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Robustness Recovery of Observer Based Multivariable Control Systems

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An approach for robustness recovery of the observer-based control system is presented. The approach is developed by adding a loop with appropriate constant matrix to the observer-based closed-loop system. It will be shown that if there exists an added-loop matrix M satisfying $F=MC$ for a feedback gain F and output matrix C, then the observer-based control systems have the same loop transfer functions as full-state feedback implementations, in other words, the former has the same relative stability and robustness as the latter.

Introduction

Optimal control theory has many positive features such as robustness properties including guaranteed classical gain margins of-6db to $+\infty$ and phase margins of 60^0 in all channels, and sensitivity reduction guaranteed by the circle condition. This result is valid, for the full-state feedback case, however it is not completely satisfactory in the case that observers or Kalman filters are used in an implementation. Much researches concerning observer or LQG based multivariable control system design have been done[Doyle and Stein(1979, 1981), Lehtomaki et al.(1981), Lee and Chen(1987)]. Doyle and Stein(1979) showed that if a sufficiently large spectral intensity matrix is selected, then the LQ regulator guaranteed stability margins will be recovered at the input of the plant provided that the minimum-phase assumption holds. Their method is essentially the dual of a sensitivity recovery method suggested by Kwakernaak(1969). Lehtomaki et al.(1981) showed that if the system model embedded within the Kalman filter is always the same as the true perturbed system, the LQ and Kalman-filter guaranteed stability margins will apply to LQG controllers at the input and output of the plant. Lee and Chan(1987) showed that if there exists a constant matrix M such that 1) the filter gain $G_f=BM$ and 2) the regulator gain $G_r=kMC(k\geq 1)$ then the LQ guaranteed minimum stability margins will be recovered at both input and output of the plant as $k\rightarrow\infty$.

In this paper, we shall show a method of designing observer (or LQG) based multivariable control system recovered at both input and output. This method is similar to the one of Lee and Chen in view of the conditions for the filter gain and the regulator gain but its approach is basically different, which will be shown in the following section. The LQ guaranteed minimum stability margins at both input and output of the plant are recovered just in the case of $k=1$. That is, we need not the condition $k\rightarrow\infty$ which was shown by Lee and Chen.

Observer-based controllers

We consider the multivariable control-loop as shown in Fig.1. The plant is an nth order linear system with m inputs and p outputs :

$$G(s) = C(sI - A)^{-1}B \tag{2.1}$$

where the matrix pairs (A,B) and (A,C) are controllable and observable respectively. The control-loop system of Fig.1(a) is driven with full-state feedback which is given by solving the general optimal regulator problem, and the one of Fig.1(b) is done with a full-order observer with various points of the loop marked. The loop transfer functions at break points k=1,2,3,4 are denoted $T_k(s)$. It is clear that this overall control loop includes linear-quadratic Gaussian controllers as special cases. Further, for the more realistic control scheme, it also allows dynamic elements such as integrators and lead-lag elements in the closed-loop.

The properties of the loop transfer functions taken with respect to the loop breaking point x were introduced by Kwakernaak and Sivan, Doyle and Stein, Lehtomaki et al. as the following.

Property 2.1

1. The closed-loop transfer function matrices from command r to state x are identical in both implementation of Fig.1.
2. The loop transfer functions broken at ① are identical in both implementations.
3. The loop transfer functions broken at ② are generally different in the two implementations, but they are identical if the observer dynamics satisfy :

$$K(I + C(sI - A)^{-1}K)^{-1} = B[C(sI - A)^{-1}B]^{-1} \tag{2.2}$$

for all values of the complex variable s.

Lemma 2.1 [**Lehtomaki et.al.(1981)**]

If the matrix P satisfies the matrix algebraic Riccati equation :

$$A^TP + PA + Q - PBR^{-1}B^TP = 0 \tag{2.3}$$

with $R > 0$ and $Q \geqq 0$, then

$$(I + T_u(s))^*R(I + T_u(s)) = R + H(s) \tag{2.4}$$

where * means the conjugate transpose and

$$T_u(s) = F(sI - A)^{-1}B = R^{-1}B^TP(sI - A)^{-1}B \tag{2.5}$$

$$H(s) = [(sI - Q)^{-1}B]^*(Q + 2Re(s)P) \tag{2.6}$$

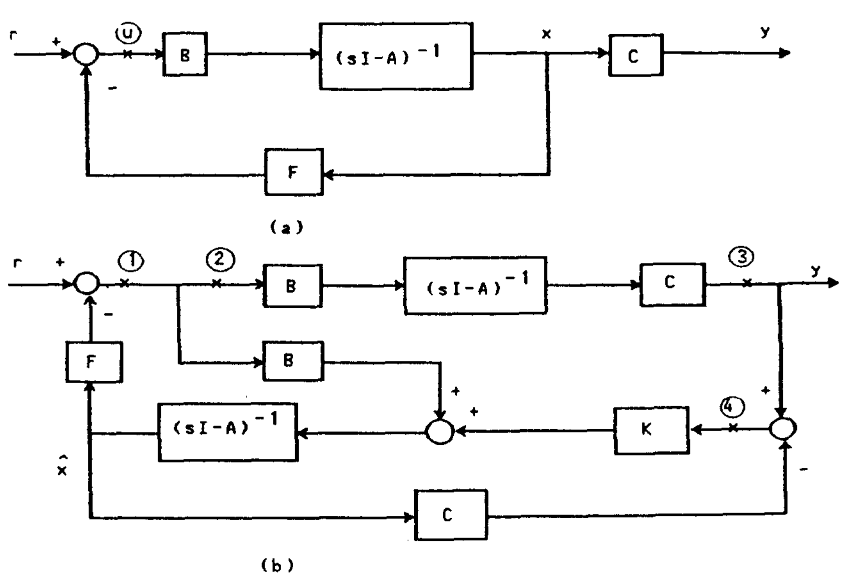

Fig.1 LQG regulator.

$[(sI-A)^{-1}B]$

Furthermore, if $Q>0$, B has full rank and $P\geq 0$, then (2.4) implies that

$$(I+T_u(s))^*R(I+T_u(s))>R,\ s\epsilon\ D_r \quad (2.7)$$

Alternatively, if $\det(jwI-A)\neq 0$ for all w and $P\geq 0$ then (2.4) implies that

$$(I+T_u(s))^*R(I+T_u(s))\geq R,\ s\epsilon\ D_r \quad (2.8)$$

where D_r is Nyquist contour of radius r.

Using the return difference matrix of the closed-loop :

$$T_O=I+F(sI-A)^{-1}B, \quad (2.9)$$

eq. (2.7) becomes with $R=I$

$$T_O^*(s)\,T_O(s)>I. \quad (2.10)$$

This relation means that if the return difference of a system in Fig.1(a) is larger than R=1, that is, the circle condition is satisfied, then the robustness of the system is guaranteed.

From property 2.1, we see that the loop transfer functions broken at ① and ④ satisfy the following inequalities

$$(I+T_1(s))^*(I+T_1(s))\geq I \quad (2.11)$$

$$(I+T_4(s))^*(I+T_4(s))\geq I \quad (2.12)$$

for all $s\epsilon\ D_R$. Thus it is noted that at points ① and ④ the LQ and Kalman filter minimum guaranteed stability margins can be applied. In the next section, we will introduce a method of recovering the robustness of the loop transfer functions broken at points ② and ③.

Robustness recovery

In this section the problem to be considered, of course, is that "how can we make the loop transfer functions at points ② and ③ shown in Fig.1(b) equal to the ones for the full state feedback implementation (Fig.1(a)) ? ". That is if $T_2=T_u$ and $T_3=T_y$ where

$$T_u=F(sI-A)^{-1}B \quad (2.5)$$

$$T_y=C(sI-A)^{-1}K \quad (3.1)$$

$$T_2=F(sI-A+BF+KC)^{-1}KC(sI-A)^{-1}B \quad (3.2)$$

$$T_3=C(sI-A)^{-1}BF(sI-A+KC+BF)^{-1}K \quad (3.3)$$

then the closed-loop with observer based implementation will have the same robustness to the optimal full state feedback system shown in Fig.1(a). Unfortunately, the problem stated in the above is not easily solved by the closed-loop system as it is shown in Fig.1(b), however several implicit methods exist. If we add a loop with appropriate constant matrix to Fig.1(b) as shown in Fig.2, then we may achieve our object which will be seen in the following theorem.

Theorem 3.1

Consider the closed-loop system shown in Fig 2. Suppose that there exists a posoitive definite solution P which satisfies the Riccati equation (2 3). If there is a matrix M such that

$$MC=F \quad (3.4)$$

where F is given by

$$F=R^{-1}B^TP, \quad (3.5)$$

then, the loop transfer function taken at point ②

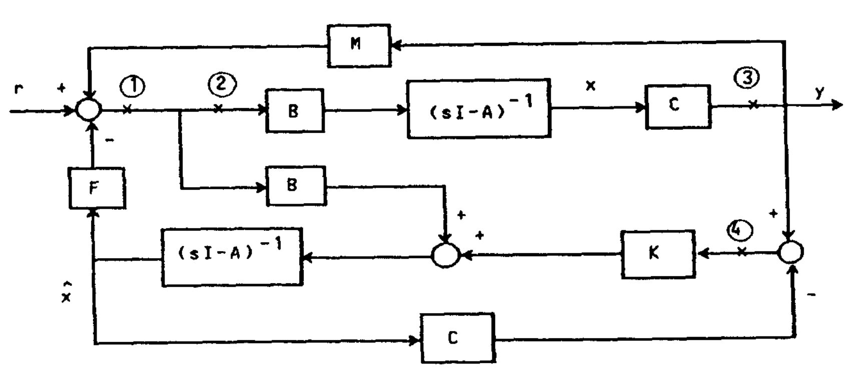

Fig.2 LQG regulator with robustness recovery.

(Fig.2) with the observer gain

$$K = BM \tag{3.6}$$

will be equal to the one $T_u(s)$.

[Proof] Consider the relation $T_u = T_2$ from Fig. 1(a) and Fig.2 :

$$F(sI-A)^{-1}B = [I+F(sI-A+KC)^{-1}B]^{-1}[M+F(sI-A+KC)^{-1}K]C(sI-A)^{-1}B \tag{3.8}$$

In eq.(3.8), if there exists a matrix M such that MC=F, then, the loop transfer function (taken at point ②) will be equal to the one of Fig.1(a) by taking the observer gain as K=BM since KC=BF=BMC.

Dually for theorem 3.1, we can obtain easily the following corollary.

Corollary 3.1

If there exists a matrix M such that

$$K = SC^T\Theta^{-1} = BM \tag{3.9}$$

where S is a positive definite solution of the equation :

$$AS+SA^T+\Sigma - SC^T\Theta CS=0,\ \Sigma \geqq 0,\ \Theta \geqq 0, \tag{3.10}$$

then, the loop transfer function taken at point ③ will be equal to the one $T_y(s)$ where F=MC.

[Proof] The loop transfer function taken at point ③ is given by the following equation :

$$T_3 = C(sI-A)^{-1}B[I+F(sI-A+KC]^{-1}B]^{-1}[M+F(sI-A+KC)^{-1}K]$$

The proof is similar to one of theorem 3.1.

Corollary 3.2

K, F obtained by theorem 3.1 and corollary 3.1 satisfy respectively the properties :

$$Re(\lambda_i I - A + KC) < 0 \tag{3.11}$$

$$Re(\lambda_i I - A + BF) < 0 \tag{3.12}$$

Consider the above results for the case of LQG based control system. Then we can see in the following how to determine the weighting matrices Q, R which yield for the colsed-loop system to have LQ guaranteed stability margins. This result has been similarly shown by Lee and Chen (1987). They showed that the LQ guaranteed margins of the loof transfer functions $T_2(s)$ and $T_3(s)$ are obtained as $k\rightarrow\infty$ in the form of the gain matrix F=kMC. That is, they showed

$$\lim_{k\to\infty} T_2(s) = MC(sI-A)^{-1}B \tag{3.13}$$

$$\lim_{k\to\infty} T_3(s) = C(sI-A)^{-1}BM \tag{3.14}$$

But in the following, we shall show that when k=1, the transfer functions $T_2(s)$ and $T_3(s)$ wil have the same guaranteed stability margins as those of $T_u(s)$ and $T_y(s)$ respectively.

Theorem 3.2

Suppose that the assumption of corollary 3.1 are satisfied and the following matrices Q and R obtained from eq.(3.10)

$$Q = S^{-1}\Xi S^{-1},\ R=[M\Theta M^T]^{-1} \tag{3.15}$$

are given as the weighting matrices of the Riccati equation (2.3) where $P=S^{-1}$. Then the feedback gain given by the positive definite solution of eq.(2.3)

$$F = R^{-1}B^TP \tag{3.16}$$

gives us that

1) it minimize the cost functional :

$$J = \int_0^\infty (x^TQx+u^TRu)\,dt, \tag{3.17}$$

2) the loop transfer matrices $T_2(s)$ and $T_3(s)$ have the same guaranteed stability margins as those of $T_u(s)$ and $T_y(s)$ respectively.

[Proof] Using $P=S^{-1}$ in eq.(3.10), we have

$$PA+A^TP+S^{-1}\Xi S^{-1} - C^T\Theta^{-1}C=0 \tag{3.18}$$

By using the relations $R=[M\Theta M^T]^{-1}$ and

$$F^TRF = S^{-1}BR^{-1}B^TS^{-1} = PB(M\Theta M^T)B^TP = C^T\Theta^{-1}C \tag{3.19}$$

since $C=\Theta M^TB^TP$, we can write eq.(3.18) as

$$PA+A^TP-F^TRF = -S^{-1}\Xi S^{-1} \tag{3.20}$$

which satisfies 1). The part 2) will be proven by using the relation (3.19) and theorem 3.1.

[Remark] A method of obtaining an analytic solution of the matrix equation F=MC or K=BM is well known by Pringle and Rayner (1971).

Penrose(1955), and Munro and Vardulakis (1973).

A pxn matrix C_g^1 is said to be a g_1-inverse of the nxp matrix C if

$$CC_g^1C = C$$

It is well known by the results of Penrose (1955) that if a g_1 inverse of C is chosen by

$$C_g^1 = C^T(CC^T)^{-1}$$

as the right-inverse of C, then a solution to MC=F is given by

$$M = FC_g^1$$
$$= FCD^T(CC^T)^{-1}$$

The method of Pringle and Rayner(1971), which has been used to a pole shifting problem by Munro and Vardulakis(1973), does not require either the double matrix multiplication or the martix inversion, and if the consistency condition given by $F = FC_g^1C$ is not satisfied for a particular g_1-inverse C_g^1 and matrix F, it permits us to investigate other g_1-inverse of C.

A Simple Example

To illustrate the proposed robustness recovery method, consider the system :

$$x = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} x + \begin{pmatrix} 0 \\ 1 \end{pmatrix} u$$

$$y = [1\ 1]x$$

By solving the Riccati equation (2.3) with the weighting matrices Q=diag[10,10] and R=1, we obtain the feedback matrix :

$$F = [4.32\ 4.32]$$

A solution of satisfying F=MC is given by M=4.32. Then the loop transfer matrices $T_2(s)$ is equal to $T_u(s)$.

Conclusions

In this paper, we have proposed a method of designing observer(or LQG) based multivariable control system with robustness recovery. This approach is an explicit robustness recovery method, since the weighting matrix can be given by a finite value which has not been used for the robustness recovery in the traditional methods. But for the case that there is not a matrix M such that F=MC(or K=BM), at present, the solution does not be obtained. It is supposed that with the weighting matrix of a finite value there will not be any different method of obtaining an explicit robustness recovery except this approach.

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Robustness Recovery of Observer Based Multivariable Control Systems

관측기를 이용한 다변수 제어계의 로바스트성 회복

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최적제어이론은 −6db에서 무한대까지의 게인여유와, 60° 의 위상여유, 그리고 원조건(Circle Condition)에 의해 보증되는 감도감소등과 같은 로바스트성에 대한 긍정적인 면들을 가지고 있다. 이러한 결과는 전상태 피드백 경우일때는 타당하지만, 실현문제에 있어, 관측기나 칼만필터가 사용되어질 경우에는 위의 조건이 완전히 만족하지 않는다.

본 논문에서는 관측기를 이용하여 다변수 제어계를 설계할 경우, 입.출력부분에서 안정여유가 회복될 수 있도록하는 하나의 방법을 제안했다.

플랜트 입력과 출력에서 LQ가 보증하는 최소안정여유는 K=1의 경우에서도 회복될 수 있음을 보였다.

로바스트성 회복을 위해 본 논문에서는 종래의 방법(K=∞의 조건)과는 달리 유한값으로 하중행렬이 주어질수 있으므로 본 방법은 하나의 명료한 로바스트성 회복법이라 할 수 있다.

F=MC(or K=BM)를 만족하는 행렬 M이 존재하지 않을 경우, 그 해는 얻어질 수 없다. 그러나 K를 유한값으로 취했을 경우, 이 방법 이외의 또 다른 방법으로서는 입력과 출력에서 로바스트성 회복을 가지게 할수 없을 것으로 생각된다.