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# Edge Detection By Fusion Using Local Information of Edges

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***Abstract*— This paper presents a robust algorithm for edge detection based on fuzzy fusion, using a novel *local edge information* measure based on Renyi's $a$-order entropy. The calculation of the proposed measure is carried out using a parametric classification scheme based on local statistics. By suitably tuning its parameters, the local edge information measure is capable of extracting different types of edges, while exhibiting high immunity to noise. The notions of fuzzy measures and the Choquet fuzzy integral are applied to combine the different sources of information obtained using the local edge information measure with different sets of parameters. The effectiveness and the robustness of the new method are demonstrated by applying our algorithm to various synthetic computer-generated and real-world images.**

## I. INTRODUCTION

Edge detection is a fundamental task in digital image processing and machine vision. An edge is defined as the boundary between two regions with relatively distinct gray-level properties. Most edge detection techniques are based on the computation of a local derivative operator. However, due to the presence of noise in most real-world images, it is usually difficult to have an accurate estimation of the gradient. Therefore, edge detection techniques based solely on gradient operators are usually inefficient in the presence of impulse noise in images.

Fuzzy sets theory [1] provides a flexible framework to cope with the ambiguity and vagueness often present in digital images. In addition, fuzzy sets theory offers the ability of incorporating expert knowledge and human intuition into digital image processing systems.

In this paper we present a robust edge detection scheme based on a novel measure of local information of edges and fuzzy fusion. The proposed method successfully retrieves true edges while at the same time eliminates the influence of noise.

## II. FUZZY MEASURES AND FUZZY INTEGRALS

### A. Fuzzy Measures

Let $X$ be an arbitrary set and $\Omega$ a sigma-algebra of subsets of $X$. A set function $g : \Omega \to [0,1]$ defined on $\Omega$ is a fuzzy measure if it satisfies the following conditions:

1) $g(\emptyset) = 0$, $g(X) = 1$ (Boundary conditions).
2) If $A, B \subset \Omega$ and $A \subset B$, then $g(A) \leqslant g(B)$ (Monotonicity).
3) If $F_n \in \Omega$ for $1 \leqslant n < \infty$ and the sequence $\{F_n\}$ is monotone (in the sense of inclusion), then $\lim_{n\to\infty} g(F_n) = g(\lim_{n\to\infty} F_n)$.

Sugeno [2] introduced the so called $\lambda$-fuzzy measure satisfying the following additional property:

$$g(A \cup B) = g(A) + g(B) + \lambda g(A)g(B) \tag{1}$$

for some $\lambda > -1$, where $A, B \subset X$, and $A \cap B = \emptyset$.

Let $X = \{x_1, x_s, \ldots, x_n\}$ be a finite set and let the fuzzy densities of the $\lambda$-fuzzy measure be defined as $g^i = g(\{x_i\})$. With boundary condition $g(X) = 1$, the value of $\lambda$ can be found by solving the following equation:

$$\lambda + 1 = \prod_{i=1}^{n} \left(1 + \lambda g^i\right). \tag{2}$$

It has been proved [3] that for a fixed set of $g^i$, $0 < g^i < 1$, there exists a unique root of $\lambda > -1$, and $\lambda \neq 0$ using (2).

### B. Choquet Integral

Let $(X, \Omega)$ be a measurable space and let $h : X \to [0,1]$ be an $\Omega$-measurable function. The Choquet integral [4] of the function $h$ with respect to a fuzzy measure $g$ is defined by:

$$\int_X h(x) \circ g(\cdot) = \int_0^{+\infty} g(A_a) da, \tag{3}$$

where $A_a = \{x \mid h(x) > a\}$.

If $X$ is a discrete set, the Choquet integral can be computed as follows:

$$e = \sum_{i=1}^{n} \left[h(x_i) - h(x_{i-1})\right] g_i^n, \tag{4}$$

where

$$h(x_1) \leqslant h(x_2) \leqslant \cdots \leqslant h(x_n), \tag{5}$$

with $h(x_0) = 0$, and

$$g_i^j = \begin{cases} g(\{x_i, x_{i+1}, \ldots, x_j\}), & i \leqslant j, \\ 0, & \text{otherwise.} \end{cases} \tag{6}$$

It should be mentioned that the Choquet integral reduces to the Lebesgue integral for probability measures. Choosing the appropriate fuzzy densities is very crucial for the application of the fuzzy integral to image processing problems.

## III. Proposed Method

### A. Local Edge Information Measure

Several information-theoretic measures have been developed and applied to image processing and pattern recognition problems. For the edge detection it is expected that edge points will carry more information than non-edge ones, since objects' contours are the primary sources of stimulation of the human vision.

In [5] Renyi defined the $a$-order entropy $H_a$ of a probability distribution $(p_1, p_2, \ldots, p_k)$ as follows:

$$H_a = \frac{1}{1-a} \ln \left( \sum_{n=1}^{k} p_n^a \right), \quad (7)$$

where $a$, with $a \neq 1$, is a positive real parameter. It should be mentioned that the $a$-order entropy is a one-parameter generalization of the Shannon's entropy $H_S$, since $\lim_{a \to 1} H_a = H_S$.

In order to calculate the local edge information measure for the $(i,j)$-th pixel, we use a sliding $w \times w$ window centered at the $(i,j)$-th pixel. We consider the sub-image at each window position as an independent information source with $k$ $(\neq 1)$ possible symbols $s_1, s_2, \ldots, s_k$, with $s_1 = 0$ and $s_k = L-1$, where $L$ is the number of gray levels in the image. The distance between two sequential symbols, i.e. $s_t$ and its preceding symbol $s_{t-1}$, is $s_t - s_{t-1} = \frac{L-1}{k-1}$. The pixels inside the window $W$ are assigned to their nearest source symbol $s_n$ according to their gray levels, using the following rule:

$$\mathfrak{L}_n = \{ g_{ij} | d(g_{ij}, s_n) < d(g_{ij}, s_m), \text{ for each } s_{m \neq n} \} \quad (8)$$

with $m, n = 1, ..., k$. $d(\cdot)$ is a distance measure, $g_{ij} \in W$ is the gray level of the $(i,j)$-th pixel and $\mathfrak{L}_n$ is the set of pixels inside the window $W$ classified as the $n$-th symbol $s_n$ of the source. Let us denote by $\mathfrak{L}$ the set of pixels belonging to window $W$. It is evident that:

$$\mathfrak{L} = \bigcup_{n=1}^{k} \mathfrak{L}_n \quad \text{and} \quad \mathfrak{L}_n \cap \mathfrak{L}_{m \neq n} = \emptyset \quad (9)$$

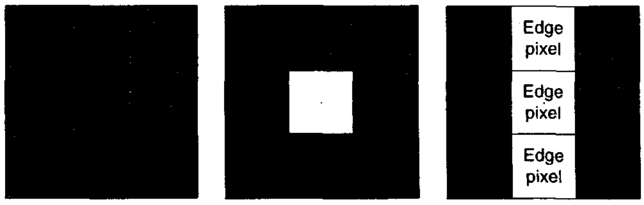
The estimated probability $p_{s_n}$ of the $n$-th symbol $s_n$ of the source, computed as frequency of occurrence, is given by:

$$p_{s_n} = \frac{\|\mathfrak{L}_n\|}{\|\mathfrak{L}\|} = \frac{\|\mathfrak{L}_n\|}{w^2} \quad (10)$$

where $\| \cdot \|$ is the cardinality of a set. The *local edge information* measure for the $(i,j)$-th pixel is defined as:

$$H_a^k(i,j;w) = \frac{1}{(1-a)\ln k} \ln \left( \sum_{n=1}^{k} p_{s_n}^a \right). \quad (11)$$

If a pixel's gray level is equidistant from two symbols, it is assigned to the symbol with the smallest index. The local edge information measure depends directly on the number $k$ of possible symbols. For the problem of edge detection we consider as an intuitive lower bound of $k$ the number of three symbols (classes). This is justified if we consider the case of the window being centered at an edge. Then three classes are involved, one corresponding to the edge itself and two more corresponding to the regions to both sides of the edge. As the number of classes increases, the measure becomes in general more sensitive to secondary edges of the image, but at the same time to the presence of noise. Therefore, there is a tradeoff between detailed edge extraction and noise suppression. Since (11) is an entropy measure that reaches its maximum value when all symbols are equiprobable, the number of classes cannot exceed $w^2$. This is a mathematical constraint that practically can be overridden in order to control the sensitivity of the measure. The main difference between the entropic local edge information measure and typical edge operators, i.e. Sobel or Prewitt that are just different approximations of the derivative at a pixel location, is that the proposed measure reflects the ability of pixels in a region to be classified into different classes.




(a) (b) (c)

Fig. 1. Sliding window centered at (a) smooth region, (b) noisy pixel and (c) edge.

Equation (11) has some interesting properties. Consider the three possible situations shown in Fig. 1, where the number of symbols is set to $k = 3$. When the window is centered at a pixel belonging to a region of constant intensity, the local edge information measure assigned to the center pixel is zero regardless of the value of parameter $a$. This is due to the fact that all symbols' probabilities are zero except for the one that corresponds to the class of the constant intensity level which has probability one. In the case when the center pixel is an impulse noise pixel, then its gray level will be significantly different from its neighbors. This means that the symbols' probabilities involved will be close to one and zero, thus assigning a small value to the local edge information measure. When the center pixel is located at a true edge which divides two smooth regions, it is evident that the three symbols' probabilities are equal, which results in the maximization of the measure. The dependence of the local edge information measure to the number $k$ of possible symbols can be clearly seen in Fig. 2. As the number of symbols increases new types of edges emerge, due to the different ways the pixels are classified to their nearest source symbols.

Using Renyi's entropy instead of the one proposed by Shannon, has the advantage that by varying the parameter $a$, we can control the behavior of entropy to different probability


Fig. 2. (a) Initial image. Edges detected using the local edge information measure with (b) $k = 3$ and (c) $k = 9$ possible symbols. Parameter $a$ was fixed to $a = 20$ and the window size was set to $3 \times 3$ pixels.

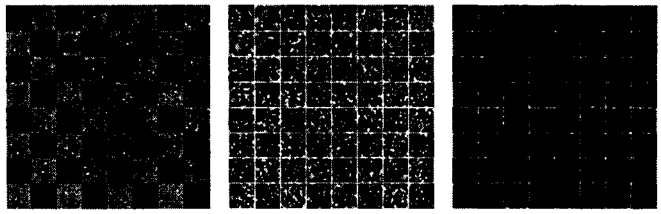



Fig. 3. (a) Initial computer-generated image contaminated with "salt & pepper" noise with density $d = 0.03$. Edges detected using the local edge information measure with (b) $a = 0.2$ and (c) $a = 20$. The number $k$ of possible symbols was fixed to $k = 3$ and the window size was set to $3 \times 3$ pixels.

values. When $a \rightarrow 0$, small probability values have large influence to the entropy value, thus making the local edge information measure more sensitive to noise. As $a$ increases, the influence of small probability values decreases, making the measure immune to noise while producing thinner edges. The behavior of the local edge information measure to changes of the parameter $a$ is illustrated in Fig. 3.

### *B. Edge Detection by Fusion*

Fuzzy integrals have been widely used for classifier fusion in many applications [6]. In [7] and [3] fuzzy integrals have been applied for image processing and computer vision tasks. Fuzzy integrals are generalized mean operators ranging between min and max. Their main characteristic is that fuzzy integrals are weighted operators whose weights are defined not only on different attributes, but also on all the subsets. This allows the representation of importance and interaction between attributes [8]. Moreover, it is believed that fuzzy integrals are the only operators at present which can model this type of interaction [9].

By varying the set of parameters of the local edge information measure, that is the number of possible symbols $k$, the parameter $a$ of Renyi's entropy and the size $w$ of the sliding window, different edge maps can be obtained that contain different types of edges, while at the same time the influence of noise in the produced edge map can be controlled. Since suppression of noise is a desired property for every edge detector, we select large values for the parameter $a$, a choice that is justified by the analysis performed in Section III-A.

In the first stage of the proposed algorithm, a set of edge images is obtained using different sets of parameters for the calculation of the local edge information measure. The parameters are selected in such a way that the resulting edge maps to contain different types of edges, i.e. strong and/or weak edges. It should be mentioned that according to the properties of the local edge information measure low values of $k$ extract primary edges, while for large values of $k$ both strong and weak edges can be extracted.

In order to calculate the fused edge map from the set of edge images, the only information required is the selection of the fuzzy densities which stand for the degree of importance assigned to each of the images of the set. The higher the value of the fuzzy density assigned to an image, the more important the influence of this image is to the final fused edge map. The fusion of the data sources obtained after applying the local edge information measure with different set of parameters can be implicitly expressed as:

$$e(i,j) = \mathcal{F}[H_{a_1}^{k_1}(i,j;w_1), H_{a_2}^{k_2}(i,j;w_2), \ldots, H_{a_n}^{k_n}(i,j;w_m)], \quad (12)$$

where $e(i,j)$ is the gray level of the $(i,j)$-th pixel of the fused edge image, $\mathcal{F}$ is a fusion operator, and $H_{a_l}^{k_l}(i,j;w_l)$, $1 \leqslant l \leqslant m$, are the gray levels of the $(i,j)$-th pixels of the images derived by the local edge information measure with parameters $k_l$, $a_l$, and $w_l$. Due to the definition (11) of the local edge information measure the $H_{a_l}^{k_l}(i,j;w_l)$, for $1 \leqslant l \leqslant m$, is in the range $[0,1]$.

Applying the discrete Choquet fuzzy integral as the fusion operator, the $(i,j)$-th pixel of the fused edge image $e(i,j)$ is given by:

$$e(i,j) = \sum_{l=1}^{m} \left(H_{a_l}^{k_l}(i,j;w_l) - H_{a_{l-1}}^{k_{l-1}}(i,j;w_{l-1})\right) g(A_m), \quad (13)$$

where $H_{a_0}^{k_0}(i,j;w_0) = 0$.

### *C. Outline of the Algorithm*

1) Given an image $f$.
2) Given the parameters $a_1$, $k_1$, and $w_1$, compute the local edge information image $H_{a_1}^{k_1}$.
3) Repeat step (2) with different sets of parameters in order to obtain the images $H_{a_2}^{k_2}, H_{a_3}^{k_3}, \ldots, H_{a_m}^{k_m}$.
4) Given fuzzy densities, compute the fused edge image $e$ by applying the fusion operator to the images $H_{a_1}^{k_1}, H_{a_2}^{k_2}, \ldots, H_{a_m}^{k_m}$, as described by (13).

## IV. Experimental Results

The proposed method has been tested using various synthetic computer-generated and real-world images with and without or presence of noise. For the fusion stage of our algorithm we have considered three different information sources obtained using three different sets of parameters for the calculation of the local edge information measure. For the simulation we have used gray-scale images of size $256 \times 256$ pixels with 8 bits-per-pixel gray-tone resolution.

Fig. 4 demonstrates the performance of the proposed fusion scheme based on the local edge information measure. Figs.

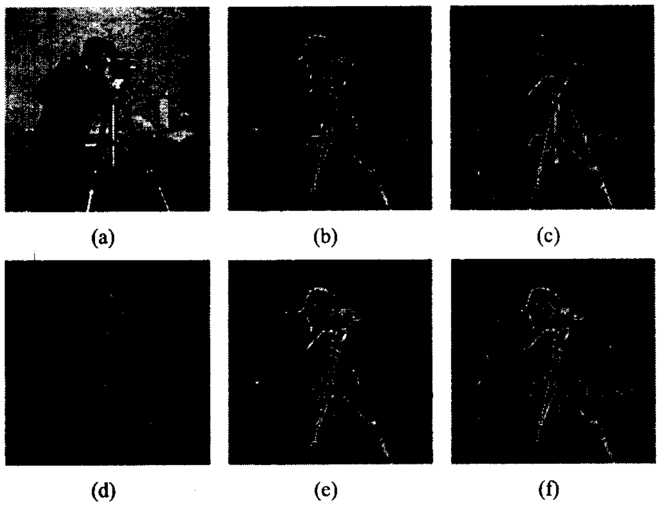


Fig. 4. (a) Initial image. Edges detected using the local edge information measure with (b) $k = 3$, (c) $k = 5$ and (d) $k = 9$ possible symbols. Fused images using fuzzy densities (e) $g^1 = 0.9$, $g^2 = 0.5$, $g^3 = 0.3$ and (f) $g^1 = 0.8$, $g^2 = 0.3$, $g^3 = 0.9$. Parameter $a$ was fixed to $a = 20$ and the window size was set to $3 \times 3$ pixels.

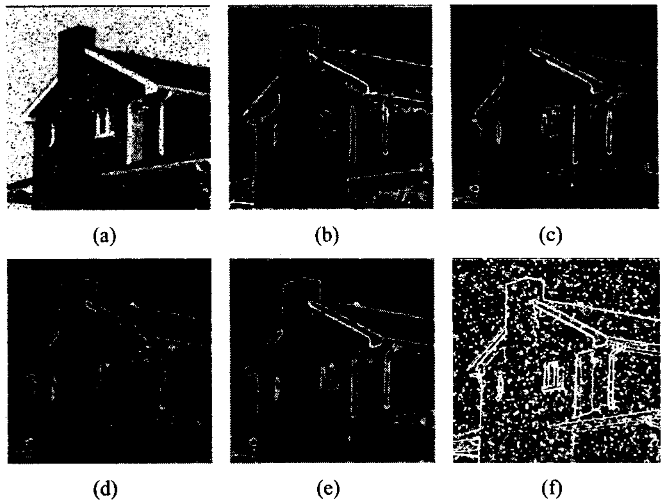


Fig. 5. (a) Initial image contaminated with "salt & pepper noise" with density $d = 0.05$. Edges detected using the local edge information measure with (b) $k = 3$, (c) $k = 4$ and (d) $k = 6$ possible symbols. (e) Fused image using fuzzy densities $g^1 = 0.5$, $g^2 = 0.6$, $g^3 = 0.9$. (f) Edge image obtained using the Sobel operator. Parameter $a$ was fixed to $a = 20$ and the window size was set to $5 \times 5$ pixels.

4(b)-4(d) show the intermediate images derived using the local edge information measure with $k = 3, 5$ and 9 possible symbols respectively. The parameter $a$ of Renyi's entropy was fixed to $a = 20$ and the window size was set to $3 \times 3$ pixels. Two different sets of fuzzy densities were used in order to extract different types of edges from the initial image. Fig. 4(e) illustrates the result of the fusion of the intermediate images with fuzzy densities $g^1 = 0.9$, $g^2 = 0.5$, and $g^3 = 0.3$. The fuzzy densities were chosen in such a way that the fused image to highlight primary edge but at the same time to contain a small portion of edges of other types. In Fig. 4(f) the fuzzy densities were set to $g^1 = 0.8$, $g^2 = 0.3$, and $g^3 = 0.9$, in order to preserve both the primary edges contained in the image of Fig. 4(b) but at the same time we considered as more important the edges contained in the image of Fig. 4(d). One observes that the proposed method successfully extracts different types of edges. Finally, in order to demonstrate the robustness of the presented method to the presence of noise, we applied the proposed method the the noisy image of Fig. 5(a). The image was contaminated with "salt & pepper" noise with density $d = 0.05$. Figs. 5(b)-5(d) illustrate the images derived after applying the local edge information measure. In order to suppress the noise, we have selected a window size of $5 \times 5$ pixels and we have set the parameter $a$ to $a = 20$. The number $k$ of possible symbols was set to $k = 3, 4$, and 6 respectively for the three intermediate images. Fig. 5(e) is the fused image using fuzzy densities $g^1 = 0.5$, $g^2 = 0.6$, and $g^3 = 0.9$. The result after applying the Sobel operator to the image of Fig. 5(a) is shown in Fig. 5(f). From the comparison of the images obtain using the proposed approach and the Sobel edge detector, one observes that the presented algorithm exhibits high immunity to noise while preserving the edges of the image. On the contrary, the image derived by the Sobel operator contains a large amount of noise.

## V. Conclusion

In this paper we present a robust algorithm for edge detection based on fuzzy fusion. Moreover, we introduce a novel descriptor of edges using the local edge information measure based on Renyi's $a$-order entropy. The proposed method is parametric and can be suitably tuned to extract different types of edges from an image, according to the parameters of the local edge information measure and the selected fuzzy densities for the fusion stage. Furthermore, by adjusting the parameters the proposed method exhibits robustness to noise.

Our future works involve an automated selection of the fuzzy densities directly from the input data. Finally, we are going to extend the notion of the local edge information measure in the context of image denoising and contrast enhancement.

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