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〈**Original**〉

Calculation of the Coupled Photon-Electron Slowing Down Energy Spectrum in a Homogeneous, Infinite Tissue Equivalent Material.

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Abstract

A somewhat detailed energy spectra in terms of the track length resulting from coupled electron-photon slowing down are calculated throughout the ICRU standard tissue with uniformly distributed gamma sources of ^{60}Co and ^{137}Cs, respectively. The calculation was accomplished by utilizing the latest available cross-section data as input to a carefully optimized computer code.

In this report, the calculational method is described in detail. Furthermore, results of calculations are given in graphical form.

The results show that the energy spectrum defined in terms of differential track length has about same shape although the energies of gamma source are different. The discontinuity in the energy spectrum appears at the energy of $T=(1/T_0+2/m_0c^2)^{-1}$, because a primary photon can not be degrade to a point below this energy.

I. INTRODUCTION

Spatially and angularly independent electron-photon transport in a uniformly homogeneous medium can be calculated in a straightforward manner by direct numerical integration. Spencer and Fano[1] developed a numerical technique for calculating electron slowing down, independent of position and direction, i.e., this is a mathematical procedure with only one variable, energy(T), while the general procedure has six variables. In the procedure, it is assumed that the system has a homogeneous and infinite medium with uniformly distributed sources. They considered bremsstrahlung losses on spectrum but they neglected the Compton and photoelectrons generated by bremsstrahlung photons.

The method developed by Spencer and Fano for the calculation of the spectra has been applied extensively by McGinnies[2], and in his calculation he considered the accumulation of secondary knock on electrons and neglected the bremsstrahlung effect.

Similar methods have been applied to a slowing down of photon energy[3]. After then H.L. Beck[4] has coupled these two methods in a systematic way and solved the entire electromagnetic cascade procedure including all major sources and interactions.

By using Beck's method, we considered only energy spectrum in a medium. EGATL code of coupled electron and gamma track length program has been utilized as a calculational tool, and a trapezoidal rule integration was chosen for simplicity in computer programming.

Source energies of 0.66MeV (^{137}Cs) and 1.17, 1.33MeV (^{60}Co) were considered, and tissue eqivalent material was chosen as representative material. This material composed as follows; 76.2% oxygen, 11.1% carbon, 10.1% hydrogen and 2.6% nitrogen.

The solution of this problem may find direct application to the analysis of the chemical and biological actions of gamma rays, and the biological effects of gamma rays are in fact the biological effects of their secondary charged particles.

II. THEORY

II-1. Gamma Ray Slowing Down Equation

If the source strength is expressed in photons/g, then the photon track length, $Y_\gamma(T)$, is equivalent to the flux in photons/cm²-MeV. As well known, the integral equation describing the conservation of photon track length is[4~6]

$$[\mu(T)/\rho]\cdot Y_\gamma(T)=\int_T^{T_0}\sigma_c(T,T')Y_\gamma(T')dT' + S(T)+\delta(T-T_0) \quad (1)$$

where, μ/ρ : mass attenuation coefficient

$S(T)$: additional sources of photons

$\delta(T-T_0)$: monoenergetic sources

$\sigma_c(T,T')dT$: probability per unit track length that a photon of energy T' will be Compton scattered to the interval between T and $T+dT$.

Distintion may also be made between those photons which have suffered no collisions in coming from the source and those which have made at least one collision. Thus $Y_\gamma(T)$ can be written as the sum of two components, namely,

$$Y_\gamma(T)=Y^s(T)+\delta(T-T_0)/\mu(T_0)/\rho \quad (2)$$

Substituting back into equation (1),

$$[\mu(T)/\rho]\cdot Y^s=\int_T^{T_0}dT'\sigma_c(T,T')Y^s(T') + \sigma_c(T,T_0)/[\mu(T_0)/\rho]+S(T) \quad (3)$$

Solutions of equation (3) have been obtained by numerical integration, i.e., we can evaluate equation (3) numerically for $Y_\gamma(T)$ using previously calculated Y_e's and Y_γ's at higher energies.

II-2. Electron Slowing Down Equation

We assume that electrons are produced within a uniform infinite medium with initial energy T_0. Then we inquire about the resulting spectrum, $Y_e(T)$, of electrons. The expected value $Y_e(T)dT$ of the track length is the solution of[4,7]

$$\int_T^{T_0}dT'Y_e(T')K(T',T)=\int_T^{T_0}S(T')dT' \quad (4)$$

where, $Y_e(T)$: electron track length at the energy of T'

$K(T',T)$: probability per unit path length that the electrons kinetic energy drops from T' to below T.

Rewriting equation (4), we obtained the followings;

$$\begin{aligned}Y_e(T)F_c(T_0,T)=&\int_T^{T_0}S(T')dT'\\&+\int_{2T}^{T_0}dT'[K_s(T',T)-K_b(T',T)]Y_e(T')\\&-\int_T^{T+\Delta}dT'[Y_e(T')K_c(T',T)-Y_e(T)\overline{K}_c(T',T)]\\&-\int_T^{T+\Delta}Y_e(T')K_b(T',T)dT'\end{aligned} \quad (5)$$

where, $F_c(T_0,T)=\int_T^{T_0}dT'\overline{K}_c(T',T)$

$\Delta=T_0-T$ or T

$\overline{K}_c(T',T)$: Spencer-Fano modified cross section.

The first term in equation (5) must include all sources of electrons. Fig. 1 shows the flow diagram of conversion of energy in the course of photon energy absorption in a medium, i.e., $S(T)$ includes electrons from continuous energy sources and electrons from gamma ray undergoing pair production, Compton scattering and photoelectric absorption. We thus rewrite the source term in equation (5).

$$\begin{aligned}\int_T^{T_0}S(T')\,dT'=&\int_T^{T_0}S_0(T')dT'\\&+\int_T^{T_0}dT'\left[\int_{T'}^{T_0}Y_\gamma(E)\,\sigma_{cs}(E,T')dE\right]\\&+2\int_T^{T_0}dT'\left[\int_{T'}^{T_0}Y_\gamma(E)\sigma_{pp}(E,T')dE\right]\\&+\int_{T+B.E.}^{T_0}Y_\gamma(T')\sigma_{pe}(T')dT'\end{aligned} \quad (6)$$

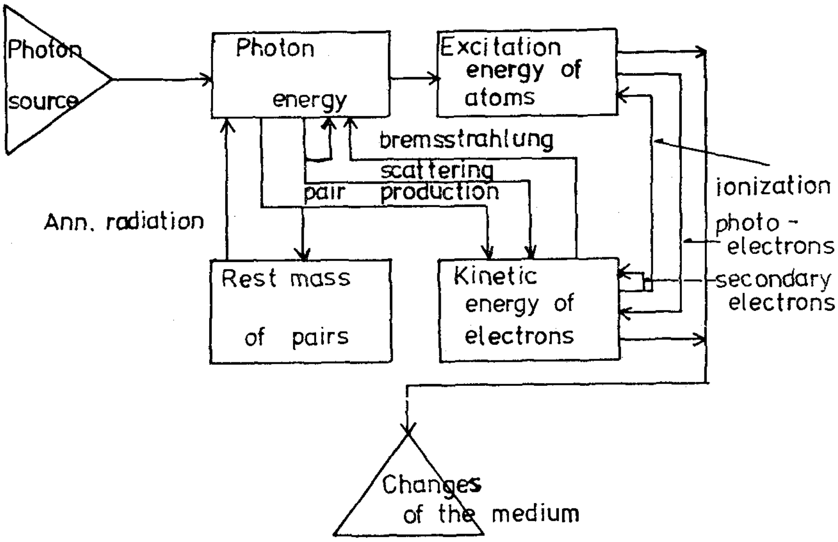

Fig. 1. Flow diagram of conversion of energy in the photon energy absorption in medium.

where, $Y_{\gamma}(E)$: photon track length at the energy of E

$B.E.$: binding energy

$\sigma_{cs}(E, T')$, $\sigma_{pp}(E, T')$: probability of a photon energy E producing an electron of kinetic energy T' by Compton scattering and pair production, respectively.

The second term on the right of equation (5) is a source of secondary electrons due to scattering. Since the energy loss is limited to $\tau \leq T/2$ for electrons of energy T, the K's are 0 if $T > T_0/2$.

The integrand in the third integral in equation (5) is small even though each of the terms become quite large for small energy losses. The last integral represents the majority of bremsstrahlung losses.

III. NUMERICAL METHOD

When we compute the values corresponding to a large number of values of T, it is often advantageous to make a numerical evaluation of the integrals. Thus we also determined $Y(T)$ step-by-step iteration scheme as follows: Having determined $Y(T)$ at $T_0, T_1, T_2, \cdots\cdots T$, we estimated what $Y(T_n)$ will be.

As T close to T_0, both $Y_{\gamma}(T)$ and $Y_e(T)$ are varying rapidly, thus two different quadrature schemes were used in solving equations (3) and (5). In order to calculate the quadrature weights and to achieve good accuracy over a wide range of source energies, the integral from T_0 to $T_0/2$ was divided into 80 energy groups and 24 groups were used for each subsequent factor of 2 reduction in energy. It is advantageous to evaluate the integrals on a logarithmic scale of energy values. The reasons for this are: 1) The integration can proceed owing to the smaller and smaller intervals of T even if the population of secondary electrons increase so rapidly at low energies that the solution diverges; 2) the special behavior at $T'=2T$ can be taken into account very simply by choosing ξ according to $\xi^n=1/2$.

In the gamma ray slowing down equations, (1) the probability of photon of energy T', scattering down to T and of a photon energy E' producing an electron of kinetic energy T becomes 0 whenever $T' > T/(1-2T)$ and $T > 2E'^2/(1+2E')$, respectively. We thus distinguish $T' > T/(1-2T)$ from $T' < T/(1-2T)$.

$$
\begin{aligned}
[\mu(T_N)/\rho]\cdot Y_{\gamma}^{s}(T_N) = \sum_{n=1}^{N-1} & W_{n,N}(T_n)\sigma_c(T_N, T_n)Y_{\gamma}^{s}(T_n) \\
& +\sigma_c(T_N, T_0)/[\mu(T_0)/\rho] \\
& +S(T_N) \\
& +W_{N,N}(T_N)\sigma_c(T_N, T_N) \\
& \quad Y_{\gamma}^{s}(T_N) \qquad (7)
\end{aligned}
$$

for $T_{N-1} < T_N/(1-2T_N)$.

$$[\mu(T_N)/\rho]\cdot Y_\gamma^s(T_N)=\sum_{n=1}^{N-1} W_{n,N}(T_n)\sigma_c(T_N, T_n)Y_\gamma^s(T_N) + \sigma_c(T_N, T_0)/[\mu(T_0)/\rho] + S(T_N)+4CT_N^2\cdot Y_\gamma^s(T_{N-1})/[(1-2T_N)^2 U] \qquad (8)$$

for $T_{N-1}>T_N/(1-2T_N)$.

Where, $C=0.15Z/A$

$U=T_{N-1}-T_N$

$W_{n,N}(T_n)$: quadrature weight.

In the electron slowing down equation, (5), we replace the integral from $T'=T_{N-1}$ to $T'=T_N$ for the third integral and for the last integral since the integrals in equation (5) become very large and bremsstrahlung cross section diverges in the critical range $T'\approx T$. As a result, we finally obtained the followings;

$$\begin{aligned} Y_e(T_N)=1/\{&E_c(T_0, T_N)-W_{N,N}(T_N)[2C(m_0c^2)] \\ &/[\mu\beta^2(T_N)]-\mu[K_b(T_{N-1}, T_N)/2 \\ &+(3/4)U\sigma_b(T_{N-1}, U]\} \\ &\times\sum_{n=1}^{N} W_{n,N}(T_n)S(T_n) \\ &+\sum_{n=1}^{n'} W_{n,N}(T_n)[K_s(T_n, T_N) \\ &-K_b(T_n, T_N)]\cdot Y_e(T_n) \\ &-\sum_{n=n''}^{N-1} W_{n,N}(T_n)[Y_e(T_n)K_c(T_n, T_N) \\ &-Y_e(T_N)\overline{K}_c(T_n, T_N)] \\ &-\sum_{n=n''}^{N-1} W_{n,N}(T_n)Y_e(T_n)Y_e(T_n)K_b(T_n, T_N) \\ &-\left[\frac{(2Cm_0c^2)W_{n,N}(T_N)}{U(T_{N-1})}\right] \\ &\cdot Y_e(T_{N-1})+U[K_b(T_{N-1}, T_N)/2 \\ &+U\sigma_b(T_{N-1}, U/4)Y_e(T_{N-1}) \end{aligned} \qquad (9)$$

where, n' : number of energy group corresponding to $2T$

n'' : number of energy group corresponding to $T+\Delta$.

The photon and electron fluxes at any T_N are determined by numerically integrating equations (7), (8) and (9), using the values of fluxes previously calculated for higher energies. The integration was started at T_0 but Y_e diverges at T_0 so we start our integration at $T_1/T_0=0.9999$.

The flow chart to calculate EFLUX, GFLUX, EFLU, GFLU, RAD, ENLOSS in the EGATL system is presented in Fig. 2.

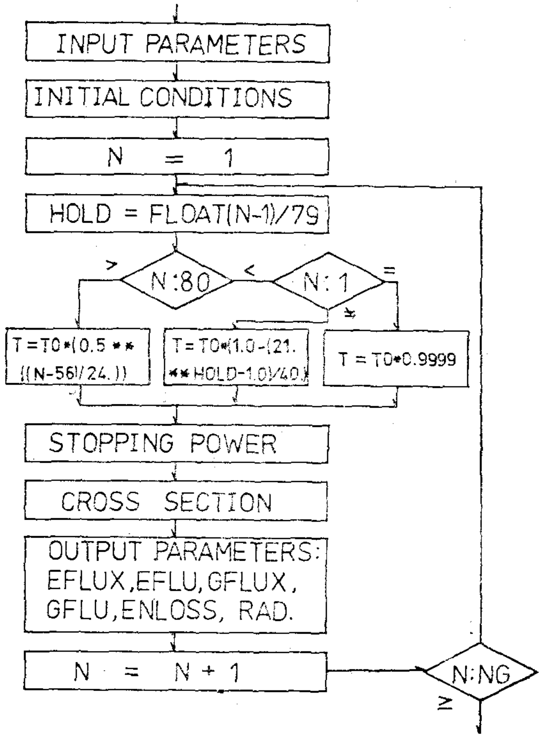

Fig. 2. Flow chart in EGATL system.

Where, GFLUX : differential gamma track lengths
EFLUX : differential electron track lengths
GFLU : integral gamma track lengths
EFLU : integral electron track lengths
RAD : integral radiation yield
ENLOSS : integral restricted energy loss.

IV. RESULTS AND DISCUSSION

Solutions of equation (7), (8) and (9) have been obtained by numerical integration. The results of our calculations summarized in Fig. 3 through 7.

Fig. 3 is a plot of differential track lengths vs. energy, with the primary energy $T_0=0.66$MeV. In the figure the differential track length of g/cm^2-MeV is presented as a function of the kinetic energy in MeV units. We notes that Y_T rises with decreasing T until a certain point when a rather rapid decrease sets in. The rise is due to the fact that successive Compton scatterings of a photon decrease its energy by decreasing amounts. Rather sharp drop at the low energy end is due to the absorption by photoelectric effect. The photon track lengths starts at

0.66 MeV and rises a peak at about 96KeV. The creation of photons by direct scattering from the source photons produce spectra that are discontinuous at an energy T such that $(mc^2/T)-(mc^2/T_0)=2$, since by virtue of the Compton relation because a primary photon cannot be degraded to a point below this energy. For this reason there is a discontinuity in Y_T at $T=(1/T_0+2/mc^2)^{-1}$. Once scattered photons produce the discontinuities observed at about 0.18 MeV in Fig. 3.

Figures 4 and 6 show the integral photon and electron track lengths in tissue equivalent material calculated using photonelectron transport code EGA-TL. Plotted curve is the total number of particles (photons) calculated in units of g/cm² versus kinetic energy, MeV. The curves are nearly constant at low energies.

Figure 5 illustrates the effect of different primary energies of ^{50}Co in the same medium. The curves all have the same shape except for energies close to the source energy. This indicates that the photon distribution in the medium is not affected by the

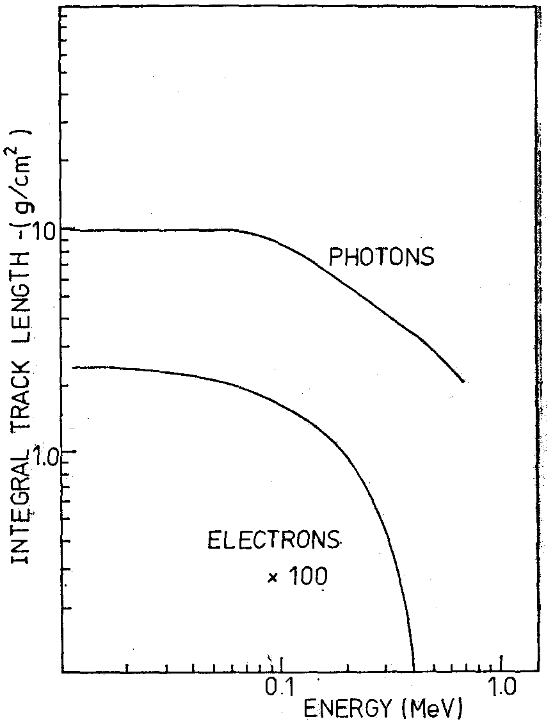

Fig. 4. Integral photon and electron track lengths for 0.66MeV photon source in medium.

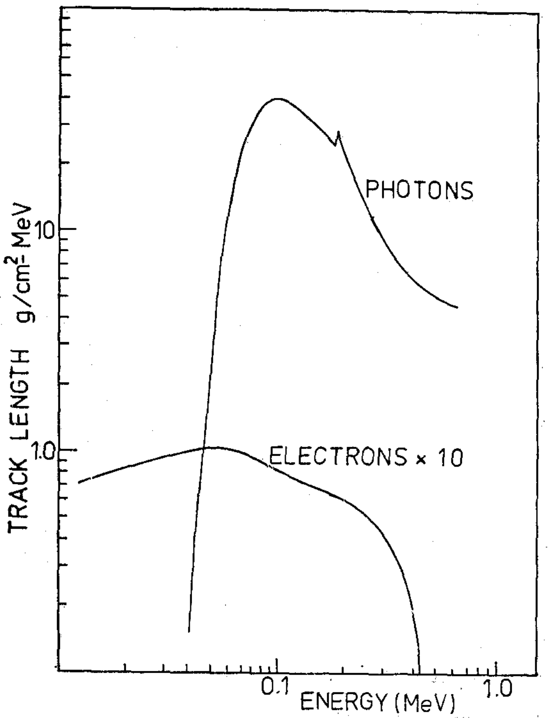

Fig. 3. Differential track length spectra for 0.66 MeV photons in tissue equivalent material.

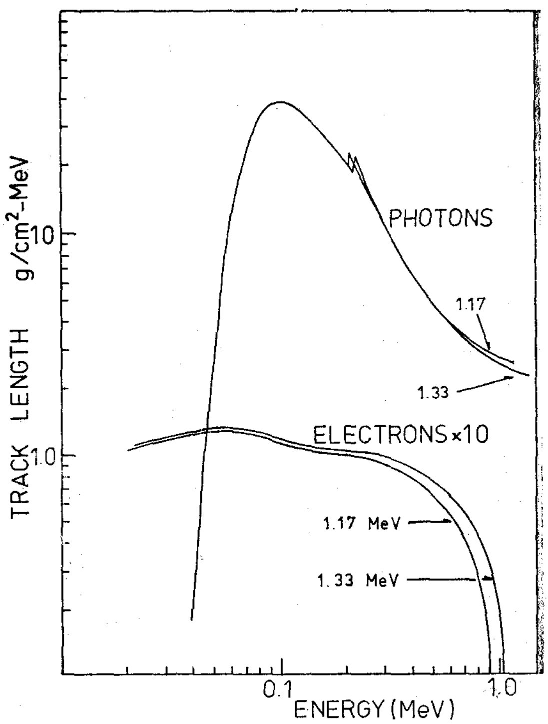

Fig. 5. Differential track length spectra for 1.17 and 1.33MeV photons.

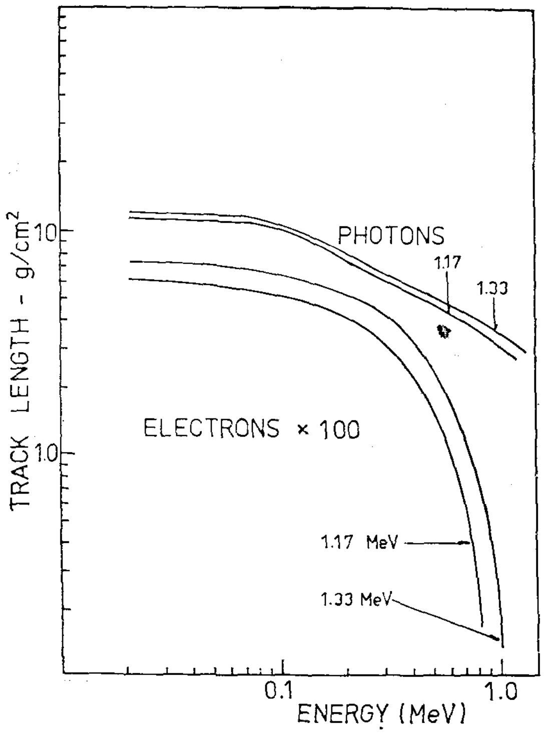

Fig. 6. Integral track lengths for 1.17 and 1.33 MeV photons in tissue equivalent material.

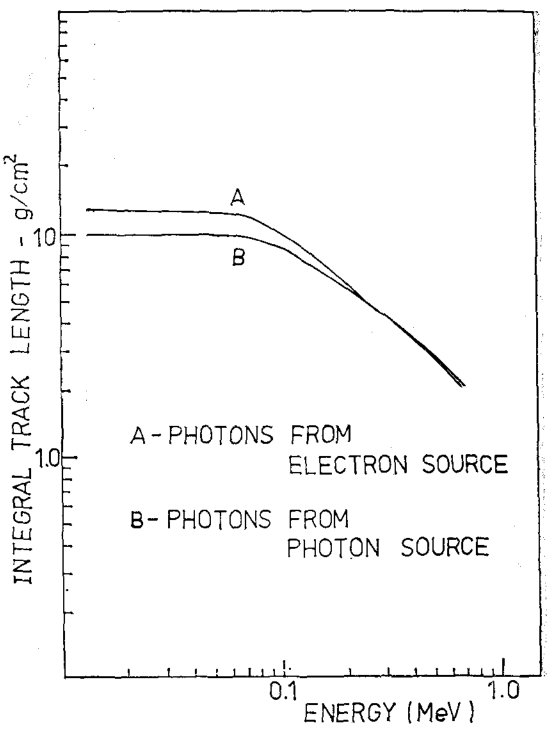

Fig. 7. Integral photon track lengths for 0.66MeV photon and electron source in same medium.

initial source energy. The discontinuities discussed above are observed at about 0.205 MeV and 0.215 MeV for 1.17MeV and 1.33MeV photons, respectively.

Finally in figure 7, the integral photon track lengths for a 0.66MeV photon source and for a 0.66 MeV electron are compared. We found that the total number of photons above given energy converge to about same ratio, independent of the type of source. In the figure, the curves indicate that electron sources give slightly more photons than photon sources.

The method presented in this report has many applications, including the determination of slowing down spectra in certain media with uniformly distributed sources, parametric studies of the relative importance of various physical effects on the transport process, estimates of radiation yields, and verification of more complex transport codes. The present calculations are much faster to carry out than those required for more general transport method and yet treat the physical processes more exactly and in more detail than that is often possible in more general methods.

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<論文>

무한평판 조직등가 물질에서 광자—전자결합 감속 에너지 스펙트럼의 계산

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요 지

Co-60과 Cs-137의 감마선원이 균일하게 분포된 ICRU의 표준 조직등가 물질에서 광자의 감속과 이 감속과정에서 생성된 전자의 감속을 결합시킨 에너지 분포를 비적길이의 함수로서 계산하였으며, 계산은 최적 전산코드의 입력으로서 최근의 핵단면적 데이터를 사용하였다.

본 논문에서는 이론적 계산방법을 상세히 기술하였으며 계산 결과는 그림으로 나타내었다. 그 결과, 미소 비적길이의 함수로서 정의되는 에너지 분포는 상이한 에너지의 감마선원에 대해 동일한 형태로 나타나며, 초기광자는 어느 에너지 이하로 감속되지 않기 때문에 $T=(1/T_0+2/m_0c^2)^{-1}$의 에너지에서 불연속이 나타난다.