

# INVERSE HALFTONING OF COLOR IMAGE USING KALMAN FILTER

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## ABSTRACT

In this paper, it proposes the technique to restore from a binary image in the color image. The color image is composed of three element images of red, green and blue. Therefore, the color image is first divided into a red, green, and blue element, and the Inverse Halftoning[2] ~ [4] is processed to each element images. Finally, each element images is collectively displayed. In that case, the Kalman filter was applied to the Inverse Halftoning for the restoration accuracy improvement of the image. As a result, it was possible to restore it in the color image as well as the time of a monochrome image. Moreover, the result that the restoration accuracy had improved even when which combining with the technique by using the Kalman filter for the Inverse Halftoning so far came out.

**Keywords:** Inverse Halftoning, Kalman filter

## 1. PREFACE

The Inverse Halftoning is processing that restores the image made binary by the half tone processing to former gray-scale image. Pixel value of the image that is to from 0 to 255 is displayed only by two colors of binary of 0 or 255 black or ,in a word, or white on the boundary of the threshold as the half tone processing mentioned here. And, when the image is made binary, it is an expression technique as mixing the noise, and doing the one that is not a continuous step, etc. using the mistake of eyes of man who shows it. The one that is called the dither method and a random dither as a past technique is enumerated in the half tone processing. And, because the noise is mixed when the half tone was processed, it is necessary to remove it though the Inverse Halftoning returns the image made binary thus to former step value. Even perfection has not lived yet as a past technique of the Inverse Halftoning though the restoration of the image of the level that is only these past techniques can expect though a smoothing filter, a median filter, and Gaussian filter, etc. are proposed. Because there is an evil such as growing dim of the edge part of the image to remove even the high frequency elements ,in a word, necessary information in the image other than the noise when it tries to obtain a smooth image by the Inverse Halftoning. Oppositely, the removal of the noise becomes insufficient when it tries to emphasize the edge part of the image, and the great result is not obtained so much as restoration accuracy of the image. The removal of the noise and the emphasis of the edge relate closely in the restoration of the image like this, and the design of

processing where it can be difficultly achieved is expected as for doing the two highly accurate at the same time.

Then, it proposes to apply the Kalman filter[5],[6] to the Inverse Halftoning in this paper. The effectiveness is shown in the field of the aerospace engineering and the control engineering as for the Kalman filter, and it is being applied to fields of the civil engineering and economics, etc. now. Additionally, it has aimed at the improvement of the restoration accuracy in various fields of the multimedia such as voice recognition[6] and GPS[7] under the noise environment because the Kalman filter is used because the effectiveness is reported.

Moreover, it experimented on the object that processed a Inverse Halftoning to the color image of the bit map in this thesis. A monochrome image becomes the image of 8bit that queues up the pixel in from 0 to 255 step. However, the color image becomes an image composed of red, blue, green of from 0 to 255 step, and total 24bit of 8bit respectively. Therefore, the obstacle verified whether to the restoration by the way of displaying it resolving the image of each element, giving of each the Inverse Halftoning, and bringing the end deflecting together in one.

## 2. KALMAN FILTER

### 2.1 Modeling of image

The next expression is used for the model of the image that applies the Kalman filter and the image after the filter by the technique in the past is applied.

$$X(t) = FX(t-1) + W(t-1) \quad (1)$$

Here, F is  $9 \times 9$  state transition procession, X(t) is a state variable, and W(t-1) is an internal noise. Moreover, t shows time. State variable X assumes  $X_i(i=0,1,2,\dots,8)$ , and becomes arrangement shown in Figure 1.

State variable  $X_i(t)$  ( $i=0,1,2,3,4$ ) should be assumed to be the one that pixel value at time of the previous state changed as it is, and  $X_i(t)$  ( $i=5,6,7$ ) be presumed by the linear sum of pixel value indicated in Table 1. This "Transition of the pixel by the transition at time" corresponds to "Movement of the pixel by the lusterware scanning".  $X_8(t)$  is used as presumption pixel value of the image that applies the Kalman filter. It is necessary to presume  $X_8(t)$  by the linear sum of an arbitrary pixel of  $X_i(i=0,1,2,\dots,8)$ .

Next, the next expression is used for the model of the observation image.

$$Y(t) = HX(t) + V(t) \quad (2)$$

Y is an observation image here, H is  $4 \times 9$  observation

procession, and  $V$  is an observation noise.  $HX(t)$  is as follows.

$$HX(t) = [X_5(t) \quad X_6(t) \quad X_7(t) \quad X_8(t)]^T \quad (3)$$

## 2.2 Algorithm of Kalman filter

The algorithm when the Kalman filter is applied from expression (1) and (2) to the system is as follows.

(1) Filter equation

$$\hat{X}(t+1/t) = F\hat{X}(t/t) \quad (4)$$

$$\hat{X}(t/t) = \hat{X}(t/t-1) + K(t)[Y(t) - H(t)\hat{X}(t/t-1)] \quad (5)$$

(2) Kalman gain

$$K(t) = P(t/t-1)H^T [HP(t/t-1)H^T + Q_v]^{-1} \quad (6)$$

(3) Presumption error margin covariance procession

$$P(t+1/t) = FP(t/t)F^T + Q_w \quad (7)$$

$$P(t/t) = P(t/t-1) + K(t)HP(t-1) \quad (8)$$

(4) Initial condition

$$\hat{X}(0/-1) = \hat{X}(0) \quad (9)$$

$$P(0/-1) = E \left\{ \left[ X(0) - \hat{X}(0/-1) \right] \left[ X(0) - \hat{X}(0/-1) \right]^T \right\} \quad (10)$$

Here, the Kalman gain,  $Q_v$ , and  $Q_w$  are the dispersion matrices of observation noise  $V(t)$  and internal noise  $W(t)$  in  $K(t)$  respectively.

$X_0$	$X_1$	$X_5$
$X_2$	$X_8$	$X_6$
$X_3$	$X_4$	$X_7$

Fig.1 Configuration of state variable

Table 1 Filtered pixel

Filtered pixel	Support
$X_5(t)$	$X_i(t-1)(i=1,5,6)$
$X_6(t)$	$X_i(t-1)(i=5,6,7)$
$X_7(t)$	$X_i(t-1)(i=4,6,7)$

## 2.3 Presumption of parameter

The method of presuming state transition procession  $F$ , dispersion matrix  $Q_v$ , and  $Q_w$  that is a necessary parameter when the above-mentioned algorithm is achieved is as follows.

It thinks about element  $F_{5i}(i=1,5,6)$  of  $F$  of the fifth line and five line five row element  $Q_{w55}$  of  $Q_w$  in the beginning. Because  $X_5(t)$  is presumed from Table 1 by the linear sum of  $X_1(t-1)$ ,  $X_5(t-1)$ , and  $X_6(t-1)$ , the following expressions are obtained.

$$X_5(t) = \begin{bmatrix} F_{51} & F_{55} & F_{56} \end{bmatrix} \begin{bmatrix} X_1(t-1) \\ X_5(t-1) \\ X_6(t-1) \end{bmatrix} + W_5(t) \quad (11)$$

$X_5(t-1)$  is multiplied by both sides.

$$\begin{aligned} X_5(t)X_5(t-l) &= \begin{bmatrix} F_{51} & F_{55} & F_{56} \end{bmatrix} \begin{bmatrix} X_1(t-1) \\ X_5(t-1) \\ X_6(t-1) \end{bmatrix} X_5(t-l) \\ &\quad + W_5(t)X_5(t-l) \end{aligned} \quad (12)$$

Next, the expected value is taken.

$$\begin{aligned} j_i(l) &= E[X_j(t)X_i(t-l)] \\ Q_{wji}(l) &= E[W_j(t)X_i(t-l)] \end{aligned} \quad (13)$$

It becomes the following expressions from expression (12) and (13).

$$55(l) = \begin{bmatrix} F_{51} & F_{55} & F_{56} \end{bmatrix} \begin{bmatrix} 15(l-1) \\ 55(l-1) \\ 65(l-1) \end{bmatrix} + Q_{w55}(l) \quad (14)$$

(1) is the following here.

$$l = \begin{cases} 1 & l = 0 \\ 0 & l \neq 0 \end{cases} \quad (15)$$

Moreover, if a similar operation is done putting  $X_1(t-1)$  and  $X_6(t-1)$  on both sides of expression (11), it becomes the following.

$$51(l) = \begin{bmatrix} F_{51} & F_{55} & F_{56} \end{bmatrix} \begin{bmatrix} 11(l-1) \\ 51(l-1) \\ 61(l-1) \end{bmatrix} \quad (16)$$

$$56(l) = \begin{bmatrix} F_{51} & F_{55} & F_{56} \end{bmatrix} \begin{bmatrix} 16(l-1) \\ 56(l-1) \\ 66(l-1) \end{bmatrix} \quad (17)$$

It is expression (16) as for  $l=0$  and 1, and if  $l=1$  is substituted for (17), it becomes the following in expression (15).

$$\begin{bmatrix} 55(0) & 51(1) & 55(1) & 65(1) \\ 51(1) & 11(0) & 51(0) & 61(0) \\ 55(1) & 15(0) & 55(0) & 65(0) \\ 56(1) & 16(0) & 56(0) & 66(0) \end{bmatrix} \times \begin{bmatrix} 1 \\ -F_{51} \\ -F_{55} \\ -F_{56} \end{bmatrix} = \begin{bmatrix} Q_{w55} \\ 0 \\ 0 \\ 0 \end{bmatrix} \quad (18)$$

$F_{5i}(i=1,5,6)$  and  $Q_{w55}$  are obtained from this expression. Each element of procession F and the corner element of  $Q_w$  can be requested in the same way.

Next, the method of presuming off-diagonal element  $Q_{wji}(j \neq i)$  of  $Q_w$  is described. If the expected value is taken putting  $X_i(t)$  on both sides of expression (11), it becomes the following about  $Q_{w5i}(i=6,7,8)$ .

$$Q_{w5i} = 5i(0) - \begin{bmatrix} F_{51} & F_{55} & F_{56} \end{bmatrix} \begin{bmatrix} 1i(L) \\ 5i(L) \\ 6i(L) \end{bmatrix} \quad (19)$$

However,  $i$  in this expression is 6, 7, and 8.  $Q_{wji}$  ( $j \in [6,8], i \in [5,8], j \neq i$ ) can be obtained by a similar technique. Only because it is that the pixel at time of the previous state changes to the pixel that corresponds more than Figure 1 as it is, it becomes  $Q_{wji}=0$  about  $Q_{wji}$  ( $j \in [5,8], i \in [0,4]$ ). Because statistic concerning the original picture image is an unknown, the value of  $\rho_{ji}(l)$  of expression (13) becomes an unknown. The following relational expressions consist when neither  $X_j(t)$  nor  $X_j(t-l)$  show the same pixel.

$$\begin{aligned} \rho_{ji}(l) &= E[X_j(t)X_i(t-l)] \\ &\cong E[Y_j(t)Y_j(t-l)] \end{aligned} \quad (20)$$

Assumption that former signal is irrelevant to the noise here etc. are used.

Moreover, the value of expression (20) becomes decentralization of the original picture image when  $X_j(t)$  and  $X_j(t-l)$  show the same pixel, and the method of the description as follows makes presumption possible. First of all, it is necessary to express the pixel count used to presume  $X_8(t)$  by nine, in a word, the linear sum of  $X_i(t-l)$  ( $i=0,1,\dots,8$ ) in expression (1). In general, there is a character with a high correlation between the adjoining pixels in the image. When internal noise  $W_8$  is distributed in expression (1), considering this character, and element  $Q_{w88}$  of procession  $Q_w$  of the eighth row of eight lines reaches a very small value in a word, it is assumable. Therefore, the presumption value of the decentralization of the original picture image is set for the value of  $Q_{w88}$  to become small.

Next, it is necessary to obtain  $Q_v$  of the observation noise, that is, decentralization  $\sigma_v^2$ . When the decentralization of pixel value of the observation image is assumed to be  $\sigma_v^2$  by  $\rho_{ii}(0)$ 's being presumed, this is obtained from the

following expressions.

$$\sigma_v^2 = \sigma_y^2 - \rho_{ii}(0) \quad (21)$$

Therefore, dispersion matrix  $Q_v$  of the observation noise is as follows.

$$Q_v = \begin{bmatrix} \sigma_v^2 & 0 & 0 & 0 \\ 0 & \sigma_v^2 & 0 & 0 \\ 0 & 0 & \sigma_v^2 & 0 \\ 0 & 0 & 0 & \sigma_v^2 \end{bmatrix} \quad (22)$$

### 3. COLOR INVERSE HALFTONING

It is composed of a red, green, of 8bit blue element image for the color image. Therefore, it restored it by processing the Inverse Halftoning of each element. Figure 2 shows the flow of the restoration processing.

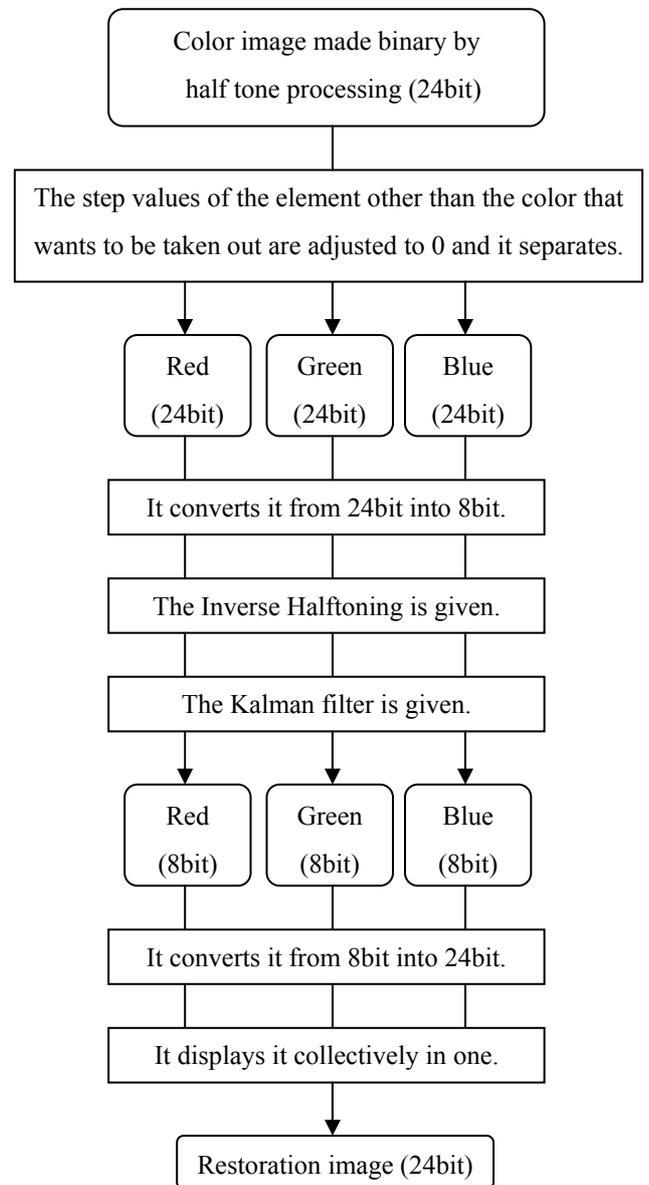


Fig.2 Flow of color Inverse Halftoning

## 4. RESULTS

Ideal reproduction image "Balloon" (256×256 pixels and 256 steps) used to experiment is shown in Figure 3, and the original picture image made binary by the random dither[1],[10] is shown in Figure 4. And, the Kalman filter was compared by using a smoothing filter, Gaussian filter, and a median filter for the experiment this time with the image that gave the Inverse Halftoning once putting it. Figure 5 is a result of giving the smoothing filter twice. Figure 6 is a result of giving the Gaussian filter once. Figure 7 is a result of giving the Smoothing filter once and Gaussian filter once. Figure 8 is a result of giving the Smoothing filter once and Median filter once. Figure 9 is a result of giving the Gaussian filter once and Median filter once. (a) of Figure from 5 to 9 results only the traditional Inverse Halftoning. (b) of Figure from 5 to 9 results processing the Kalman filter to (a) five times. The weight putting of a noteworthy pixel of the smoothing filter was assumed to be 1 in this paper. Moreover, the reciprocal of the extension of Gaussian filter was calculated by 0.52.



Fig.3

Ideal reproduction image



Fig.4

Error margin diffusion image



(a)



(b)

Fig.5 Smoothing filter(twice)



(a)



(b)

Fig.6 Gaussian filter



(a)



(b)

Fig.7 Smoothing filter and Gaussian filter



(a)



(b)

Fig.8 Smoothing filter and Median filter



(a)



(b)

Fig.9 Gaussian filter and Median filter

Moreover, PSNR was used as a method of evaluating the restoration accuracy of the image. The calculation of PSNR is requested by the following expressions.

$$PSNR = 10 \log \frac{N \times M \times T^2}{\sum_{x=0}^N \sum_{y=0}^M \{f(x,y) - f'(x,y)\}^2} \quad (23)$$

$f(x,y)$  is coordinates  $(x,y)$  step density of the ideal processing image.  $f'(x,y)$  is coordinates  $(x,y)$  step density of the processing image.  $N$  and  $M$  are each lengths of the image and pixel counts on side.  $T$  is the maximum step value of the image. PSNR is generally used as a technique to which the image objectively evaluates how to have been deteriorated before it compresses it and after it compresses it in the field of the image compression etc. The evaluation result approaches 0dB as infinity and deterioration become cruel when it is shown with dB, and there is no deterioration between images evaluating it two at all. It is said that it becomes easy to be noticeable of deterioration when falling below 30dB as a standard. PSNR is assumed that there is a difference that understands usually looking enough if differing by 0.2dB, and assumed that the distinction with the original picture is difficult by PSNR 40dB. Table 2 shows the restoration evaluation by PSNR.

## 5. CONCLUSION

The restoration of the level also that is which technique was able to be confirmed and to confirm made without trouble in the color in this thesis though the Inverse Half toning was processed to the color image. And, when the Kalman filter was put, the improvement of the restoration accuracy was able to be confirmed in any combination as shown in Table 2. Especially, accuracy has improved by Gaussian filter and as much as 2.5dB. However, because data is taken only by the image that gave the random dither this time, it is thought that there is a possibility that the combination with higher restoration accuracy is found by taking and comparing data in the image that gives other half tone processing. Moreover, it is thought that the possibility of the accuracy improvement rises in doing the Inverse Halftoning to which a reverse-random dither is not done this time. Moreover, the Inverse Halftoning was processed after it had converted it into 8bit once in this paper. However, if it is possible to process it like 24bit, the time of the calculation can be considerably saved. These are future tasks.

Table2: Image evaluation by PSNR

	The Kalman filter isn't used.	Five times of Kalman filter use.
Smoothing filter (twice)	34.241[dB]	34.476[dB]
Gaussian filter	30.060[dB]	32.564[dB]
Smoothing filter and Gaussian filter	34.393[dB]	34.447[dB]
Smoothing filter and Median filter	30.951[dB]	31.762[dB]
Gaussian filter and Median filter	33.420[dB]	33.873[dB]

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