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Application of the Intensity of Use of Mineral Consumption Forecasting

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ABSTRACT : This study found that that dynamics of intensity of use and economic theory of derived demand can both be accommodated through an extensive translog demand model. The basic idea in this recognition is that the skewed life cycle empirical pattern of intensity of use plotted against per capita income is of lognormal form and this lognomal intensity of use model can be mathematically transformed into an eqivalent simple translog intensity of use model. Empirical results showed that this extensive traslog model, which is a flexible function and includes both the classical case of fixed coefficients and the dynamic case of varying coefficients of the explanatory variables, gave better forecasts than the original intensity of use model and other conventional models.

INTROUDUCTION

Generally speaking, techniques used for long-range forecasting of mineral demand have been rather simplistic, such as extrapolation of a growth rate or a time trend in demand itself or the multiplication of a measure of intensity of use (mineral demand/GNP) of the mineral by projected income. Hence, these methods are viewed as simplistic, because the multi-facted dynamics that affect demand across time and which operate at different rates across time are treated as one, and the sum effect of these dynamics is represented by the time path of a single trend mode. One or both of the aforementioned methods had been commonly used until a few years ago when professor Malenbaum showed that intensities of use for many mineral commodities exhibited a life cycle pattern, much like a skewed normal curve, e. g., lognormal, when intensity of use is plotted against per capita income (see Fig. 1 and 2). For the U.S., these same data showed that for many mineral commodities intensity of use peaked many years ago and has been declining since.

While the above historical account shows evolution in methods of long range forecasting to more improved methods, there are some bothersome deficiencies and inconsistencies :

1. technology and prices are ignored in the Malenbaum approach ; on the other hand traditional econometric models that are more comprehensive, hence conceptually appealing, have not contained the dynamics in intensity of use indicated by historical evidence and empirical relations.
2. Useful statistical methods also have been ignored, e. g. rigorous estimation of intensity of use relations and recursive and iterative approaches to minimize uncertainty and error of prediction of mineral demands for both ex post and ex ante forecasting.

The subject of this study is improved methods for long term forecasting of minerals demand. Intensity of use, both in its simple, original form and as described by richer economic relations is one such method, particularly when intensity of use is estimated using rigorous statistical methods.

GENERAL DESCRIPTION OF THE IU PROBLEM

Malenbaum (1977) defines intensity of use as $\frac{x_t}{y_t}$, and describes this measure as a function of per capita income $y_t = \frac{y_t}{N}$:

$$\frac{x_t}{y_t} = f(y_t) = IU_t,$$

where

N_t = population
x_t = mineral demand
y_t = income

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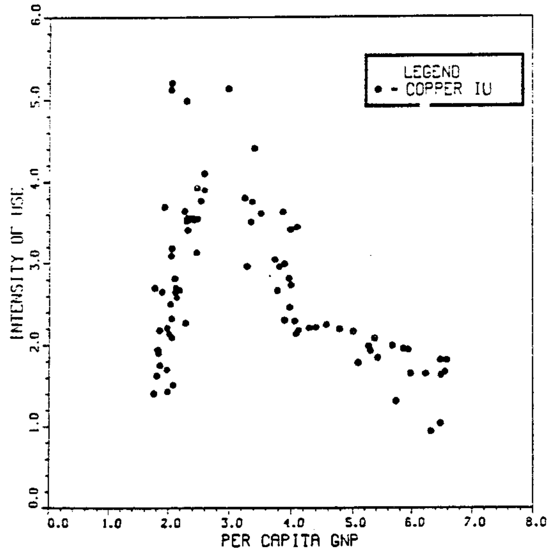


Fig. 1. Trend of U. S. copper intensity of use, 1901-1983 (short tons per millon 1972 dollars).

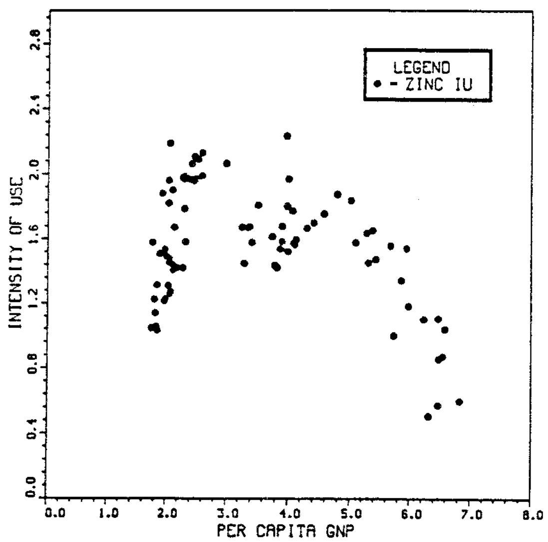


Fig. 2. Trend of U. S. zinc intensity of use, 1901-1984 (short tons per millon 1972 dollars).

While the change in IU over time has a pronounced effect on and must be accounted for in long-term demand predictions, the simple conceptual model of this change as proposed by Malenbaum is not appealing in terms of its economics. For example, although Malenbaum's procedure has been applied widely to mineral commodity forecasting, there are some concerns about the neglect of determinants of the dependent variable (IU) other than income, e.g., materials-saving technological change and materials substitution. When minerals are introduced into industries, technological changes may accelerate, and markets may be captured rapidly from the competing materials. In time, technological impacts on costs diminish and new materials tend to take over.

The empirical evidence for the pattern in intensity of use presented by Malenbaum is especially compelling when performing long-term forecasting. But, the difficulty of employing intensity of use to make long-term forecasts belies the seeming simplicity of the empirical measure. For example, Malenbaum's extrapolations of his own constructed plots of IU were simple, seemingly prudent, extensions of informally estimated linear relations, based upon the most recent years of economic theory and rigorous statistical methods to improve long term mineral forecasting.

STATISTICAL-EMPIRICAL MODELS

Rigorous statistical methods* are employed in this study to model simple intensity of use, as Malenbaum defined it, and to test the models by ex post forecasting on the last few years of data ; data which purposefully were withheld from the data base for model development. Special effort was devoted to identify objectively the model best indicated by the data, based upon slope characteristics of the data and of known models. These methods require the simultaneous selection of data "cut" and model form. Then, data transformations were employed to enable fitting of the selected model by least squares estimation procedures. When these transformations were not sufficient, as in the case of the Gompertz curve, a least squares estimation was approximated by Gomes polynomials. The objective at this level was to examine formal mathematical models as models of intensity of use and then to employ these models to estimate intensity of use and demand on a test period, thereby permitting the examination and comparisons of various models for ex post forecasting.

* This empirical investigation differed from those by Roberts (1985) and Roberts and Harris (1984) in that the latter employed methods of computer search to fit complex functions, while this research employed rigorous statistical methods.

The most frequently selected model is the log-normal IU :

$$IU_t=\frac{K}{\sigma\sqrt{2\pi}}e^{-\frac{1}{2}\left(\frac{\ln y_t-\mu}{\sigma}\right)^2},$$

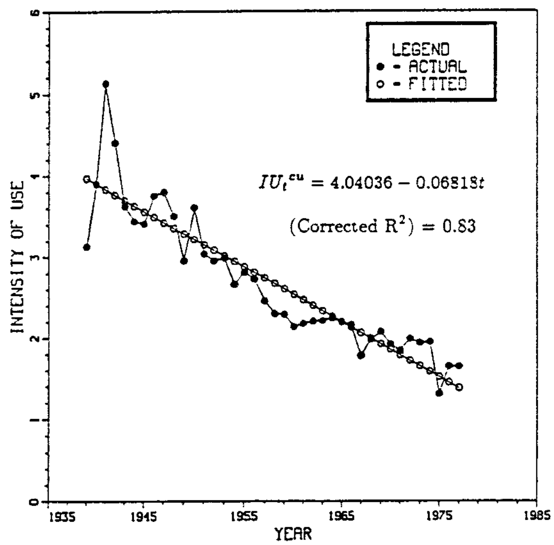


Fig. 3. Actual and fitted copper intensity of use using the method of least squares.

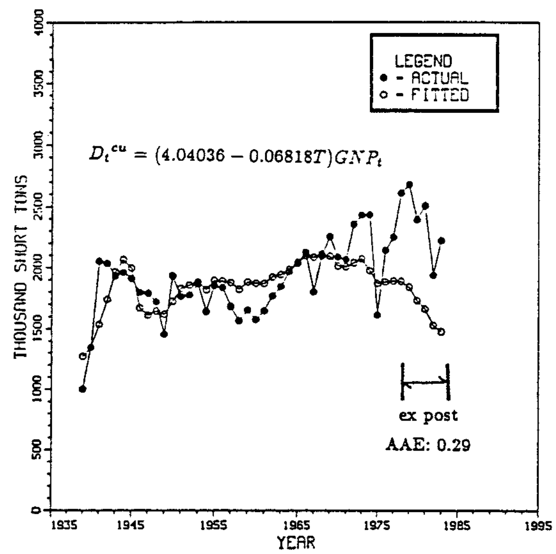


Fig. 4. Actual and predicted copper consumption using the model of least squares IU, 1939-1977 (ex post : 1978-1983).

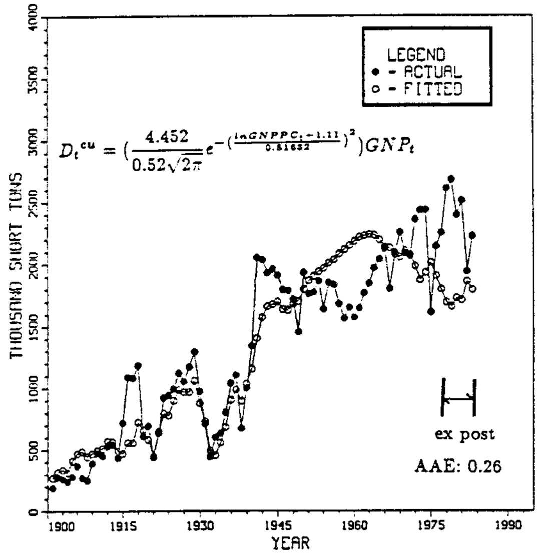


Fig. 5. Actual and predicted copper consumption using the model of lognormal IU, 1901-1977 (ex post : 1978-1983).

where

K, μ and σ are parameters estimated in the fitting process.
y_t=per capita income.

Other models investigated and sometimes fitted include the linear (last part of time seires), normal, and derivative logistic. All of these were fitted by least squares or by methods that approximate least squares estimation, e.g., Gemes polynomials. Some very excellent fits were found, and many of these performed quite well in the test period.

The lognormal life cycle model for a mature industry, even when the fit was very good, occasionally produced forecasts that are not believable. The projection of a flat (constant) IU may seem like a conservative forecast, but when income, Yt, is increasing exponentially, even at a low compound rate, such a projection results in exponentially increasing demand. The important point here is that when IU has peaked and is well down the tail of the life cycle, forecasts of IU are predicted upon the fit of the lognormal tail, which exhibits a gentle slope in that region, and there is a very fine line between a fit which produces exponentially increasing demand and one which produces decreasing demand.

For the purpose of demonstration and comparison, intensity of use of U.S. copper was forecast by a time trend which was estimated by a least squares linear relationship (Fig. 3). This linear trend model is of interest because of its use by the U.S. Bureau of Mines for forecasting mineral demand. Although the equation of this model appears satisfactory in the base period, forecast demands clearly missed tracing the ex post demands (see Fig. 4). A lognormal model of IU and y_t ($GNPPC_t$) was fitted to the intensity of using data transformations and least squares (Fig. 5). Although the fitted lognormal model performed a bit better than the linear trend model for the expost period, its performance still leaves much to be desired. Findings such as these prompted the examination of models with more economic structure.

ECONOMICS AND THE INTENSITY OF USE MEASURE

Examination of Malenbaum's definition of intensity of use as an economic model leads to a simple income demand model as the closest parallel :

$$d_t = Ay_t^{\alpha}$$

where d_t = per capita consumption of metal ; y_t = per capita income.

Given $IU_t = \frac{\frac{D_t}{pop_t}}{\frac{Y_t}{pop_t}} = \frac{d_t}{y_t}$ the above model implies

$$IU_t = Ay_t^{\alpha-1}$$

Although this model is a proper statement of Malenbaum's concept of intensity of use, it is not conformable with the full range of empirical data, because by this model. depending upon the magnitude and sign of α, IU is either an increasing or decreasing exponential function of y, except when $\alpha = 1$, for which IUt is constant. Consider this model along side one that is conformable with empirical evidence : a transformed, generalized equivalent of the lognormal :

$$IU_t = Ay_t^{\alpha_1} e^{\alpha_2(\ln y_t)^2}$$

where $\ln A = \ln K - \ln\sigma - \ln(\sqrt{2\pi}) - \frac{\mu^2}{2\sigma^2}$

$$\alpha_1 = \frac{\mu}{\sigma^2}$$

$$\alpha_2 = \frac{1}{2\sigma^2}$$

and

$$\ln IU_t = \ln A + \alpha_1 \ln y_t + \alpha_2 (\ln y_t)^2$$

The above equation reveals that the empirically consistent model consists of the income demand model multiplied by an exponential function of $\ln y_t$. Further insight is gained by nothing that $y_t^{\alpha_2 \ln yt}$ is equal to $e^{\alpha_2(\ln y_t)^2}$. Therefore, we could write the lognormal model of IU as follows :

$$IU_t = Ay_t^{\alpha_1 + \alpha_2 \ln yt}$$

and

$$d_t = Ay_t^{\alpha_1 + 1 + \alpha_2 \ln yt}$$

where

$$d_t = IU_t \cdot y_t = \text{per capita demand}$$

This is revealing, for it shows that the lognormal is also a translog function. When viewed in the light, improvement on the lognormal IU model leads to the use of a translog income (per capita) demand model.

Even though this translog income demand model is flexible and conformable with empirical evidence, it is deficient in economic structure. Maximization of an aggregate production function subject to constraint by the cost function yields as the very minimum model for derived demand across time the following :

$$D_t = f(P_t,\ P_{st},\ Y_t,\ \tau_t)$$

where

P_t, P_{st} = own factor price and substitute factor price, respectively,

Y_t = income,

τ_t = technical change in the mineral using process.

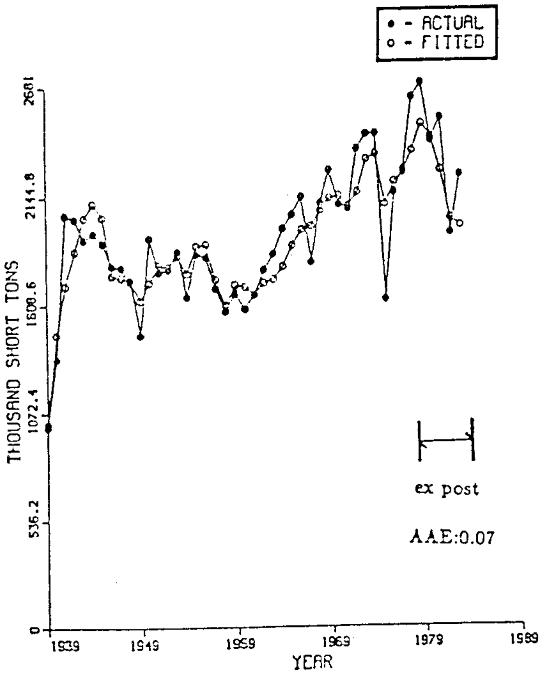
Identifying f( ) to be the flexible translog function would result in the following relation :

$$D_t = \alpha 1 Y_t^{\beta y(\ )} P_t^{\beta p(\ )} Ps_t^{\beta ps(\ )} \ \tau_t^{\beta \tau(\ )}$$

Suppose we divide both sides of the equation by Y_t.

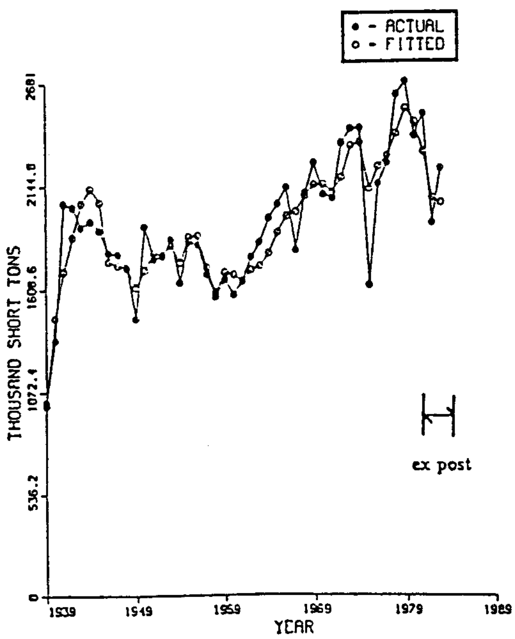
$$IU_t = \frac{D_t}{Y_t} = \alpha Y_{yt}^{\beta yo-1} \ P_t^{\beta p(\ )} Ps_t^{\beta ps(\ )} \tau_t^{\beta t(\ )}$$

This shows that extended IU and per capita demand translog models differ only by the exponent of Y : $\beta_{yo}$for demand and β_{yo}-1 for IU. Of course, for any particular commodity, some of these coefficients may be zero, giving a simpler estimated model for that particular com-



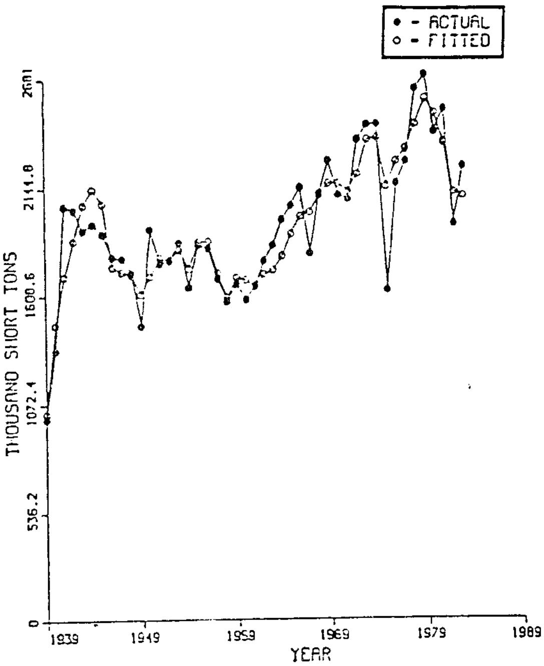
$D_t^{cu} = 7.837 y_t^{0.387 \ln r} \tau_t^{0.072 \ln p_{cu} - 0.2609 \ln r}$

Fig. 6. Actual and predicted copper consumption using the model of translog IU, 1939-1977 (ex post: 1978-1983).



$D_t^{cu} = 7.84 y_t^{0.41 \ln r} \tau_t^{0.067 \ln p_{cu} - 0.26 \ln r}$

Fig. 7. Actual and predicted copper consumption using the model of translog IU, 1939-1980 (ex post: 1981-1983).



$D_t^{cu} = 7.91 y_t^{0.417 \ln r} \tau_t^{0.063 \ln p_{cu} - 0.26 \ln r}$

Fig. 8. Actual and predicted copper consumption using the model of translog IU, 1939-1983.

modity. By employing stepwise regression, an equation can be selected that contains only a few variables, while in others there may be several, that are significant. Fig. 6 shows the very good fit of the translog model for copper for the base period (1939-1977) and the ex post (1978-1983) forecasts by the model:

$$D_t = 7.83 Y_t^{0.3871 \ln \tau_t} \tau_t^{0.0721 \ln Pcu_t - 0.2609 \ln \tau_t}$$

As indicated in the figure, this model performed very well in the ex post period. Fig. 7 shows the same model, but bit to data for 1939-1980, with the ex post period being 1981-83, and Fig. 8 shows the same model fitted to all of the data.

One approach to long-term forecasting is to include only those explanatory variables that can be projected with less error than direct projection of the dependent variable. Basically, that philosophy is consistent with Malenbaum's simple intensity of use measure. Although such an approach seems logical and practical, it generally is not good procedure, for it leads to the neglect of prices and technology and to a distorted view of the effect, say, of income, for all of the variation in demand is attributed to either income or random effects.

A far better procedure is to construct a more complete economic model, e. g., one with prices and technology, so that changes in consumption during the base period can be appropriately allocated to the most relevant economic factors. Even if none of the variables except income were projected, the presence of the variables would mitigate an otherwise exaggeration of the effect of income on consumption. Besides, the presence of the other variables permits contingency and sensitivity analyses and the description of selected scenarios.

Initial research on the extended translog demand model employed time as a proxy for technology, a practice that is common in econometric models and convenient for long-term forecasting because of the known value of future time. The extended translog model was fitted to data on several mineral commodities. In general, the model fit the data very well, and ex post testing showed it to be superior to extrapolation of trend models, simple lognormal, and simple translog models. However, some cases were found that produced "wild" forecasts.

COMPREHENSIVE TESTING AND COMPARISION OF MODELS

The scope of this paper includes ex post and ex ante testing and comparison of forecasts by various models over the few most recent years, as has been demonstrated for the linear trend model for IU, the lognormal IU model, the extended translog model of IU, and a trend tracing of the translog IU model. For example, for the ex post period of 1978 to 1983, errors for the models were found to be as follows :

Linear trend of IU	AAE = 0.29
Lognormal IU model	AAE = 0.26
Extended translog IU model	AAE = 0.07

As stated earlier, the extended translog IU model and its trend-traced version perform much better than the other two models. Certainly, such performance on the recent five year period would encourage the use of the extended translog IU model, either with trend tracing or without. This may indeed be a robust conclusion ; however, such testing is deficient in two regards :

Table 1. Ex post and ex ante forecasts : actual versus predicted copper consumption using the extensive translong IU model.

YEAR	ACTUAL	PREDICTED EXPOST (1978-1986)	PREDICTED EXANTE (1978-1986)	PREDICTED EXPOST (1981-1986)	PREDICTED EXANTE (1981-1986)
1967	1803.00				
1968	2097.00				
1969	2258.00				
1970	2089.00				
1971	2068.00				
1972	2359.00				
1973	2433.88				
1974	2436.08				
1975	1617.07				
1976	2145.08				
1977	2254.20				
1978	2611.35	2349.02	2347.14		
1979	2680.79	2481.27	2378.73		
1980	2397.50	2416.49	2330.16		
1981	2511.04	2252.39	2352.88	2314.44	2423.91
1982	1941.15	2013.28	2401.92	2069.87	2480.29
1983	2226.65	2068.88	2338.04	2130.89	2425.24
1984	2322.60	2117.95	2292.24	2198.70	2387.66
1985	2313.20	2135.90	2064.25	2222.59	2150.36
1986	2213.20	2147.30	2143.12	2239.64	2236.55
AAE		0.0654	0.0827	0.0487	0.0851

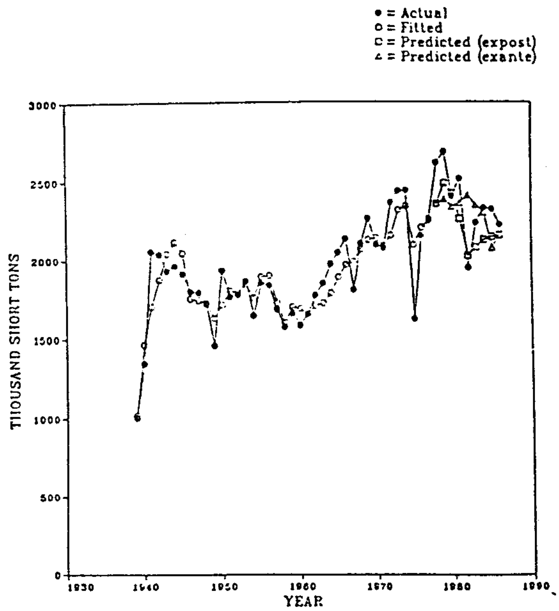


Fig. 9. Actual versus predicted values of U. S. copper consumption, ex post : 1978-1986, ex ante : 1978-1986.

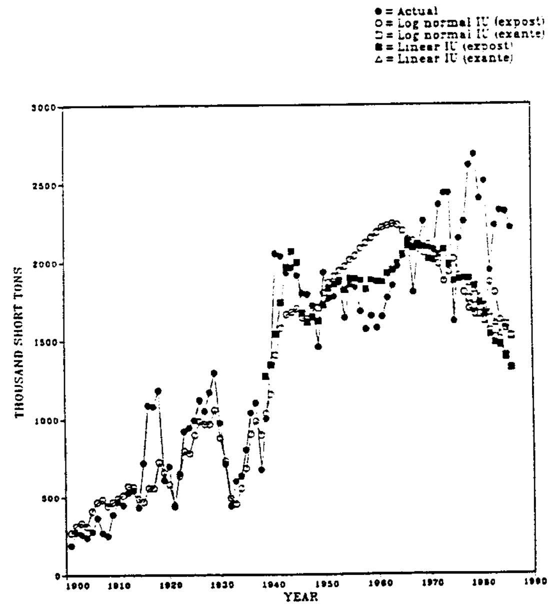


Fig. 11. Actual versus predicted values of U. S. copper consumption, ex post : 1978-1986, ex ante : 1978-1986

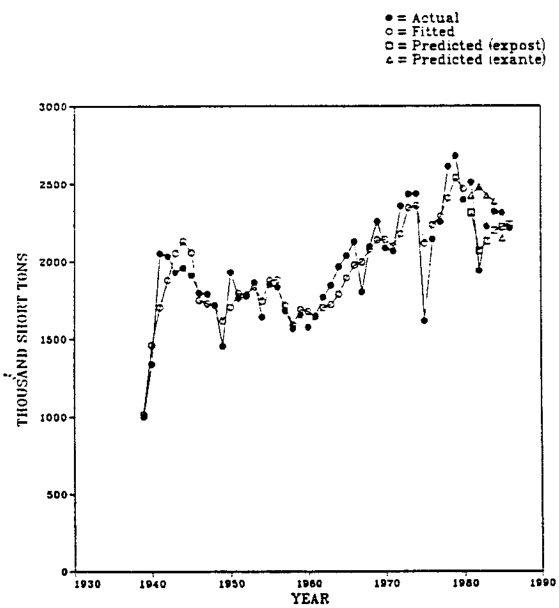


Fig. 10. Actual versus predicted values of U. S. copper consumption, ex post : 1981-1986, ex ante : 1981-1986.

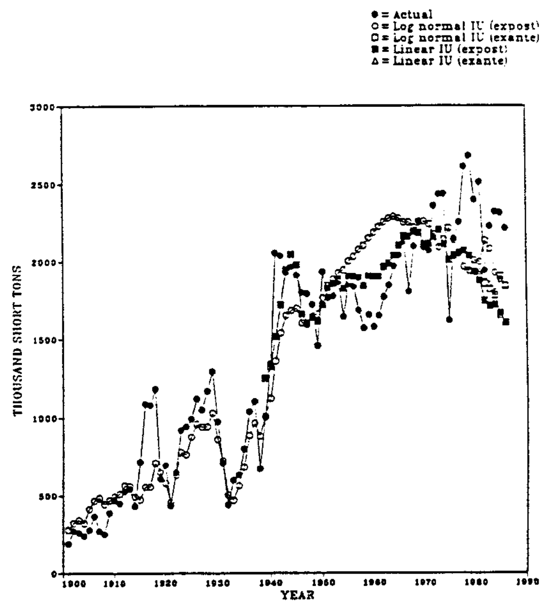


Fig. 12. Actual versus predicted values of U. S. copper consumption, ex post : 1981-1986, ex ante : 1981-1986.

1. It considers ex post forecasting performances on only one period; furthermore, that period was a time of unusual changes in markets and technologies.
2. Actual long term forecasting is not an ex post experience; rather, it is ex ante, meaning that none of the explanatory variables are known; consequently, they, e.g. GNP and population, must also be forecast.

Since actual long term forecasting is an ex ante experience, ex ante forecasting was performed on periods of 1978 to 1986, and 1981 to 1986. The estimations presented here are based on time series data of U. S. copper consumption from 1939 to 1986. In an attempt to forecast the dependent variable for the ex ante period it was necessary to first forecast the explanatory variables. Forecasts of these variables were achieved by several methods and are explained in a later section. Ex post and ex ante forecast values are given in Table 1. Fig. 9 and 10 illustrate time paths of the actual and predicted values of U. S. copper consumption. The ex post forecasts show that the extended translog IU model predicts very accurately for the ex post period, successfully locating the important turning points of the copper consumption time paths. Although the model predicted less satisfactorily ex ante than ex post, as expected, the ex ante forecasts followed the pattern of actual values fairly well. For the purpose of comparison, the linear trend model for IU and the lognormal IU model were applied for the same periods. As Fig. 11 and 12 indicate, predictions of these models are much lower than the actual consumption. The empirical results of this study confirm that predictions of copper consumption using per capita income only, which is consistent with Malenbaum's model, are inferior to those based upon income plus price and technology.

CONCLUSION

This study found that dynamics of intensity of use and economic theory of derived demand can both be accommodated through an extensive translog demand model. The first step in this recognition is that the skewed life cycle empirical pattern of intensity of use IU plotted against per capita income (y_t) is of lognormal form. The second link is mathematical; specifically, it can be shown that a lognormal intensity of use model can be transformed into an equivalent simple translog intensity of use model:

$$IU_t = Ay_t^{\alpha_0+\alpha_1 \ln y_t}$$

Noting that intensity of use is the ratio of per capita demand to per capita income($\frac{d_t}{y_t}$) permits the rewriting of the above intensity of use model as a per capita translog demand model:

$$d_t = Ay_t^{\alpha_0+1+\alpha_1 \ln y_t}$$

This model can be recognized as a dynamic version of a simple income demand model: $d_t = Ay_t^{\alpha_1}$. Such recognition constitutes the last link in the evolution of the extensive translog demand model, because given that recognition, traditional demand theory leads to the extension of the model to include other variables, such as own price, P_o, substitude price, P_s, and technical change, τ :

$$d_t = Ay_t^{\alpha_0+\alpha_1 \ln y_t+\alpha_2 \ln p_{ot}+\alpha_3 \ln p_{st}+\alpha_4 \ln \tau_t} \; P_o^{b_0+b_1 \ln p_{ot}+b_2 \ln p_{st}+b_3 \ln \tau_t} \; P_{st}^{c_0+c_1 \ln p_{st}+c_2 \ln \tau_t} \; \tau_t^{d_0+d_1 \ln \tau_1}$$

This is the full form of a translog demand model having four explanatory variables. Usually, several of the coefficients are not significant, thus estimated models are of much simpler form. Empirical testing of the model showed that generally it performs better than the other models tested, which included trend models (linear, derivative logistic, derivative Gompertz, normal, and lognormal) of demand and of intensity of use; conventional intensity of use models in which intensity of use is a function of per capita income (linear, normal and lognormal), and traditional demand models which include income and price.

Data on copper, zinc, lead and potash were used to test forecasting models. Models were compared by ex post performance on a test period. It was found that variables such as income, technological change, and relative prices all affect the demand for a mineral commodity in the United States and should be considered in making long-term forecasts. Empirical results showed that the extensive translog model, which is a flexible function and includes both the classical case of fixed coefficients and the dynamic case of varying coefficients of the explanatory variables, gave better forecasts than the original intensity of use model. The model used is small, consisting merely of four variables, but this does not seem to be an important limiting factor for the model's simulation performance. Moreover, being small, these models are easier to maintain and use, because fewer independent variables have to forecast in advance.

Although the extensive translog IU models were found to make much better forecasts than other models tested, some of the translog models were not appealing because the price of the mineral had a positive sign. Of course such a result conflicts with price theory and the theory of derived demand. Even so, such a result is not unusual in econometric analyses in which a time series of quantities traded and their prices are employed to estimate the parameters of a single equation model of demand, particularly when a long time series is employed. This is simply the familar identification problem compounded by the long time series. Simply stated, a price coefficient that is both statistically significant and of wrong sign is evidence that demand shifts have dominated the market, which has the effect that market quantities and prices define the supply curve more than the demand curve, or the effect of other variables surpasses that of own price. Of course, one remedy is to model both demand and supply, and to estimate their co-efficients simultaneously. Accordingly, additional research is needed to explore the construction of the econometric system sufficient to identify the translog demand relationship.

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鑛物資源 需要豫測 模型으로서의 使用强度 方法 應用

전 규 정

요약 : 이 研究에서는 경험적인 使用强度 方法과 派生需要의 경제적이론을 超越代數 需要 모델을 使用하여 결합하였다. 첫번째 단계로서 한 인구당 GDP와 使用强度의 실험적인 패턴인 非對稱 生命曲線을 고려하였으며, 두번째 단계로서는 이러한 非對稱 生命曲線이 수학적으로 단순한 超越代數 使用强度 모델로 바꿀수 있다는 것을 보여 주었으며, 마지막 단계로서 전통적인 需要理論이 價格, 代替效果, 技術의 變化를 포함한다는 것에 근거하여 超越 代數 需要 모델을 연장한 결과 전통적인 需要모델이나 使用强度 모델보다 더 정확하게 鑛物資源 需要를 豫測하였으며 使用强度 方法으로서도 資源의 需要를 豫測할 수 있음을 보여주었다.