which can characterize the uncertain parameters involved in the existing countermeasure models. By the application of linguistic variables and fuzzy algorithms, it is possible to provide an approximate and effective tool to describe system which is too complex or ill-defined to use precise mathematical analysis[4]. A countermeasure model is very complex system mainly due to the diversity of real environment.

In this study, a method to reduce the parameter uncertainty in a countermeasure model is proposed, using the concept of fuzzy set and system. The broad spectrum of the parameter values expressing the characteristics of shielding and evacuation is represented by linguistic variables. A linguistic variable is defined as one whose values are sentences in natural or artificaial languages as large, good, and poor. The relationship between the parameters representing the site condition and the effects of these parameters on radiation exposure is represented by fuzzy conditional statements which has the form "*IF A THEN B*", where A and B have fuzzy meaning[5,6]. Fuzzy reasoning with the obtained fuzzy relations and the given site-specific input variable gives the fuzzy set to the parameter which will be estimated.

The main objective of this study is to demonstrate a potential use of fuzzy set theory in countermeasure modeling. The proposed method can be used to describe other uncertain parameters in a countermeasure model by incorporating the reliable membership function and fuzzy conditional statements based on the opinions of experts.

Model Description

The calculational procedure developed in this study includes three main distinguishing features: 1) linguistic variables; 2) fuzzy relations; and 3) fuzzy reasoning. Figure 1 shows the calculational flow diagram.

The main function of linguistic variable is to provide a systematic means for an approximate characterization of complex or ill-defined phenomenon.

Fuzzy set is a class of objects with a continuum of membership grades. If X is a collection of objects denoted generically by x, then a fuzzy set A in X is a set of ordered pairs[7]:

$$A = \{(x, \mu_A(x)) \mid x \in X, \mu_A(x) \in [0,1]\} \quad (1)$$

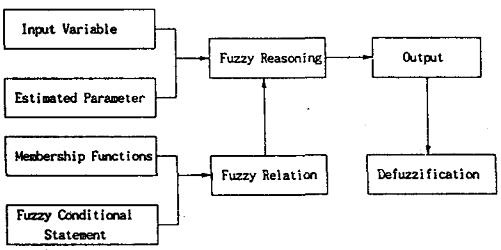

Figure 1. Calculational procedure used to describe the uncertain parameters in a countermeasure model.

where $\mu_A(x)$ is called the membership function or membership grade of x.

Fuzzy set can be modified using the linguistic hedge such as "very", "much", "more or less". The hedge "very" is used with the definition of concentration.

$$\text{CON}(A) = A^2 \quad (2)$$

The operation of fuzzy set is carried out with the membership values. Therefore, the most crucial step in the design of the problem to be solved by fuzzy set theory is the determination of the membership functions of fuzzy sets. The fuzzy set based on expert opinions with statistics is regarded as the most natural fuzzy set that can be used.

The relationship between input variables and parameter to be estimated can be represented by fuzzy conditional statements which must be determined by expert with full understanding of real phenomena[8,9]. Fuzzy relations are fuzzy subsets of A X B, that is, mappings A → B. If there is a relationship between A and B, it can be represented by the ordered pair (x, y). Here x and y are the elements of fuzzy sets A and B, respectively.

$$(x,y) \in R \quad \text{or} \quad x R y \quad (3)$$

$$\mu_R \mid A \times B \rightarrow [0,1] \quad (4)$$

The membership grade μ_R means the strength of relation, and it is useful to represent our knowledge. Using fuzzy conditional statements, fuzzy relations from each input variable to output parameter are obtained. There are several methods that have been suggested for the calculation of fuzzy relations. Two equations suggested by Mamdani and Zadeh are represented in Equations 5 and 6, respectively. The Mamdani's (Eq. 5) is used in this study.

$$\mu_R(x,y) = \mu_A(x) \wedge \mu_B(y) \quad (5)$$

$$\mu_R(x,y) = 1. \wedge (1. - \mu_A(x) + \mu_B(y)) \quad (6)$$

where $\mu_R(x,y)$ represents the strength of relation from x to y, $\mu_A(x)$ is the grade of membership of x in A, and $\mu_B(y)$ is the grade of membership of y in B. The operator $\wedge$ represents the intersection of two elements and it is treated with the operator "min":

$$\text{Intersection } A \cap B : \mu_{A \cap B}(x) = \text{Min } [\mu_A(x),\ \mu_B(x)] \quad (7)$$

In the traditional logic, the main tools of reasoning are tautologies such as an example of the modus ponens; $(A' \rightarrow (A \rightarrow B) \rightarrow B')$. It can be applied in the fuzzy set theory represented as the following logic[10].

$$\begin{array}{lll} \text{Premise} & : & x \text{ is } A' \\ \text{Implication} & : & \text{If } x \text{ is } A \text{ then } y \text{ is } B \\ \hline \text{Conclusion} & : & y \text{ is } B' \end{array} \quad (8)$$

In the above descriptions, the premise can be regarded as an input variable and the implication is equivalent to the obtained fuzzy relation. The fuzzy output B' in the conclusion can be obtained by the fuzzy reasoning with an input variable and a fuzzy relation. The "max-min" composition has become the best known and the most frequently used operation for the fuzzy reasoning.

$$\forall\ x \in A,\ y \in B,$$
$$\mu_{B'}(y) = \text{Max}\{\text{Min } [\mu_{A'}(x),\ \mu_R(x,y)]\} \quad (9)$$
$$x = f^{-1}(y)$$
$$\text{where } f^{-1}(y) \neq \phi$$

The obtained results through the fuzzy reasoning have the form of fuzzy set. And it is necessary to interpret the fuzzy result as the crisp one which can be easily understood by users. This process is carried out by the defuzzification of the obtained fuzzy set of the output parameter. The center of area method is used for the defuzzification of the fuzzy output as follows:

$$u_o = \frac{\Sigma\ \mu\ (u_j)\ x\ u_j}{\Sigma\ \mu\ (u_j)} \quad (10)$$

where u_j is the value of the j^{th} grade of the parameter to be estimated and $\mu(u_j)$ is the membership value for the u_j and u_o is the defuzzified value of the parameter.

Application Studies

There are several uncertain variables in a countermeasure model which have wide distributions of values. In most of the existing countermeasure models, uncertain variables are assumed to have representative values for the simplification. Interpolation, truncation, and assumption have been used in the simplification. These processes might result in the loss of some information which may be essential to describe the real situation.

The Fuzzy set theory provides a method which can describe the characteristics of uncertain varialbe and reduce the loss of information. These variables are represented by the fuzzy membership functions, and the effects of distribution can be handled with the fuzzy relation and the fuzzy reasoning[11].

Among the variables in a countermeasure model, the candidates for the application of the proposed method are as follows:

- Shielding factor
- Population distribution
- Conditions of road
- Evacuation speed
- Delay time
- etc.

The common characteristics of these variables are that these have distribution of values with some interval and have been simplified as some representative values. Among the protective actions, sheltering and evacuation are considered for the application of fuzzy set theory in this study.

Sheltering

As the first application, the shielding effect is estimated using the proposed method. The shielding effect is determined according to the site-specific conditions such as the material of structure, the percentile of structure with the same shielding factor, and the wind direction at the time of an accident. The site-specific conditions can be represented by the corresponding linguistic variables.

The shielding effect is defined by the following equation:

$$Se = \sum_{i=1}^{N} FP_i \times (1 - S_{fi}) \qquad (11)$$

where FP_i : the fraction of the i^{th} population group

S_{fi} : the shielding factor for the i^{th} population group

N : the number of population group

The percentile of population is derived by probabilistic treatment of human behavior, but in the case of accident the human behavior may be different according to the situations at the time of accident. Therefore, the percentile of population subjects to uncertainty. Such an uncertain variable is treated by a linguistic variable. The characteristics of variable is represented by a fuzzy membership function. The determination of membership function which can reflect the characteristics of the variable properly is important to obtain the reliable results[12, 13].

Since it is also a time-consuming and difficult work to determine a proper membership function for a variable, a simple form of membership function for the shielding conditions and the effect of shielding is used as shown in Figure 2. For computation, 11 grades for each variable

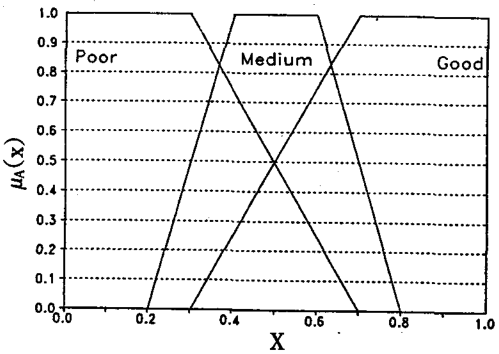

Figure 2. Membership functions specified to describe sheltering.

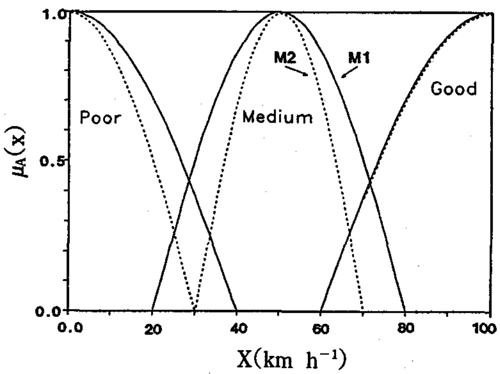

Figure 3. Membership functions specified to describe evacuation speed (M1, M2).

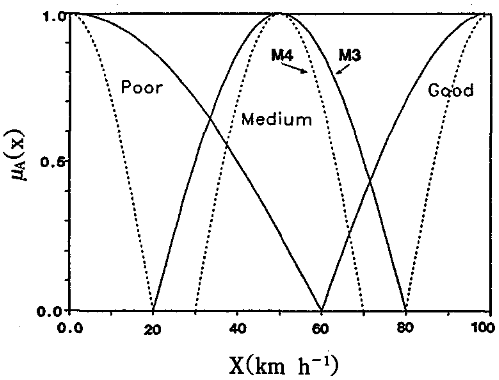

Figure 4. Membership functions specified to describe evacuation speed (M3, M4).

have been selected from the membership function.

Given the fuzzy membership function for each variable, the next step is to represent the relationship between the input variable and the parameter to be estimated by fuzzy conditional statements. The following fuzzy conditional statements have been made by engineering judgement.

If A is good	Then B is good
If A is medium	Then B is good
If A is poor	Then B is medium.

where A is the shielding condition as an input variable and B is the shielding effect.

Evacuation

The factor to be considered during evacuation are delay time, evacuation speed, percentile of population with the same evacuation speed, road condition, and shielding factors. Evacuation speed and corresponding percentile of population among these factors have large impacts on the effects of protective action. Delay time is also an influential factor, but it has no direct effect on the distribution of evacuation speed.

In the second application, the evacuation speed is considered as the parameter to be estimated. Population density and road condition have an influence on the evacuation speed. The distributions of population density and road condition are represented by linguistic variables. The membership function used to represent these linguistic variable are determined with sine curves. Four kinds of different membership functions shown in Figures 3 and 4 have been used to investigate the effect of the change of curve shape.

Table 1. Fuzzy conditional statements for evacuation.

Input		Evacuation Speed
Variable	Condition	
Population Density	Good	Slow
	Medium	Medium
	Poor	Fast
Road Condition	Good	Fast
	Medium	Medium
	Poor	Slow

Table 1 represents the fuzzy conditional statements describing the dependence of evacuation speed on population and road condition. Fuzzy relations from each input variable to the evacuation speed have been obtained with the same procedure used in the first application study. The impact of grades of these two variables G_1 and G_2 has been estimated by the composition operator as follows[14,15]:

$$\text{Impact of two variables} = (G_1 \circ R_1) \cap (G_2 \circ R_2) \qquad (12)$$

where R_1 is the fuzzy relation from population density to evacuation speed and R_2 is that from road conditions to evacuation speed, and "o" denotes the "max-min" composition as described in Equation 9.

Table 2. Fuzzy relation from shielding condition to shielding effect.

			Shielding Effect										
			SLOW			MEDIUM					FAST		
			1	2	3	4	5	6	7	8	9	10	11
shielding condition	POOR	1	0.00	0.00	0.38	0.78	1.00	1.00	1.00	0.78	0.38	0.00	0.00
		2	0.00	0.00	0.38	0.78	1.00	1.00	1.00	0.78	0.38	0.00	0.00
		3	0.00	0.00	0.38	0.78	1.00	1.00	1.00	0.78	0.38	0.38	0.38
	MEDIUM	4	0.00	0.00	0.38	0.78	1.00	1.00	1.00	0.78	0.78	0.78	0.78
		5	0.00	0.00	0.38	0.75	0.75	0.75	0.75	1.00	1.00	1.00	1.00
		6	0.00	0.00	0.38	0.50	0.50	0.50	0.75	1.00	1.00	1.00	1.00
		7	0.00	0.00	0.25	0.25	0.25	0.50	0.75	1.00	1.00	1.00	1.00
		8	0.00	0.00	0.00	0.00	0.25	0.50	0.75	1.00	1.00	1.00	1.00
	GOOD	9	0.00	0.00	0.00	0.00	0.25	0.50	0.75	1.00	1.00	1.00	1.00
		10	0.00	0.00	0.00	0.00	0.25	0.50	0.75	1.00	1.00	1.00	1.00
		11	0.00	0.00	0.00	0.00	9.25	0.50	0.75	1.00	1.00	1.00	1.00

Table 3. The defuzzified shielding effect for each shielding condition.

	Shielding Condition				
	VERY GOOD	GOOD	MEDIUM	POOR	VERY POOR
Shielding Effect	0.7974	0.7636	0.5000	0.3797	0.3428

Results and Discussion

Table 2 shows the fuzzy relation from shielding condition to shielding effect. The numerical values in the table represent the intensity of relationship between shielding conditions and shielding effect. For example, r_{63} is the relation intensity from the 6^{th} grade of shielding conditions (medium) to the third grade of shielding effect (poor). The value of r_{63} is 0.38 in Table 2. Similarly r_{69} with the value of 1.0 is the strength of relationship between medium shielding condition and good shielding effect. These results show that the obtained fuzzy relations agree well with the fuzzy conditional statements specified to describe sheltering.

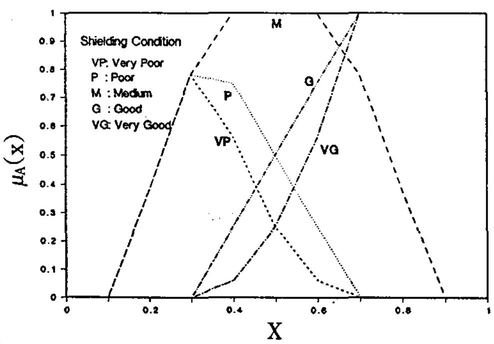

Figure 5. Shielding effects for various shielding conditions.

Fuzzy reasoning with the obtained fuzzy relation and the fuzzy set specified for input condition of sheltering gives an output fuzzy set for shielding effect. The operation of "max-min" composition is used for fuzzy reasoning. The shielding effects calculated for five shielding conditions from very poor to very good are represented with the form of fuzzy set in Figure 5.

The shielding effects obtained with the form of fuzzy set cannot be easily understood by general users. These must be interpreted as the results of the simple form by defuzzification process. The center of area method has been used for defuzzification. Table 3 shows the defuzzified shielding effects for five shielding conditions.

In the case of COSYMA[1], the shielding effect S_e for the cloudshine is calculated with the default value by the Equation 11.

$$S_e = 0.3 \times (1.-1.) + 0.3 \times (1.-0.05) + 0.15 \times (1.-0.3) + 0.15 \times (1.-0.01) + 0.1 \times (1.-1.) = 0.5385 \quad (13)$$

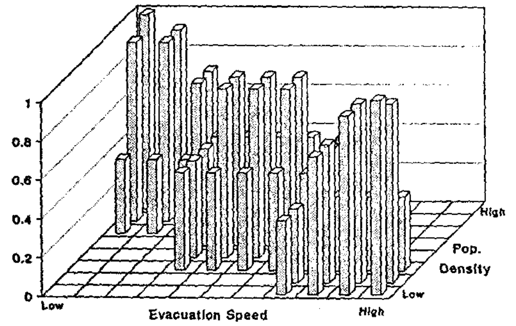

Figure 6. Fuzzy relation from population density to evacuation speed.

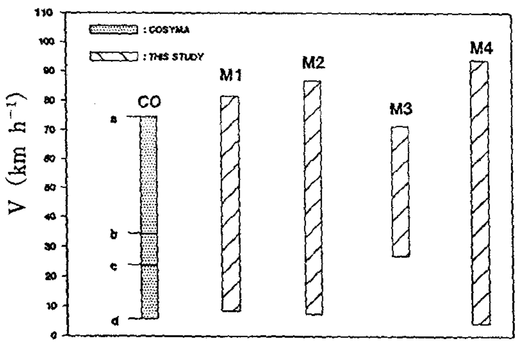

Figure 7. Intervals of evacuation speed.(CO : obtained with the values used in COSYMA ; a : PD≤100, b : 100≤PD≤500, c : 500≤PD≤1000, d : 1000≤PD, M1, M2, M3, M4 : calculated with different membership functions.)

The calculated value "0.5385" means that 53.85% of the total exposure dose is reduced by the protective action of sheltering. Comparing this value with the defuzzified shielding effects obtained with the proposed method, the shielding condition of the area considered in COSYMA is between medium and good.

As results of the second application study, the fuzzy relations corresponding to evacuation speed are obtained by the Equation 5 using the fuzzy conditional statements described in Table 1. Figure 6 shows the fuzzy relation from population density to evacuation speed, and also reflects well the fuzzy conditional statements described in Table 1.

Table 4. Input conditions for two variables.

Population Density	Road Condition
very low	very poor
low	poor
medium	medium
high	good
very high	very good

Five input conditions are defined for both the population density and the road conditions as shown in Table 4. For both input variables, the same procedure used in the first application is applied for fuzzy reasoning. Finally the fuzzy set specified for the evacuation speed is obtained with the composition operator represented in Equation 12.

In the second application, fuzzy reasoning is carried out with four membership functions shown in Figures 4 and 5. Figure 7 shows the intervals of both evacuation speeds obtained with the proposed method and the numerical values used in COSYMA. The narrower the shape of membership function, the larger the interval of evacuation speed, vice versa.

Even though simple membership functions and fuzzy conditional statements are used in this study, the results show the potential applicability of fuzzy set theory to describe uncertain parameters in a countermeasure model. It seems that the proposed method with suitable membership functions and fuzzy conditional statements based on the opinion of experts can give the reliable results on evacuation speed.

Conclusions

A method of assessing the effectiveness of protective actions by means of the concept of fuzzy set theory is proposed.

Sheltering and evacuation are considered to investigate the potential applicability of fuzzy set theory. The membership functions and fuzzy conditional statements describing these two countermeasures are made by engineering judgement.

In spite of the simple membership functions and fuzzy conditional statements, the obtained results show the potential applicability of fuzzy set theory to describe uncertain parameters in a countermeasure model.

It is necessary to develope fuzzy algorithms suitable to description of countermeasure, and to obtain realistic fuzzy membership functions and fuzzy conditional statements in parallel to get more reliable results.

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비상대응모델의 불확실한 변수에 대한 퍼지이론의 적용

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요 약

원자력시설의 비상사태시 주변주민을 보호하기 위한 비상대응행위의 효과를 평가할 필요가 있다. 비상대응행위는 지역특성과 개인 행동특성에 따라 매우 다양하게 나타날 수 있으므로, 비상대응행위를 나타내는 변수들은 특정 값을 갖기보다는 일정구간에 분포되는 값을 갖는다. 대부분의 기존 비상대응모델에서는 계산을 단순화시키기 위하여 가정을 통해 특정값을 사용한다. 단순화 과정중에 필연적으로 정보의 손실이 발생되어 결과적으로 비상대응 모델은 큰 불확실성을 포함하게 된다. 퍼지이론은 변수의 불확실성을 계산에 포함시켜 엄밀한 계산을 통해 정보손실을 최소화시키면서 계산결과를 얻어낼 수 있는 수학적인 도구를 제공해 준다. 본 연구에서는 퍼지집합, 퍼지추론, 퍼지관계 등의 이론을 응용하여 원자력시설의 비상사태시 비상대응효과를 평가할 수 있는 방법을 개발하였다. 개발된 모델의 장점은 언어변수를 이용하여 지역특성을 표현하고 전문가의 의견을 반영하여 비상대응효과를 평가하므로, 단순화 가정중에 유발되는 정보의 손실을 줄일 수 있는데 있다. 비상대응 모델내의 불확실한 변수에 대한 퍼지이론의 응용성을 개선하기 위해서는 전문가의 의견을 반영하여 변수들에 대한 적합한 멤버쉽 함수와 퍼지조건문을 확립할 필요가 있다.

Key words：*비상대응, 대피, 소개, 퍼지이론, 언어변수, 퍼지추론*