Proceedings of the 11th
KACC, October 1996

Model Predictive Control Combined with Iterative Learning Control for Nonlinear Batch Processes

Kwang Soon Lee, Won Cheol Kim, and Jay H. Lee*

Dept. of Chem. Eng., Sogang Univ., Shinsoo-1, Mapogu, Seoul 121-742, Korea
Tel: 82-2-705-8477, Fax: 82-2-3272-0319, E-mail: kslee@ccs.sogang.ac.kr
* Dept. of Chem. Eng., Auburn Univ., Auburn, AL, USA
Tel: 1-334-844-2060, Fax: 1-334-844-2063, E-mail: jhl@eng.auburn.edu

Abstract A control algorithm is proposed for nonlinear multi-input multi-output(MIMO) batch processes by combining quadratic iterative learning contol(Q-ILC) with model predictive control(MPC). Both controls are designed based on output feedback and Kalman fiter is incorporated for state estimation. Novelty of the proposed algorithm lies in the facts that, unlike feedback-only control, unknown sustained disturbances which are repeated over batches can be completely rejected and asymptotically perfect tracking is possible for zero random disturbance case even with uncertain process model.

Keywords: iterative learning control, model predictive control, batch process control, repetitive control

1. Introduction

From the viewpoint of control system design, batch processes present a unique challenge seldom found in continuous processes. In a batch process, the control problem is usually given as a tracking problem for time-varying reference trajectories defined over a finite time interval. During the course of a typical batch, process variables swing over wide ranges, and as a consequence, nonlinearity of the process is an important factor to consider in controller design. In addition, batch processes have a unique disturbance pattern. In many batch processes, disturbances are originated from the initial conditions such as feed quality. They usually last for a number of batches with the same magnitudes.

Traditionally, batch process control has relied on feedback techniques only just as in continuous process control. Feedback control is employed not only to reject disturbances, but also to find the time-varying input trajectories that correspond to the assigned output trajectories. However, feedback control has limitations in finding the input trajectories for processes with nonlinearity (or model error) and/or nonminimum-phase dynamics, and as a consequence, results in the same type of control error even for repeated disturbances. One way to overcome such limitations is to superimpose a feedforward bias signal on the feedback control signal. The feedforward bias signal should be chosen to correspond to the output reference trajectories under the nominal condition. Actually, such a feedforward signal is also necessary for designing a model-based controller using linear techniques. This is because the linear systems theory is applicable only in terms of deviations from the nominal trajectories.

The input trajectories can be calculated by inverting the process map if a process model valid over the entire operating regime is available. However, such a nonlinear map is very difficult to construct for industrial processes. This motivates us to find an alternative to the nonlinear model inversion approach. One aspect of batch operation often left unexplored is that it is repetitive. The so-called iterative learning control(ILC)[1,2] makes use of this aspect and enables us to progressively refine the input trajectories while perfectly rejecting the disturbances which are repeated over batches. The exisitng ILC techniques, however, shares a definitive flaw that they are very sensitive to output noise/disturbances and can induce excessive control actions. These are often unacceptable traits in process control applications. In addition, they are not well suited to general nonsquare MIMO processes and cannot resolve the input constraints in an optimal manner. To overcome these problems, Lee and Lee[4] have developed the so-called Q-ILC (ILC based on a quadratic criterion) algorithms for time-varying MIMO linear stochatic as well as deterministic processes with and without constraints. They have shown through rigorous proofs that all the above mentioned problems can be overcome with the proposed Q-ILC algorithms without sacrificing the desired properties of ILC.

Based on the above considerations, the objective of the present study is placed to propose a new paradigm for model-based control of batch and other transient processes, where both precise tracking and disturbance rejection are important, through combination of MPC for rejection of random disturbance and Q-ILC for rejection of repeated disturbances as well as sequential update of input trajectories. In the proposed control algorithms, the stochatic formulations of Q-ILC in ref. [4] is combined with a QDMC-type algorithm based on the state space model[3]. One of the key features of the proposed algorithm is that the so-called static gain model is used to represent the linearized model of a nonlinear batch process. The model can represent the time-varying as well as time-invariant linear dynamic systems as a linear algebraic model, and as a consequence, greatly simplify the mathematical development.

2. Process Description and Error Dynamic Model

We consider the following general MIMO nonlinear input-output model defined over N discrete-time indices, $T = \{1, \ldots N\}$.

$$y(t) = \mathsf{N}(u(t-1), \ldots, u(1), d(t-1), \ldots, d(1), x_I) + w(t) \quad (1)$$

where u, y, d and x_I denote the n_u−input, n_y−output, n_d−disturbance vectors and initial conditions, respectively: w stands for additive zero-mean random disturbance. We assume the nonlinear function N be analytic *i.e.*, defined and differentiable in a domain where the independent variables are defined.

Since the assiociated time domain has a finite time indicies, it is possible to represent the process as the following nonlinear algebraic system. For the k^{th} batch,

$$Y_k = \bar{Y}_k + W_k = \mathcal{N}(U_k, D_k, x_{I,k}) + W_k \quad (2)$$

$$\text{where} \qquad Y^T = \left[y^T(1)\ y^T(2) \ldots y^T(N)\right] \quad (3)$$

and $U, \bar{Y}, D$, and W are defined similarly.

The transition of the disturbance-free output from the k^{th} to $k+1^{th}$ batch can be represented by the Taylor series expansion.

$$\begin{aligned} \bar{Y}_{k+1} &= \mathcal{N}(U_{k+1}, D_{k+1}, x_{I,k+1}) = \bar{Y}_k + \mathbf{G}_k \Delta U_{k+1} \\ &+ \mathbf{G}_k^D \Delta D_{k+1} + \mathbf{G}_k^x \Delta x_{I,k+1} + \ldots \end{aligned} \quad (4)$$

$$\text{where} \qquad \Delta U_{k+1} = U_{k+1} - U_k \quad (5)$$

and ΔD_{k+1} and $\Delta x_{I,k+1}$ are defined similarly. Let Y_d denote the desired output sequence and define

$$\begin{aligned} \bar{E}_k &= Y_d - \bar{Y}_k \\ E_k &= Y_d - Y_k \\ V_{k+1} &= \mathbf{G}_k^D \Delta D_{k+1} + \mathbf{G}_k^x \Delta x_{I,k+1} + \ldots \end{aligned} \quad (6)$$

then we obtain a linearized state space model that describes the transition of tracking error between two successive batches.

$$\begin{aligned} \bar{E}_{k+1} &= \bar{E}_k - \mathbf{G}_k \Delta U_{k+1} + V_{k+1} \\ E_k &= \bar{E}_k + W_k \end{aligned} \quad (7)$$

Since zero-mean output disturbance vectors from two different batches can be reasonably assumed to be independent no matter how the disturbance is correlated in a batch sequence, the covariance of W_k is defined by

$$\text{cov}\left\{W_k W_j^T\right\} = \mathbf{R}^W \sigma(k-j) \quad (8)$$

Also we assume V_k can be modelled as an independent white noise vector with the covarince of

$$\text{cov}\left\{V_k V_j^T\right\} = \mathbf{R}^V \sigma(k-j) \quad (9)$$

The so-called Q-ILC algorithm (stochastic version) has been derived for the above stochastic model of (7).

Remark 1: $\mathbf{G}_k$ will have a lower block triangular structure due to causality. Time-varying as well as time-invariant linear processes can be represented by this model structure. The model of (7) is observable but may not be reachable due to time delays. $\mathbf{G}_k$ is assumed to be available after each batch by some means, *e.g.*, through identification.

Remark 2: If higher order expansion is negligible and $\mathbf{x}_I$ and $\mathbf{D}$ are same over batches, $\mathbf{V} = \mathbf{0}$, which implies they are perfectly cancelled.

The model for MPC is defined in terms of deviation variables and represents the dynamics along discrete-time indices. Deviation variables for the $k+1^{th}$ batch are defined around the trajectories from the k^{th} batch as follows:

$$\begin{aligned} \mathbf{y}_{k+1}(t) &= y_{k+1}(t) - y_k(t) \\ \mathbf{u}_{k+1}(t) &= u_{k+1}(t) - u_k(t) \end{aligned} \quad (10)$$

For the $k+1^{th}$ batch, it is straightforwardly derived from (7) or (2) that the noise-free output is represented by

$$\bar{\mathbf{y}}(t) = \mathbf{g}(t, t-n)\mathbf{u}(t-n) + \ldots + \mathbf{g}(t, t-1)\mathbf{u}(t-1) \quad (11)$$

where $\mathbf{g}(i,j)$ denotes the $(i,j)^{th}$ element of $\mathbf{G}$ and n is the number of significant FIR terms. Using the technique in Lee and Morari(1994), this model can be converted to a state space form as follows:

$$\begin{aligned} \mathbf{x}(t+1) &= \mathcal{M}\mathbf{x}(t) + \mathcal{S}(t)\delta\mathbf{u}(t) \\ \mathbf{y}(t) &= \mathcal{N}\mathbf{x}(t) + \mathbf{n}(t) \end{aligned} \quad (12)$$

where

$$\begin{aligned} \mathbf{x}^T(t) &= \left[\ \bar{\mathbf{y}}^T(t) \ \ldots \ \bar{\mathbf{y}}^T(t+n-1)\ \right] \\ \delta\mathbf{u}(t) &= \mathbf{u}(t) - \mathbf{u}(t-1) \\ \mathcal{M} &= \begin{bmatrix} \mathbf{0} & \mathbf{I} & \mathbf{0} & \ldots & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{I} & \ldots & \mathbf{0} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \ldots & \mathbf{I} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \ldots & \mathbf{I} \end{bmatrix} \\ \mathcal{S}(t) &= \begin{bmatrix} \mathbf{g}(t+1,t) \\ \mathbf{g}(t+2,t) + \mathbf{g}(t+2,t+1) \\ \vdots \\ \mathbf{g}(t+n,t) + \ldots + \mathbf{g}(t+n,t+n-1) \end{bmatrix} \\ \mathcal{N} &= \left[\ \mathbf{I} \ \ \mathbf{0} \ \ldots \ \mathbf{0}\ \right] \end{aligned} \quad (13)$$

and n is the output disturnbance composed of v and w. A standard QDMC can be derived for the above model[3].

3. Q-ILC Algorithms

The Q-ILC algorithm (Lee and Lee, 1996) is derived by solving the following stochastic minimization after each batch

$$\min_{\Delta U_{k+1}} J_{k+1} = \frac{1}{2} E\left\{\bar{E}_{k+1}^T \mathbf{Q} \bar{E}_{k+1} + \Delta U_{k+1}^T \mathbf{R} \Delta U_{k+1} \,|\mathcal{I}_k\right\} \quad (14)$$

with or without constraints as a way to attain the following ILC objective extended to general nonsquare MIMO systems.

> *Given the model of form (7), design a learning algorithm with the property that*
>
> $$E\{\|\bar{\mathsf{E}}_k\|\} = \min_{\mathsf{U}} E\{\|\bar{\mathsf{E}}\|\} \quad \text{as } k \to \infty \qquad (15)$$
>
> *under an appropriate norm definition.*

In (14), $\mathcal{I}_k$ represents the information available at the end of the k^{th} batch (e.g., output tracking error during the k^{th} batch). When there is no constraints, the problem is linear quadratic and the separation principle holds. The solution is easily obtained as a least squares solution combined with Kalman filtering. When there is constraints, however, the problem becomes nonlinear and formidable to solve. By assuming the separation principle, the problem becomes a standard quadratic programming(QP) and a suboptimal solution can be obtained.

For both cases, the following Kalman filter is used for optimum estimation of $\bar{\mathsf{E}}_k$.

$$\begin{aligned} \hat{\mathsf{E}}_{k|k} &= \hat{\mathsf{E}}_{k|k-1} + \mathbf{K}_k(\mathsf{E}_k - \hat{\mathsf{E}}_{k|k-1}) \\ \hat{\mathsf{E}}_{k+1|k} &= \hat{\mathsf{E}}_{k|k} - \mathbf{G}_k \Delta \mathsf{U}_{k+1} \\ \mathbf{K}_k &= \mathbf{P}_k \left(\mathbf{P}_k + \mathbf{R}^W\right)^{-1} \\ \mathbf{P}_{k+1} &= \mathbf{P}_k - \mathbf{P}_k \left(\mathbf{P}_k + \mathbf{R}^W\right)^{-1} \mathbf{P}_k + \mathbf{R}^V \end{aligned} \qquad (16)$$

Now the Q-ILC algorithms are summarized as follows. To explicitly signify the learning control signal, we use the superscript ^{Q}.

Unconstrained Q-ILC Algorithm

$$\Delta \mathsf{U}_{k+1}^Q = \mathsf{U}_{k+1}^Q - \mathsf{U}_k = \mathbf{H}_k \hat{\mathsf{E}}_{k|k} \qquad (17)$$

$$\text{where} \qquad \mathbf{H}_k = \left(\mathbf{G}_k^T \mathbf{Q} \mathbf{G}_k + \mathbf{R}\right)^{-1} \mathbf{G}_k^T \mathbf{Q} \qquad (18)$$

Constrained Q-ILC Algorithm

Constraints may be imposed on the input, input change (in terms of time), and the output. As far as the constraints are given by linear inequalities, they can always be reagrranged as follows.

$$\mathcal{C}_{k+1}^u \Delta \mathsf{U}_{k+1}^Q \geq \mathcal{C}_{k+1} \qquad (19)$$

Under the certainty equivalence assumption and from the Kalman filter equation, the objective (14) can be reduced to

$$\begin{aligned} \min_{\Delta \mathsf{U}_{k+1}} \bar{J}_{k+1} = \frac{1}{2} \{ & \Delta \mathsf{U}_{k+1}^T \left(\mathbf{G}_k^T \mathbf{Q} \mathbf{G}_k + \mathbf{R}\right) \Delta \mathsf{U}_{k+1} \\ & -2 \Delta \mathsf{U}_{k+1}^T \mathbf{G}_k^T \mathbf{Q} \hat{\mathsf{E}}_{k|k} \} \end{aligned} \qquad (20)$$

The above minimization subject to the constraints in (19) formulates a standard QP problem.

4. Combination of MPC

The MPC criterion can be written as

$$\min_{\mathcal{U}^M(t)} \left\{ \|\mathcal{R}(t+1|t) - \mathcal{Y}(t+1|t)\|_{\Gamma(t)}^2 + \left\|\delta \mathcal{U}^M(t)\right\|_{\Lambda(t)}^2 \right\} \qquad (21)$$

where $\mathcal{R}(t+1|t)$ and $\mathcal{Y}(t+1|t)$ denote sequences of reference trajectories and output predictions over the prediction horizon based on $\mathcal{I}_k$, respectively; $\delta\mathcal{U}^M(t)$ stands for the sequence of future control input changes (with respect to time) by MPC.

Recall that the learning control signal is added to the MPC signal through the feedforward path. Therefore, $\mathbf{u} = \mathbf{u}^Q + \mathbf{u}^M$ where $\mathbf{u}^M(t)$ and $\mathbf{u}^Q(t)$ be the MPC and Q-ILC signals (as deviation variables) at t, respectively. Based on this consideration, optimal prediction $\mathcal{Y}(t+1|t)$ can be described by for the case that the prediction horizon is equal to the control horizon

$$\mathcal{Y}(t+1|t) = \mathcal{M}^p \hat{\mathbf{x}}(t|t) + \mathcal{S}^p(t)\left(\delta\mathcal{U}^M(t) + \delta\mathcal{U}^Q(t)\right) \qquad (22)$$

where $\mathcal{M}^p$ is a $p \times n$ matrix conmposed of the first p(prediction horizon) rows of $\mathcal{M}$; $\mathcal{S}^p(t)$ is a $p \times p$ lower block triangular matrix of which the first column is the upper p blocks of $\mathcal{S}(t)$, the nonzero part of the second column is the upper $p-1$ blocks of $\mathcal{S}(t+1)$ and so on; $\delta\mathcal{U}^Q(t)$ is the sequence of $\delta\mathbf{u}^Q$ over the prediction horizon. In the above, $\mathbf{x}(t|t)$ can be obtained from the Kalman filter equation.

$$\begin{aligned} \hat{\mathbf{x}}(t|t) &= \hat{\mathbf{x}}(t|t-1) + \mathbf{L}(t)\left(\mathbf{y}(t) - \mathcal{N}\hat{\mathbf{x}}(t|t-1)\right) \\ \hat{\mathbf{x}}(t+1|t) &= \mathcal{M}\hat{\mathbf{x}}(t|t) + \mathcal{S}(t)\delta\mathbf{u}(t) \end{aligned} \qquad (23)$$

where $\mathbf{L}(t)$ is the time-varying Kalman gain.

Remark 3 : Practcally, optimal $\mathbf{L}(t)$ is hard to define due to the lack of exact knowledge of noise characteristics. In this sense, $\mathbf{L}(t)$ is considered a tuning parameter whose elements may have values from 0 (no feedback correction) to 1 (full correction) when the process has no integrating modes.

Now, by substituting (22) into (21), the MPC criterion is rearranged with respect to $\delta\mathcal{U}^M(t)$ and the combined algorithms are derived as follows.

Unconstrained Algorithm

Obtaining the least squares solution of the quadratic optimization and applying the receeding horizon strategy, we have

$$\mathbf{u}(t) = \mathbf{u}(t-1) + \mathbf{u}^Q(t) + \mathbf{K}_{MPC}(t)\mathcal{E}(t+1|t) \qquad (24)$$

where

$$\begin{aligned} \mathbf{K}_{MPC}(t) &= \mathcal{N}^p \left(\mathcal{S}^{pT}(t)\Gamma(t)\mathcal{S}^p(t) + \Lambda(t)\right)^{-1} \mathcal{S}^{pT}\Gamma(t) \\ \mathcal{E}(t+1|t) &= \mathcal{R}(t+1|t) - \mathcal{M}^p\hat{\mathbf{x}}(t|t) - \mathcal{S}^p(t)\delta\mathcal{U}^Q(t) \\ \mathcal{N}^p &= \begin{bmatrix} \mathbf{I} & \mathbf{0} & \ldots & \mathbf{0} \end{bmatrix} \end{aligned} \qquad (25)$$

Constrained Algorithm

Similarly to the constrained Q-ILC algorithm, the constrained MPC (combined with Q-ILC) becomes a standard QP problem. Derivation of the problem formation is quite straightforward and not shown here.

5. Properties of the Proposed Algorithm

Disturbance Rejection

As discussed in ***Remark 2***, unknown but repeated disturbances are automatically rejected by the learning algorithm. This is the most important benefit we can obtain from learning control. We mentioned in **Introduction** that noise sensitivity a common flaw of existing ILC algorithms. Output noise is amplified indefinitely as sampling interval decreases to zero and $\omega \to \infty$. On the other hand, the noise gain of Q-ILC is limited by

$$\begin{aligned} \|\mathbf{H}_k\|_\infty &= \left\| \left(\mathbf{G}_k^T \mathbf{Q} \mathbf{G}_k + \mathbf{R}\right)^{-1} \mathbf{G}_k^T \mathbf{Q} \right\|_\infty \\ &\leq \frac{\sigma_{max}(\mathbf{G}_k)\sigma_{max}(\mathbf{Q})}{\sigma_{min}(\mathbf{Q})} \end{aligned} \quad (26)$$

irrespective of sampling interval. In addition, in Q-ILC, noise effects are filtered by the Kalman filter.

Convergence

Since the batch process is run over a finite time horizon, the usual stability concept along time index is not important. Instead, stability along the batch index, or equivalently, convergence of E_k should be investigated. In this paper, we confine our discussion to the unconstrained case with $\mathbf{L}^T(t) = [\ \mathbf{I} \dots \mathbf{I}\ ]$ (full feedback correction in MPC) and $\mathcal{R}^T(t+1|t) = [\ \mathbf{y}_d^T(t+1) \dots \mathbf{y}_d^T(t+p)\ ]$ where $\mathbf{y}_d(t)$ denotes the deviation variable of $y_d(t)$. In this analysis, we asume the true system is (7).

Through rather tedius but straightforward manipulations, we can show from (24) the output sequence over the whole batch horizon can be represented by

$$\Delta \mathsf{U}_{k+1} = \mathbf{C}_k \mathsf{E}_{k+1} + \mathbf{D}_k \mathbf{H}_k \hat{\mathsf{E}}_{k|k} \quad (27)$$

If we write the dynamics of the total system with respect to the state estimate $(\hat{\mathsf{E}}_{k|k-1})$ and state estimation error $(\tilde{\mathsf{E}}_{k|k-1})$ after substituting the above equation into (16),

$$\begin{aligned} \tilde{\mathsf{E}}_{k+1|k} &= (\mathbf{I} - \mathbf{K}_k)\, \tilde{\mathsf{E}}_{k|k-1} + \text{noise term} \\ \hat{\mathsf{E}}_{k+1|k} &= (\mathbf{I} + \mathbf{G}_k \mathbf{C}_k)^{-1} (\mathbf{I} - \mathbf{G}_k \mathbf{D}_k \mathbf{H}_k)\, \hat{\mathsf{E}}_{k|k-1} + \\ & \quad \mathbf{F}_k \tilde{\mathsf{E}}_{k|k-1} + \text{noise term} \end{aligned} \quad (28)$$

Since (7) is completely observable, $\mathbf{I} - \mathbf{K}_k$ is asymptotically stable. Without MPC, $\mathbf{C}_k = \mathbf{0}$ and $\mathbf{D}_k = \mathbf{I}$, and it has been shown the eigenvalues of reachable modes of $\mathbf{I} - \mathbf{G}_k \mathbf{H}_k$ lie strictly inside the unit circle[4]. With MPC combined, $(\mathbf{I} + \mathbf{G}_k \mathbf{C}_k)^{-1}$ gives an effect to pull the eigenvalues toward the origin at low frequencies (by the integral action), hence enhances the convergence property. At high frequencies, $\mathbf{I} \gg \mathbf{G}_k \mathbf{C}_k$ and $\mathbf{D}_k \to \mathbf{I}$. This implies MPC has only minor effects at high frequencies. Around the crossover frequency (in the SISO sense) when MPC is tightly tuned, it may induce instability by increasing the eigenvalues. To avoid the potential instability, it is recommended to loosely tune MPC.

Robustness

It has been shown that Q-ILC allows certain amount of model error without infringing the asymptotic convergence property[4]. Allowable model error bound increases with $\mathbf{R}$. Since loosely tuned MPC provides more stability (convergence) margin at least at low frequencies, we can expect the combined algorithm can be more robust than Q-ILC alone. Although the robustness can be enhanced, MPC may degrade (asymptotically perfect) tracking performance (in the noise-free case). When there is model error, MPC predicts nonzero control error and tries to compensate it even when perfect learning signal is provided. It has been shown that in the noise-free case, asymptotically perfect tracking is possible only when the feedback control is of error-driven type and two-degrees-of-frredom controller leads to offset in the limit.[5] With MPC of current formulations, this condition is fulfilled if the prediction horizon is one or there is no model error.

6. Conlusions

An MPC algorithm combined with iterative learning control (Q-ILC) is presented for nonlinear MIMO batch process control. With the aid of Q-ILC, not only the randomly occuring disturbances can be effectively filtered but also the unknown but repetitive disturbances which are frequently observed in chemcial batch processes can be perfectly rejected. At the present formulation, MPC may degrade the tracking performance when there is model error. Development of improved algorithm based on detailed analyses on mathematical properties are being under investigation.

Acknowledgement

The authors would like to acknowledge the ARC(Korea) at POSTECH, KOSEF, LG Yonam foundation(Korea), and NSF(USA) for finacial support.

References

[1] Arimoto, S., Naniwa, T. , and Suzuki, H., "Robustness of P-type Learning Control with a Forgetting Factor for Robotic Motions", Proc. ACC, Vol. 3., 227, 1990.

[2] Moore, K.L., *Iterative Learning Control for Deterministic System*, NY, Springer-Verlag, 1993.

[3] Lee, J.H, Morari, M., and Garcia, C.E., "State-space Interpretation of Model Predictive Control", Automatica, 707, 1994.

[4] Lee, K.S. and Lee, J.H., "Model-based Refinement of Input Trajectories for Batch and Other Transient Processes", 96AIChE Meeting, Chicago, 1996.

[5] Lee, K.S., Bang, S.H., and Chang, K.S.,"Feedback-assisted Iterative Learning Control based on an Inverse Process Model," J. Process Control, Vol.4, 77, 1994.